

Algorithmic game theory – Tutorial 9*

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1 Motivation

The main goal of mechanism design is to design rules of a game so that strategic behavior of participants leads to a desirable outcome. This is not always easy and bad mechanisms can lead to undesirable situations.

Exercise 1. Consider the following auction. The auctioneer offers 50 CZK to the highest bidder. Bidding starts at a low amount (e.g., 1 CZK) and increases incrementally by 1 CZK each time. The highest bidder wins the 50 CZK, but both the highest and the second-highest bidders must pay their bids.

Exercise 2. The mayor wants to motivate bus drivers and decides to give them a bonus, which equals 10% of the cost of the bus tickets they sell. How does this change the bus traffic?

Exercise 3. During the occupation of India, the British soldiers had problems with overpopulation of cobras in Delhi. To get the situation under control, they offered a small reward to locals for each killed cobra. Can you guess what happened?

2 Mechanism design basics

In a *single-parameter environment*, there are n bidders, each bidding for a certain goods. Each bidder i has a private *valuation* v_i and there is a *feasible set* $X \subseteq \mathbb{R}^n$ corresponding to feasible outcomes. The sealed-bid auction in this environment then proceeds in three steps.

- (a) Collect bids $b = (b_1, \dots, b_n)$, where b_i is the bid of bidder i .
- (b) *Allocation rule*: Choose a feasible outcome (allocation) $x = x(b)$ from the feasible set X as a function of the bids b .
- (c) *Payment rule*: Choose payments $p(b) = (p_1(b), \dots, p_n(b)) \in \mathbb{R}^n$ as a function of the bids b .

The pair (x, p) then forms a (*direct*) *mechanism*. The *utility* $u_i(b)$ of bidder i is defined as $u_i(b) = v_i \cdot x_i(b) - p_i(b)$.

An auction is *dominant-strategy incentive-compatible (DSIC)* if it satisfies the following two properties. Every bidder has a dominant strategy: *bid truthfully*, that is, set his bid b_i to his private valuation v_i . Moreover, the utility of every truth-telling bidder is guaranteed to be non-negative. An example of DSIC auction is *Vickrey's auction* where the highest bidder is the winner and he has to pay the second largest bid.

An allocation rule x is *implementable* if there is a payment rule p such that the mechanism (x, p) is DSIC. An allocation rule x is *monotone* if, for every bidder i and all bids b_{-i} of the other bidders, the allocation $x_i(z; b_{-i})$ to i is nondecreasing in his bid z .

Theorem 1 (Myerson's lemma). *In a single-parameter environment, the following three claims hold.*

- (a) *An allocation rule is implementable if and only if it is monotone.*
- (b) *If an allocation rule x is monotone, then there exists a unique payment rule p such that the mechanism (x, p) is DSIC (assuming that $b_i = 0$ implies $p_i(b) = 0$).*
- (c) *The payment rule p is given by the following explicit formula*

$$p_i(b_i; b_{-i}) = \int_0^{b_i} z \cdot \frac{d}{dz} x_i(z; b_{-i}) dz$$

for every $i \in \{1, \dots, n\}$.

*Information about the course can be found at <http://kam.mff.cuni.cz/~cizek/>

Exercise 4. Consider a single-item auction with at least three bidders. Prove that selling the item to the highest bidder at a price equal to the third-highest bid, yields an auction that is not dominant-strategy incentive compatible (DSIC).

Exercise 5. Use Myerson's Lemma to prove that the Vickrey auction is the unique single-item auction that is DSIC, always awards the good to the highest bidder, and charges the other bidders 0.

Exercise 6. Consider single-item auction, where the winner is the bidder with the highest bid and pays the price corresponding to the second highest bid with 10% discount. For example, given bids $b = (11, 7, 10)$, the winner is bidder 1 and pays $10 \cdot 0.9 = 9$. Can you find a dominant strategy in this auction? Is it truthtelling?

Exercise 7. Consider the following single-item auction with prioritized eligibility. The seller assigns a publicly known priority level p_i to each bidder i (for example, based on loyalty or seniority). Only bidders with priority $p = \max_j p_j$ are eligible to win and the item is allocated to the highest eligible bidder. The winner pays the maximum of the second-highest bid among all eligible bidders. Is this auction DSIC? Is the auction awesome?