

Algorithmic game theory – Tutorial 10*

1 Motivation

The main goal of mechanism design is to design rules of the game so that strategic behavior by participants leads to a desirable outcome. This is not always easy, and bad mechanisms can lead to undesirable situations.

Exercise 1. Consider the following auction. The auctioneer offers 50 CZK to the highest bidder. Bidding starts at 10 CZK and increases incrementally by 5 CZK each time. The highest bidder wins the 50 CZK, but both the highest and the second-highest bidders must pay their bids.

Exercise 2. The mayor wants to motivate bus drivers and decides to give them a bonus, which equals 10% of the cost of the bus tickets they sell. How does this change the bus traffic?

Exercise 3. During the occupation of India, the British soldiers were having problems with the overpopulation of cobras in Delhi. To get the situation under control, they offered a small reward to locals for each cobra snake killed. Can you guess what happened?

2 Mechanism design basics

In a *single-parameter environment*, there are n bidders, each bidding for a certain good. Each bidder i has a private valuation v_i , and there is a *feasible set* $X \subseteq \mathbb{R}^n$ corresponding to feasible outcomes. The sealed-bid auction in this environment then proceeds in three steps.

- (a) Collect bids $b = (b_1, \dots, b_n)$, where b_i is the bid of bidder i .
- (b) *Allocation rule*: Choose a feasible outcome (allocation) $x = x(b)$ from the feasible set X as a function of the bids b .
- (c) *Payment rule*: Choose payments $p(b) = (p_1(b), \dots, p_n(b)) \in \mathbb{R}^n$ as a function of the bids b .

The pair (x, p) then forms a (*direct*) *mechanism*. The *utility* $u_i(b)$ of bidder i is defined as $u_i(b) = v_i \cdot x_i(b) - p_i(b)$.

An auction is *dominant-strategy incentive-compatible (DSIC)* if it satisfies the following two properties. Every bidder has a dominant strategy: *bid truthfully*, that is, set his bid b_i to his private valuation v_i . Moreover, the utility of every truth-telling bidder is guaranteed to be non-negative. A strategy is *dominant* if it maximizes the utility of a given bidder no matter what the other bidders do. An example of a single-item DSIC auction is *Vickrey's auction*, where the highest bidder is the winner and has to pay the second largest bid.

An auction is *awesome* if it satisfies the following three conditions:

- (a) The auction is DSIC.
- (b) If all bidders bid truthfully, then the auction maximizes the social surplus $\sum_{i=1}^n v_i x_i$.
- (c) The auction can be implemented in polynomial time.

Exercise 4. Consider a single-item auction with at least three bidders. Prove that selling the item to the highest bidder at a price equal to the third-highest bid yields an auction that is not DSIC.

Exercise 5. Consider a single-item auction, where the winner is the bidder with the highest bid and pays the price corresponding to the second highest bid with 10% discount. For example, given bids $b = (11, 7, 10)$, the winner is bidder 1 and pays $10 \cdot 0.9 = 9$. Can you find a dominant strategy in this auction? Is it truth-telling?

*Information about the course can be found at <http://kam.mff.cuni.cz/~cizek/>

Exercise 6. Consider the following single-item auction with prioritized eligibility. The seller assigns a publicly known priority level p_i to each bidder i (for example, based on loyalty or seniority). Only bidders with priority $p = \max_j p_j$ are eligible to win, and the item is allocated to the highest eligible bidder. The winner pays the maximum of the second-highest bid among all eligible bidders. Is this auction DSIC? Is it awesome?

Exercise 7. Assume there are k identical items and $n > k$ bidders. Also, assume that each bidder can receive at most one item. What is the analog of the second-price auction? Is it the j th highest bidder paying the $(j + 1)$ st highest bid for $j \leq k$? Prove that your auction is DSIC.