Algorithmic game theory — Homework 4[1]
Extensive games and mechanism design

assigned 16.12.2024, deadline 31.1.2025

Homework 1. *Construct an extensive form of the Rock-Paper-Scissors game from Table 1 and write its sequence form and the linear program for finding Nash equilibria in this game.* **[2]**

	Rock	Paper	Scissors
Rock	(0,0)	(-1,1)	(1,-1)
Paper	(1,-1)	(0,0)	(-1,1)
Scissors	(-1,1)	(1,-1)	(0,0)

Table 1: A normal form of the game Rock-paper-scissors.

Homework 2. *Assume there are k identical items and $n > k$ bidders. Also assume that each bidder can receive at most one item. Let A be the awesome auction where for this k-item setting. For a price $R \geq 1$, let A' be the following auction: initialize S to be the set of k highest bidders, then, while there is a bidder $i \in S$ with $b_i < R/|S|$, remove an arbitrary such bidder from S. Allocate an item to each bidder of S (if any), and charge each of them a price equal to the maximum between $R/|S|$ and the $(k+1)$st highest bid.*

Derive the allocation rule and the payment rule for A. Prove that whenever A obtains revenue at least R, then A' obtains revenue at least R. Prove that there exists a valuation profile for which A' obtains revenue R but A earns revenue less than R. **[3]**

Homework 3. *Let F be the uniform probability distribution on $[0, 1]$. Consider a single-item auction with two bidders 1 and 2 that have probability distributions $F_1 = F$ and $F_2 = F$ on their valuations. Prove that the expected revenue obtained by the Vickrey auction with reserve $1/2$ is $5/12$.* **[2]**

Homework 4. *Compute the virtual valuation function of the following probability distributions and show which of these distributions are regular (meaning the virtual valuation function is strictly increasing).*

(a) *The distribution given by $F(z) = 1 - \frac{1}{(z+1)^c}$ on $[0, \infty)$, where $c > 0$ is some constant,* **[1]**

(b) *Consider the probability distribution F in part (a), with $c = 1$. Argue that when bidder valuations are drawn from F, it is not necessarily the case that the expected revenue of an auction equals its expected virtual social surplus. To reconcile this observation with the theorem from the lecture about maximizing revenue, identify which assumption of this result is violated in your example.* **[3]**

[1] Information about the course can be found at http://kam.mff.cuni.cz/~cizek/