The traveling salesman problem

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Given finitely many “cities” along with the cost of travel between any two of them,

find the cheapest way of going through all the cities and coming back to the city you started out from.

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find the cheapest way of going through all the cities and coming back to the city you started out from.

The *symmetric* TSP:
Travel from A to B costs as much as travel from B to A.

What are the origins of the problem?

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Der

Handlungsreisende

wie er sein soll

und was er zu thun hat, um Aufträge zu erhalten und eines glücklichen Erfolgs in seinen Geschäften gewiß zu sein.

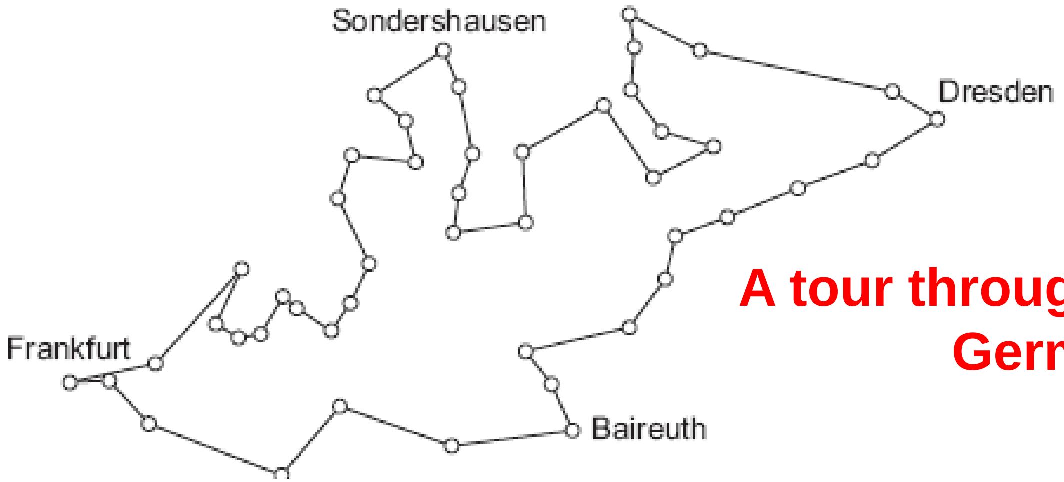
Von

einem alten Commis - Voyageur.

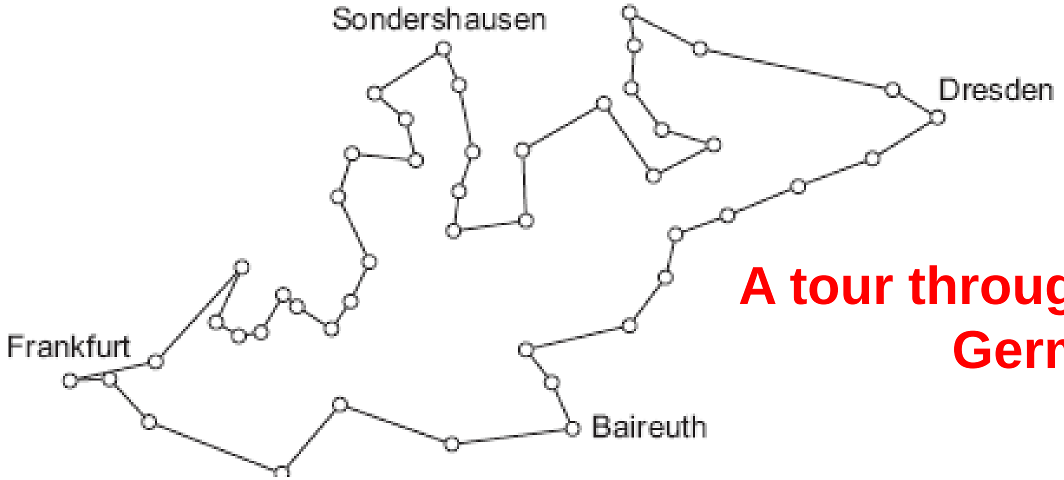
Mit einem Titelkupfer.

Ilmenau 1832,

Druck und Verlag von B. Fr. Voigt.



A tour through 47 cities in Germany



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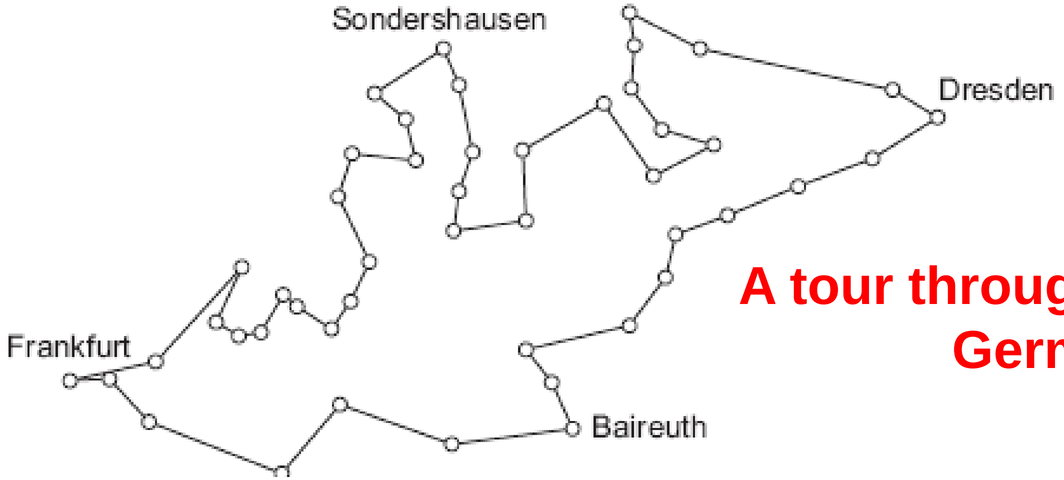
(…) Business leads the traveling salesman here and there, and there is not a good tour for all occurring cases; but through an expedient choice and division of the tour so much time can be won that we feel compelled to give guidelines about this. Everyone should use as much of the advice as he thinks useful for his application. We believe we can ensure as much that it will not be possible to plan the tours through Germany in consideration of the distances and the traveling back and forth, which deserves the traveler's special attention, with more economy.



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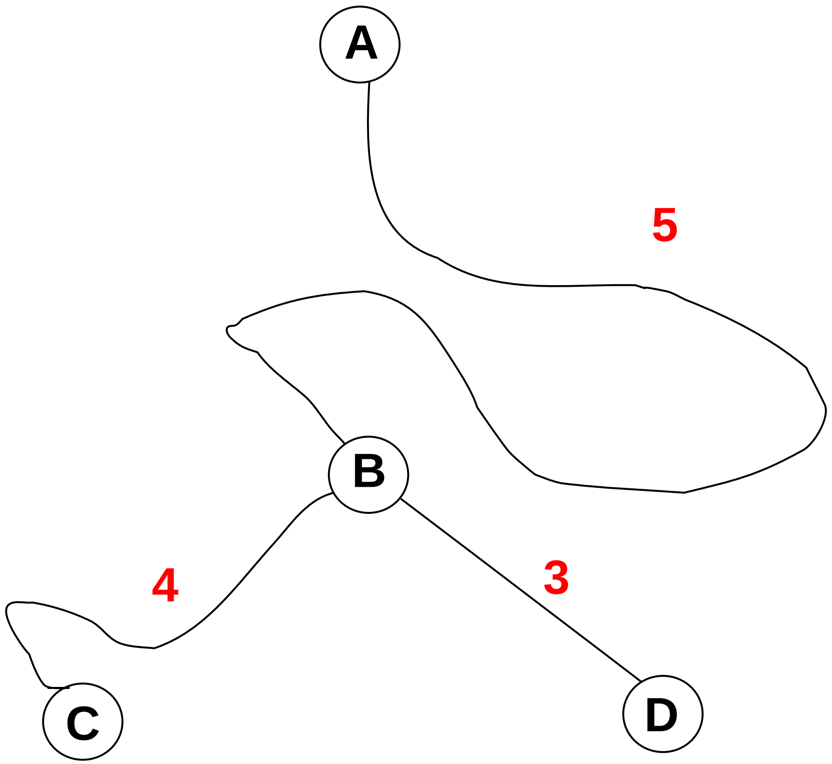

Road map

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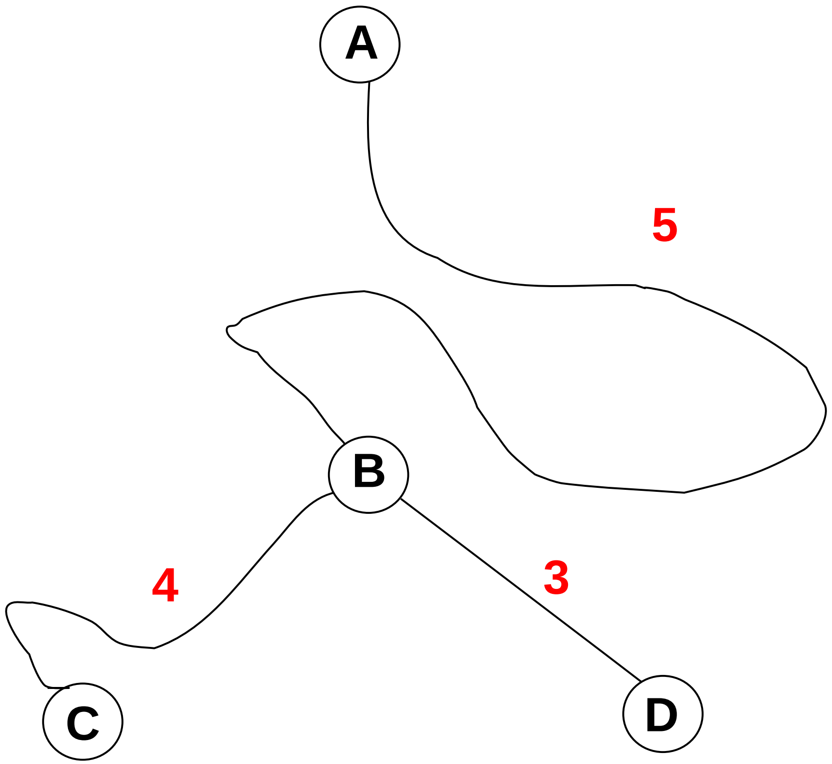

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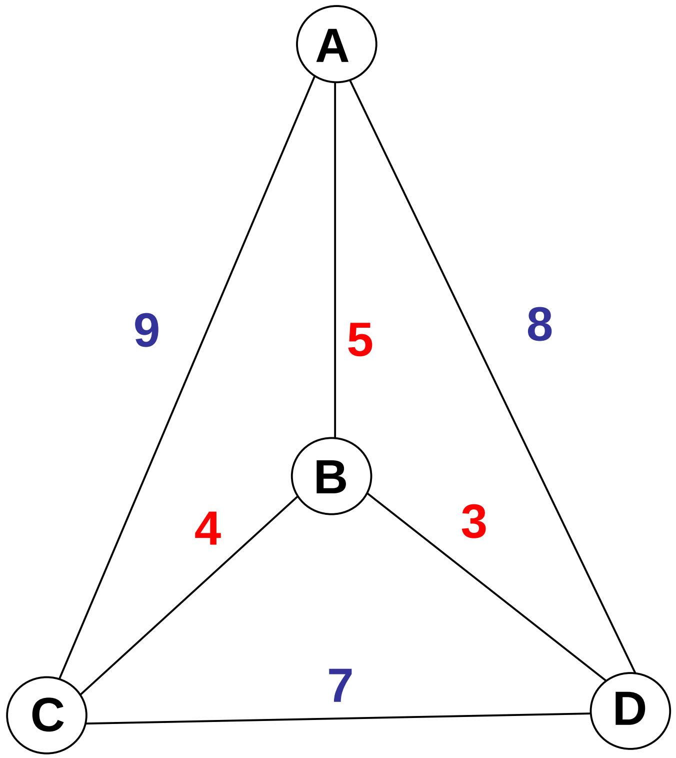

Travel costs

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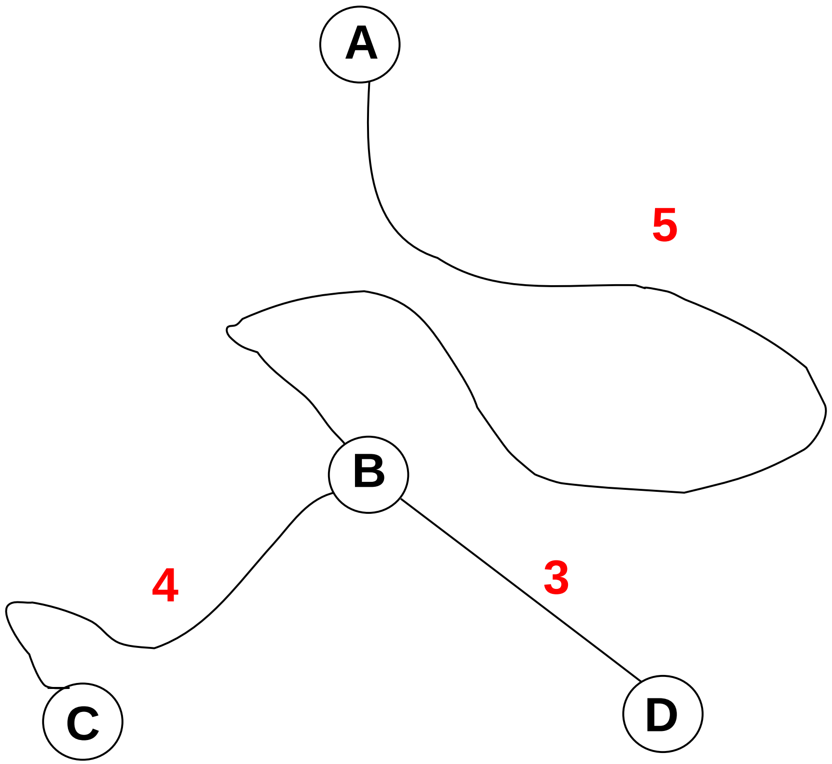

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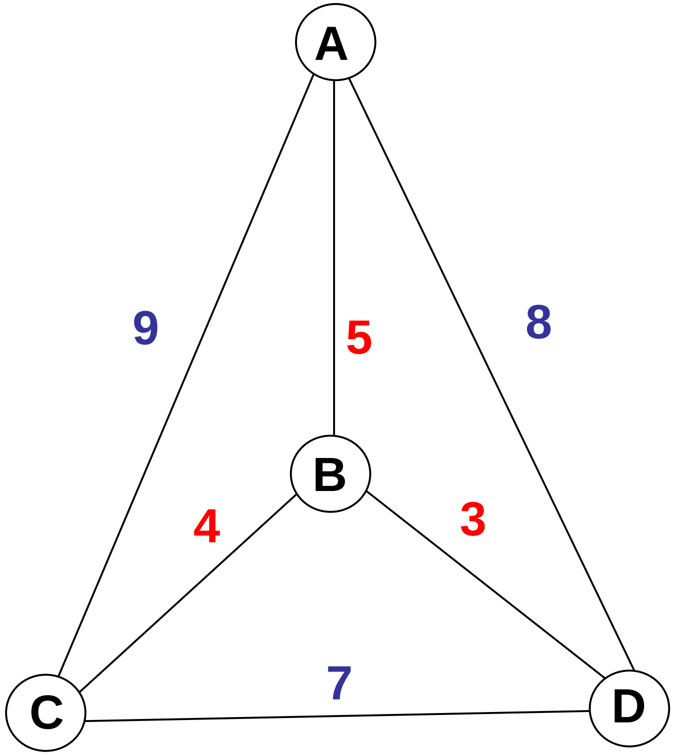

Travel costs

A-B-C-D-A

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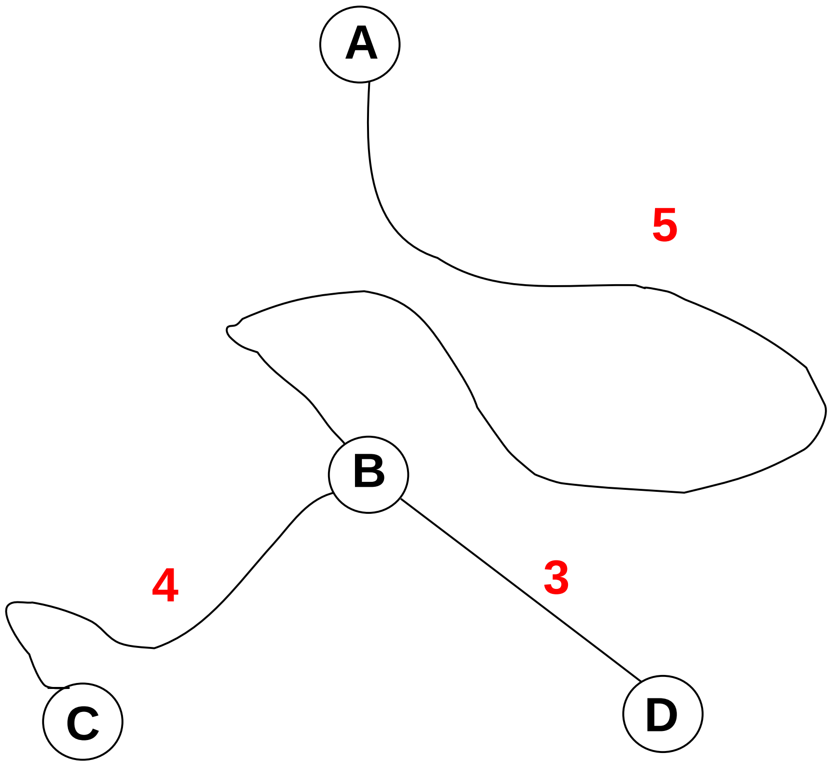

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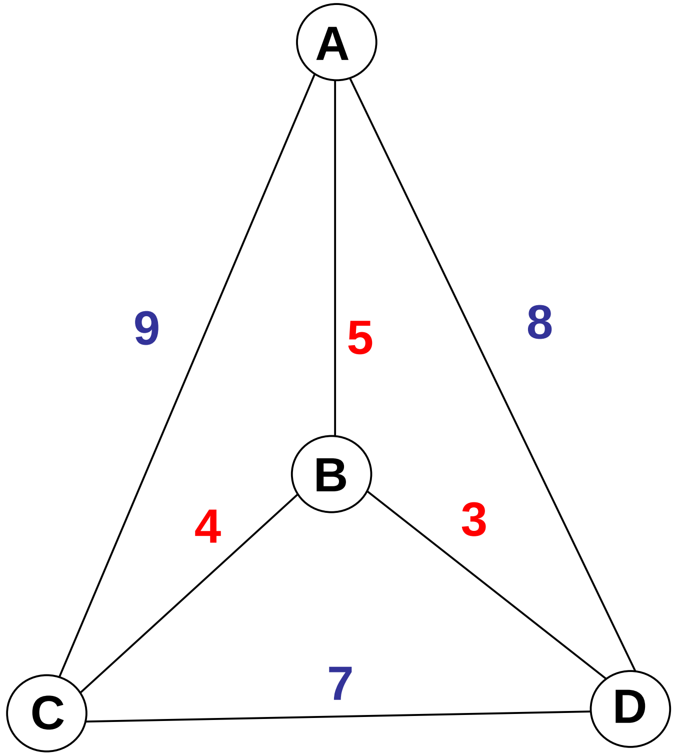

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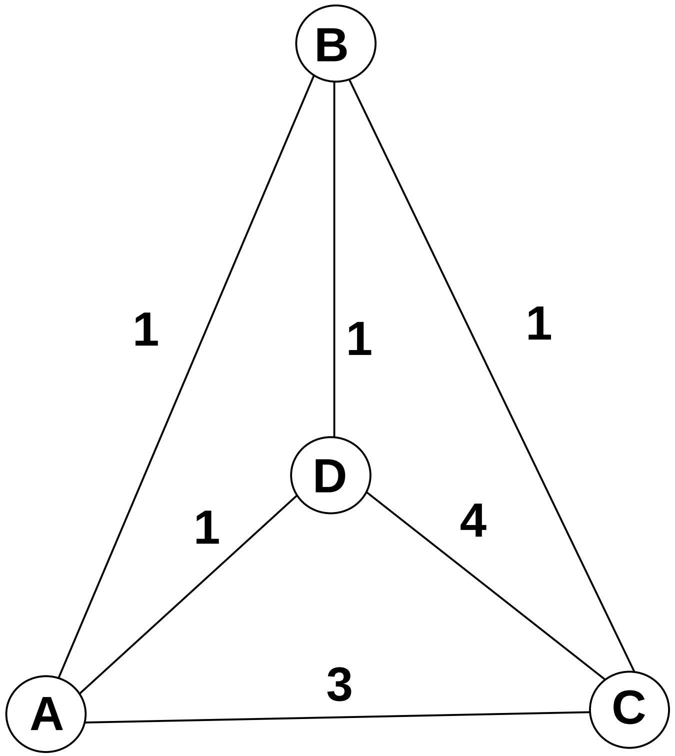

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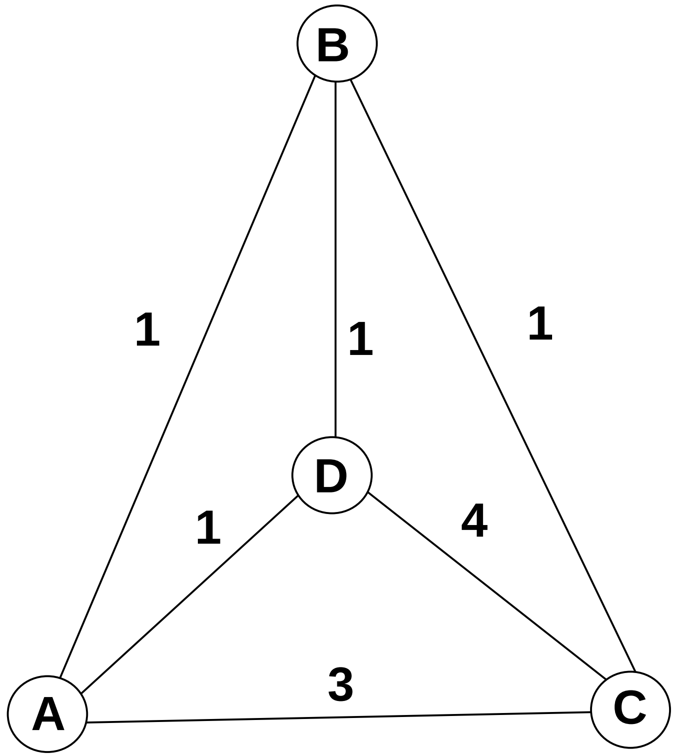
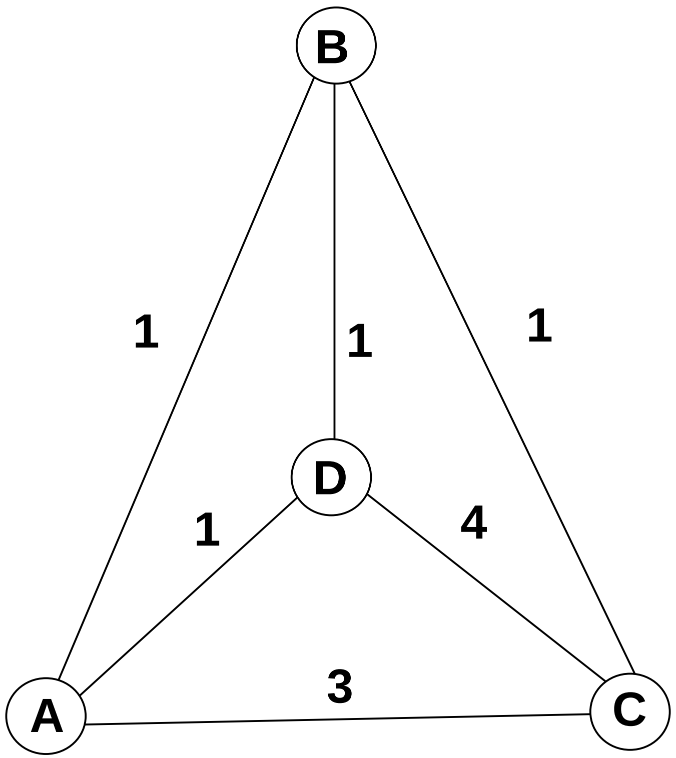
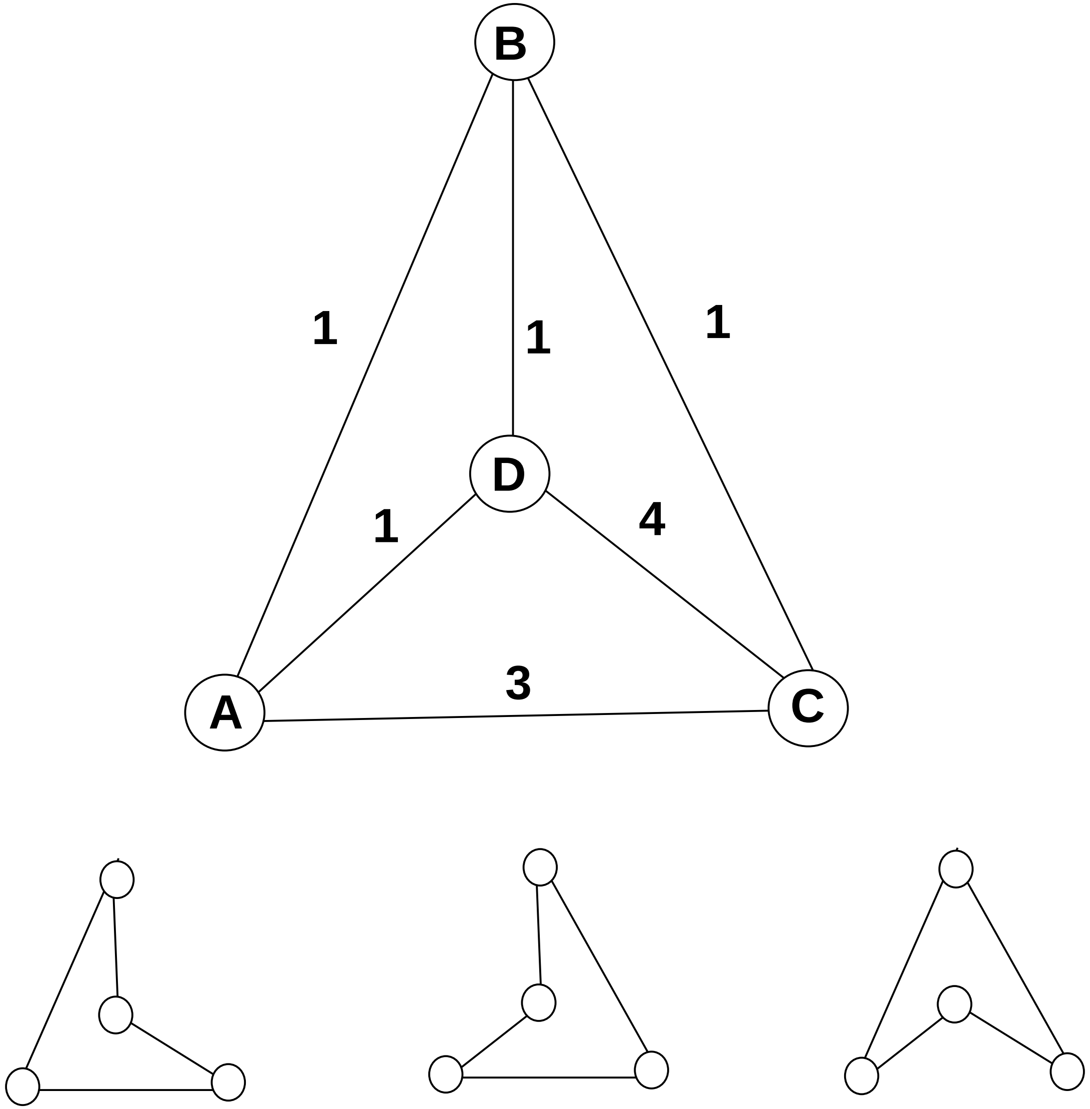


Travel costs

cost(A,C) exceeds cost(A,B)+cost(B,C)!


B
1
1
1
D
1
4
3
A
C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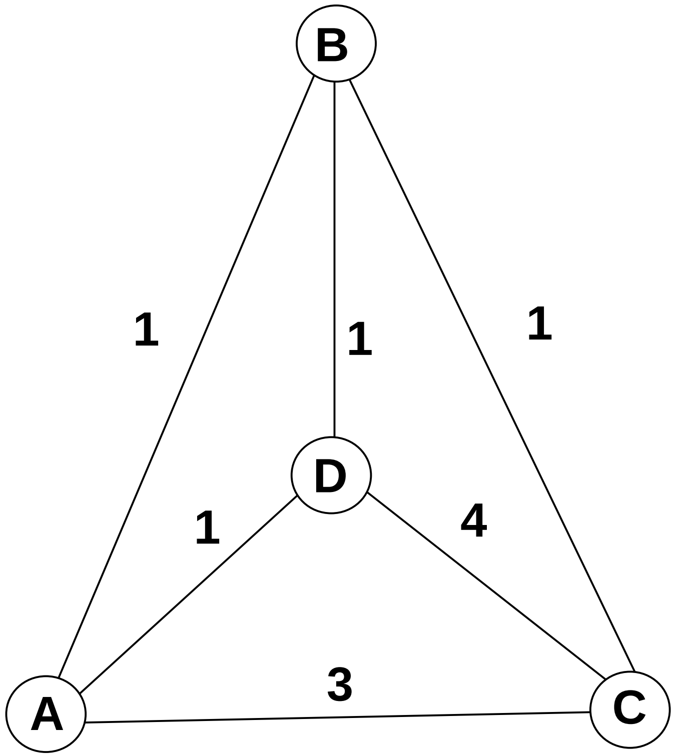
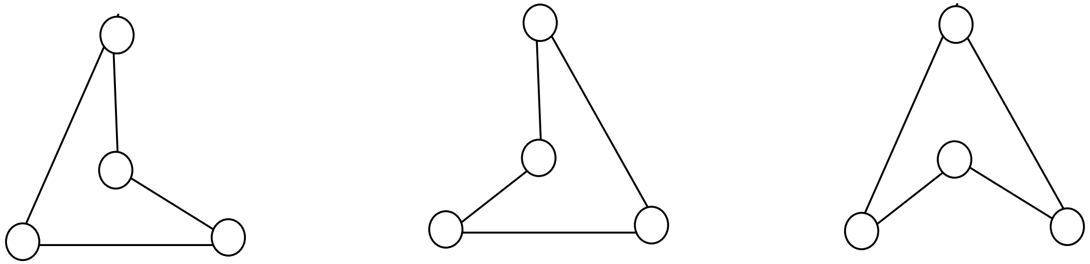
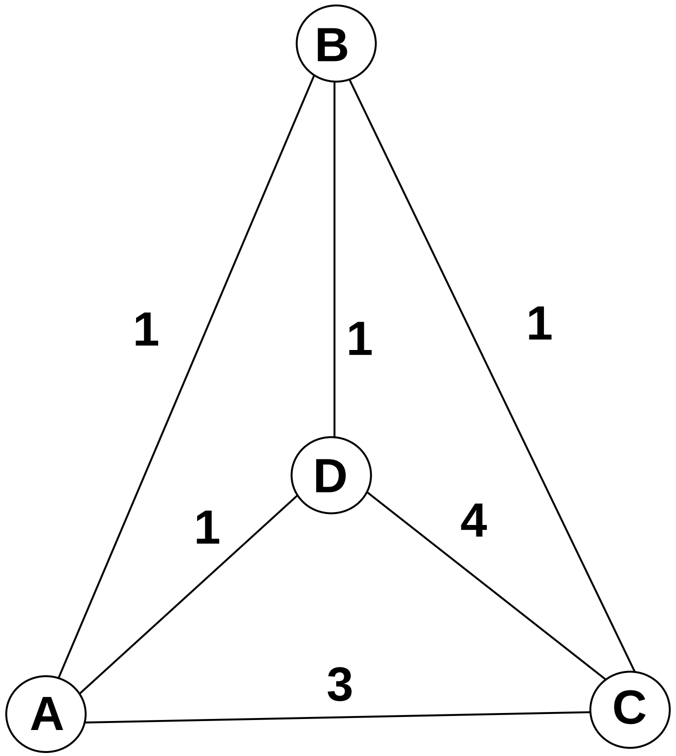
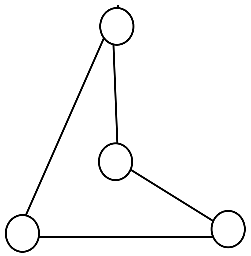




tour cost 9





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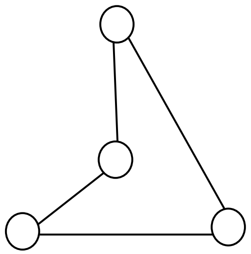
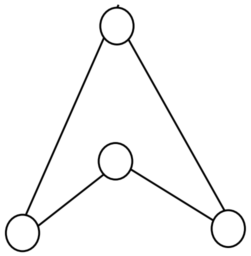
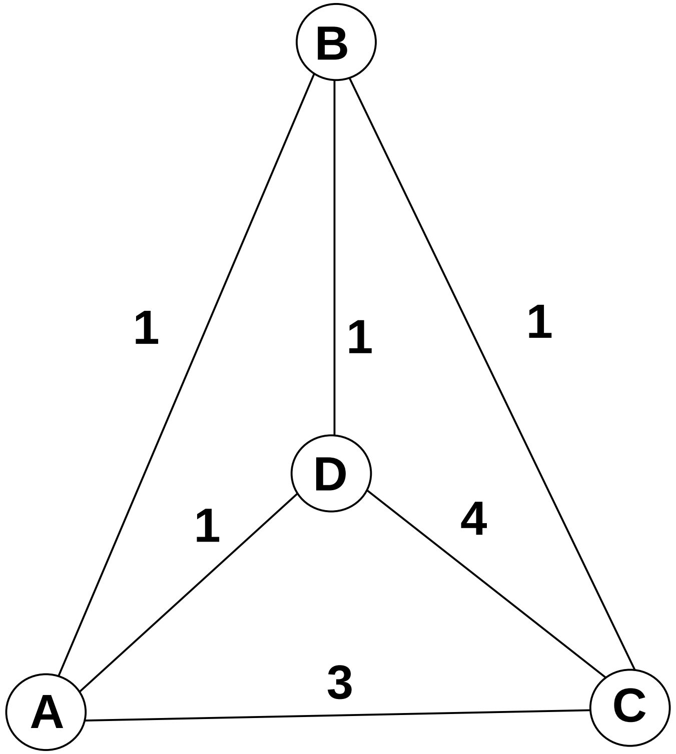
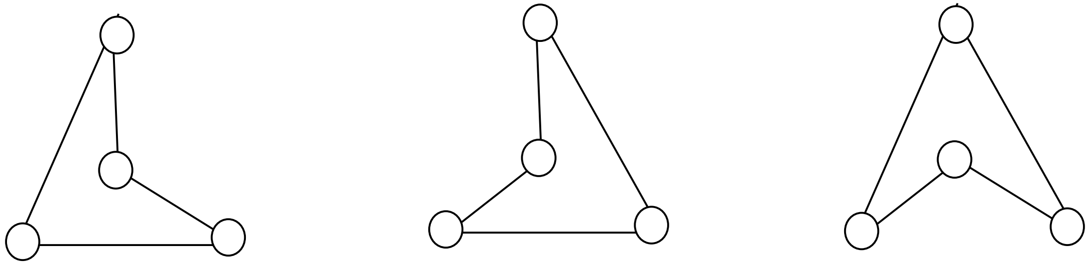


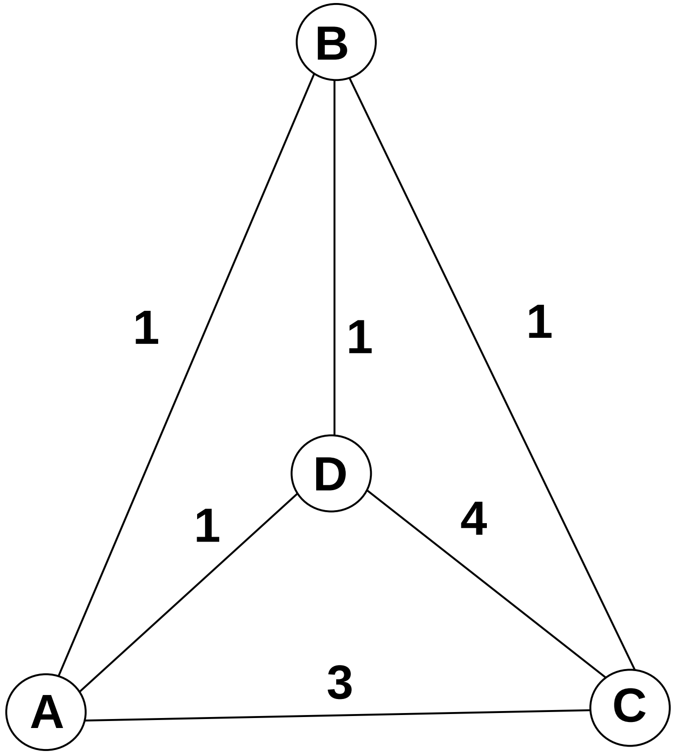
tour cost 6

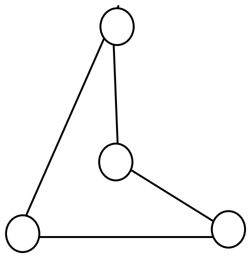






tour cost 9 tour cost 6 tour cost 7





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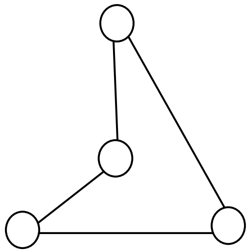
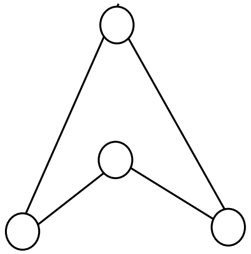


tour cost 6

OPTIMAL



tour cost 7

Great terminological confusion

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Origins of the problem

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Karl Menger
Vienna 1930

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Hassler Whitney
Princeton 1934

Origins of the problem

Harvard 1931

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(A Traveling Salesmen Problem)

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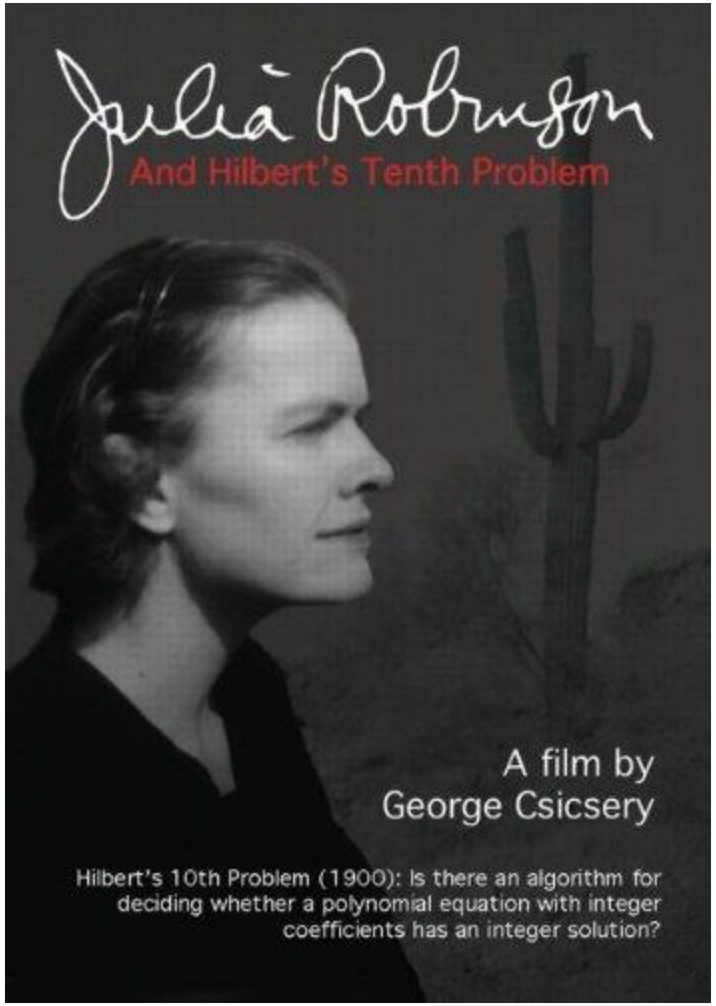

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MISCONCEPTION #7:
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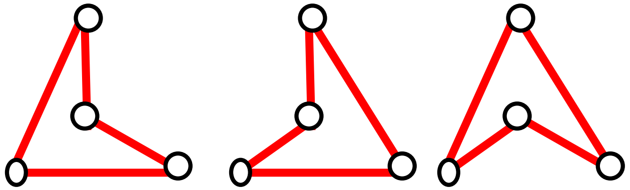

3 tours through 4 cities

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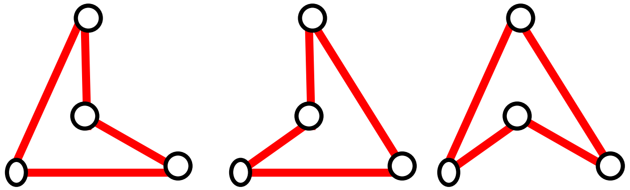

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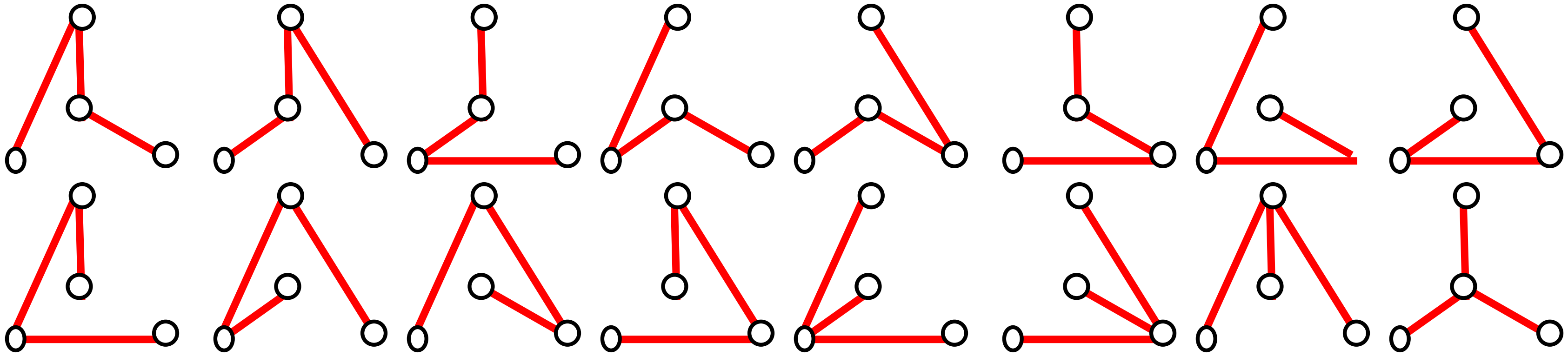

16 *spanning trees* on 4 cities

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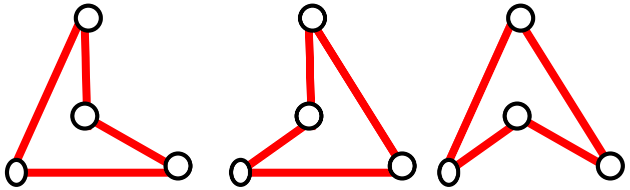

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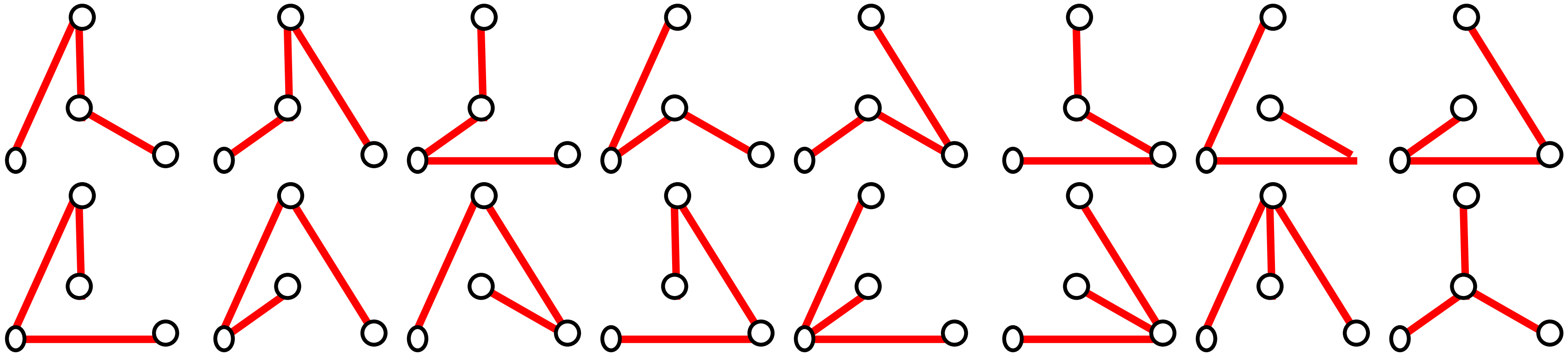
$$\frac{1}{2}(n-1)!$$



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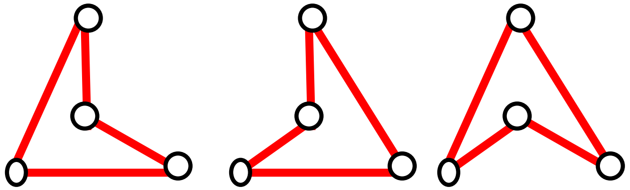

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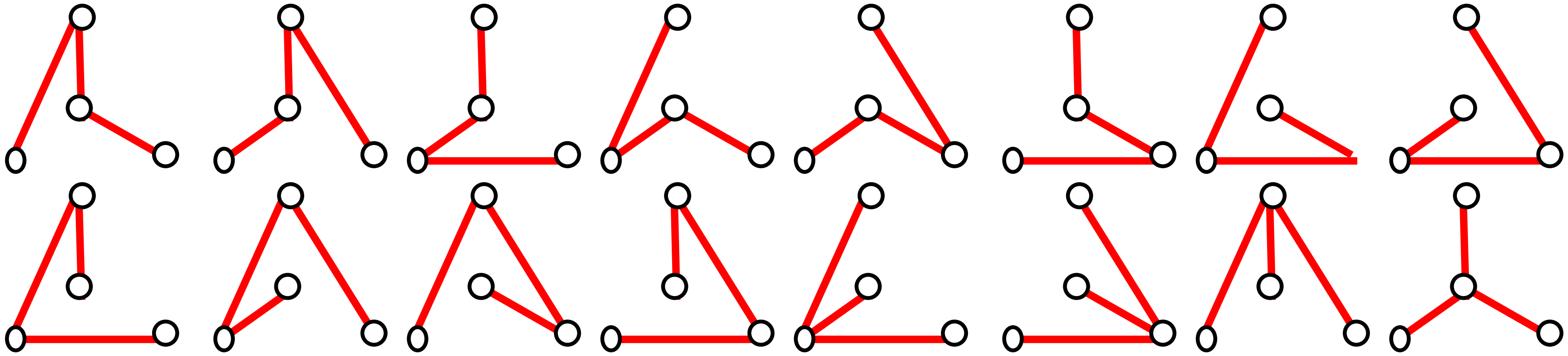

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$$n^{n-2}$$

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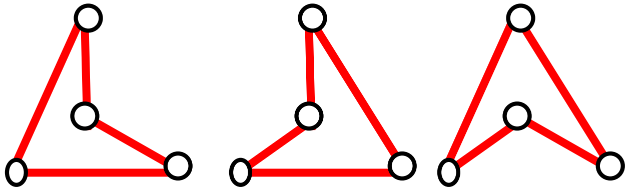
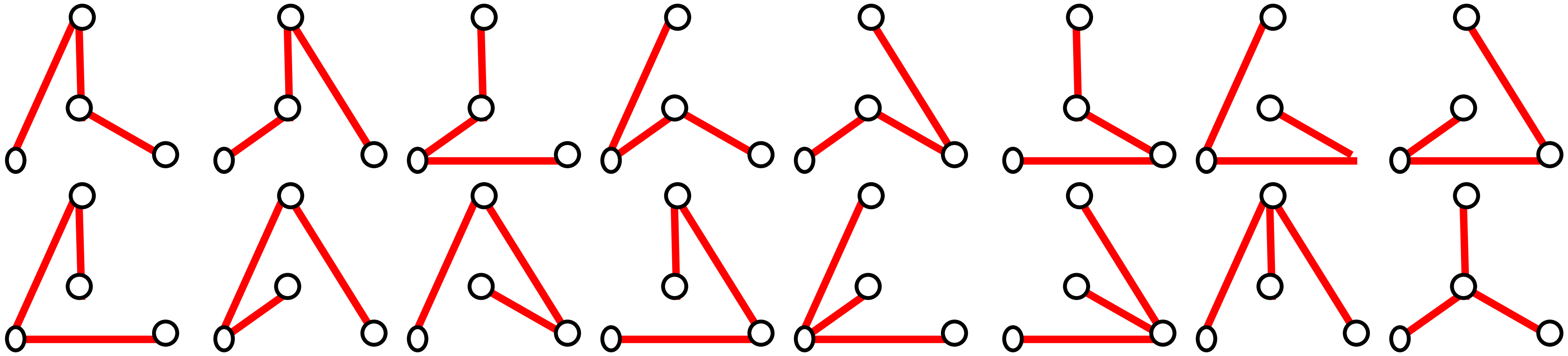


3 tours through 4 cities

$$\frac{1}{2}(n-1)!$$

$$= n^{n-2}\, e^{-n} O(n^{3/2})$$



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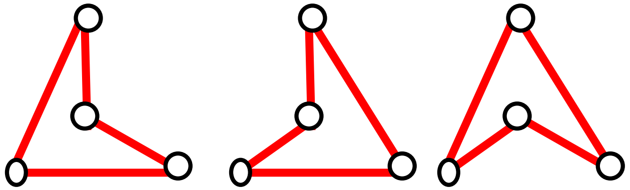

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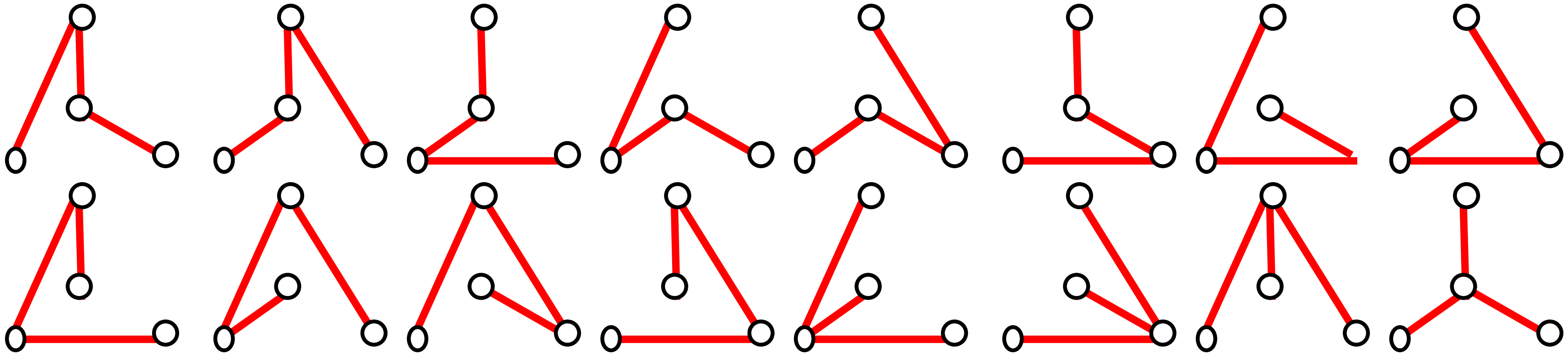
THE MINIMUM-COST SPANNING TREE PROBLEM IS EASY!



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Stephen A. Cook, *The complexity of theorem-proving procedures*. In Proc. 3rd Annual ACM Symposium on the Theory of Computing, 1971, pp.151—158.

Richard M. Karp. *Reducibility among combinatorial problems*. In R.E. Miller and J.W.Thatcher, editors, Complexity of Computer Computations, Plenum Press, 1972, pp. 85--104.

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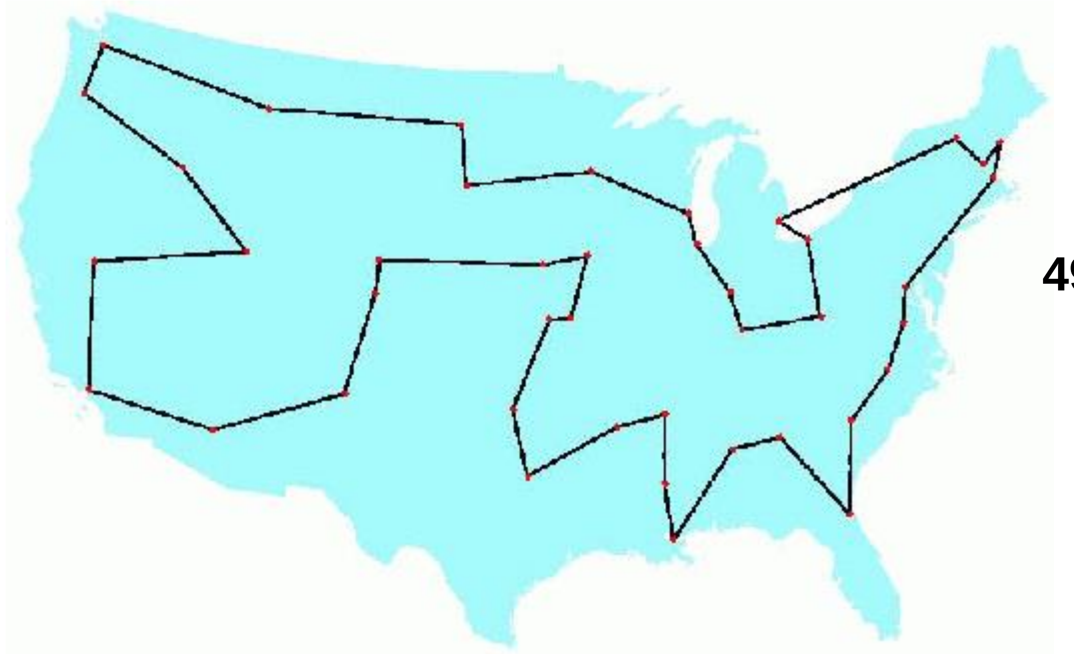

49 cities in the U.S.A.

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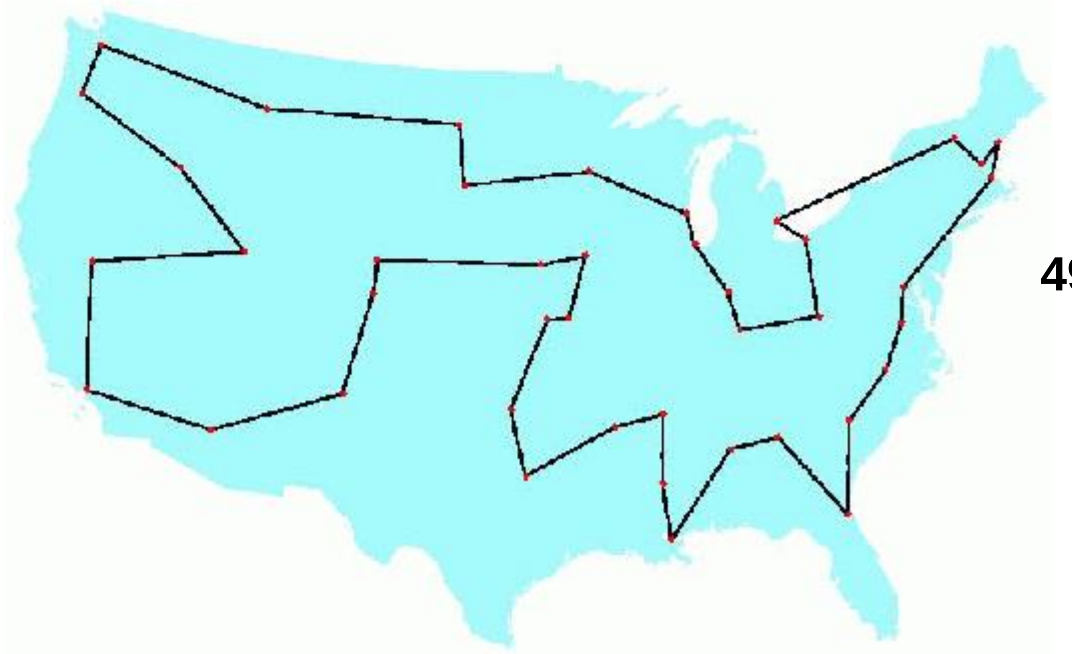

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The Big Bang

The Dantzig-Fulkerson-Johnson cutting-plane method

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applies to any problem minimize $c^T x$ subject to $x \in \mathcal{S}$

such that S is a finite subset of a Euclidean space

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such that ***S*** is a finite subset of a Euclidean space

choose a system $Ax \leq b$ of inequalities satisfied by all points of $\mathcal{S}$
repeat $x^* =$ an extreme point of $\{x : Ax \leq b\}$ that minimizes $c^T x$;
  if x^* belongs to $\mathcal{S}$
  then return x^*;

  end
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end

cutting plane

The Dantzig-Fulkerson-Johnson initial TSP box:

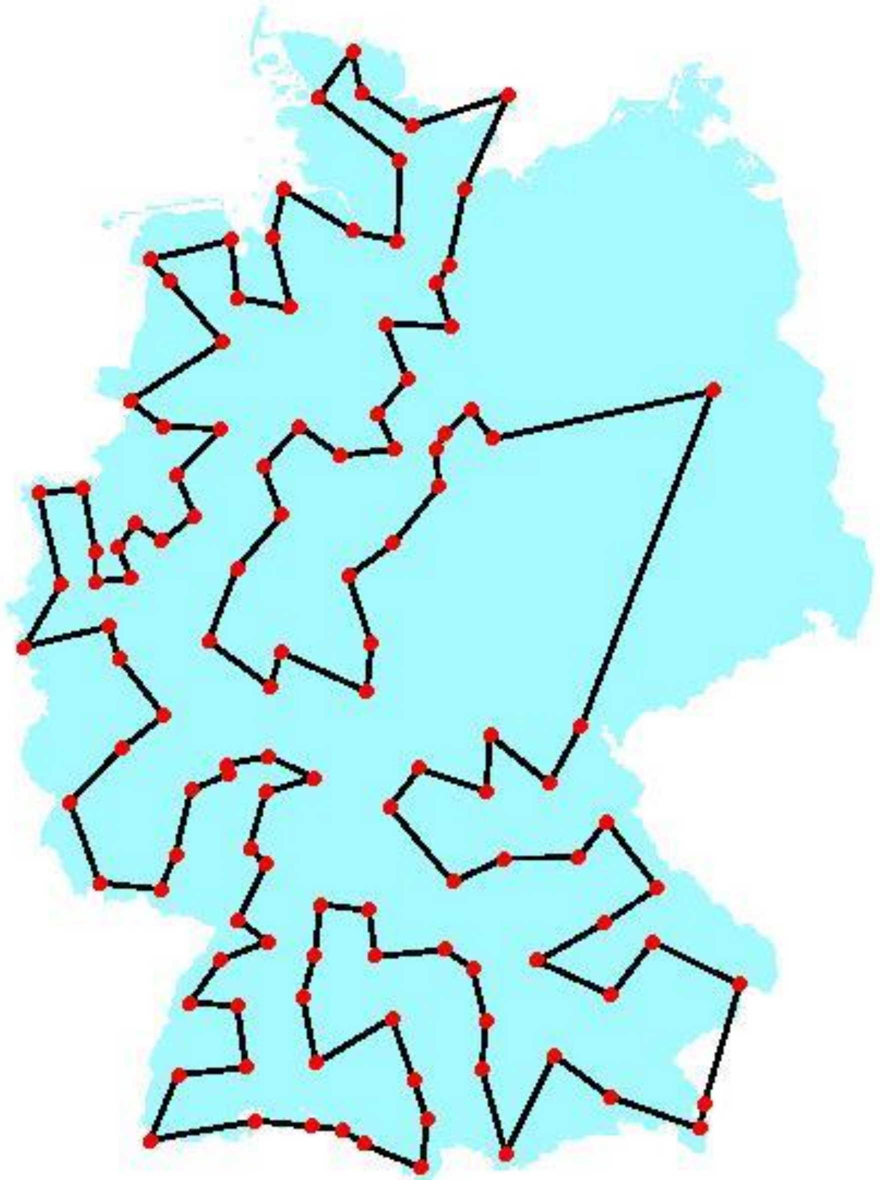
$$
\begin{aligned}
0 \le x_{vw} \le 1 & \qquad \text{for all edges } vw \\
\sum_w x_{vw} = 2 & \qquad \text{for all vertices } v
\end{aligned}
$$

1962 Procter & Gamble $10,000 contest: a 33-city instance

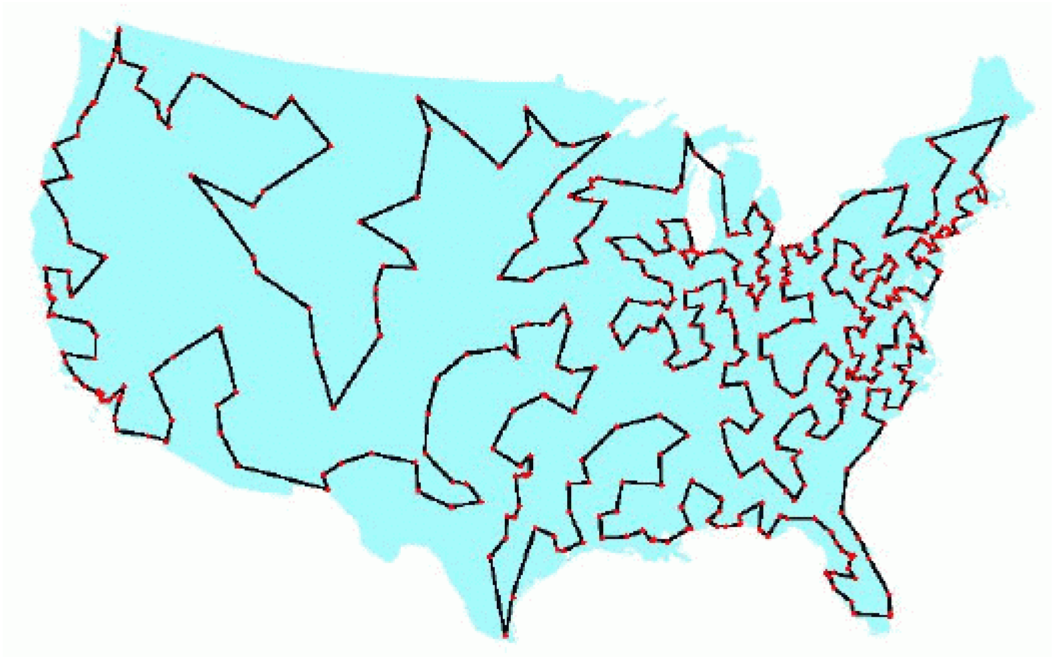
1962 Procter & Gamble $10,000 contest: a 33-city instance

Tiebreaker: Write a short essay on one of Procter & Gamble's products

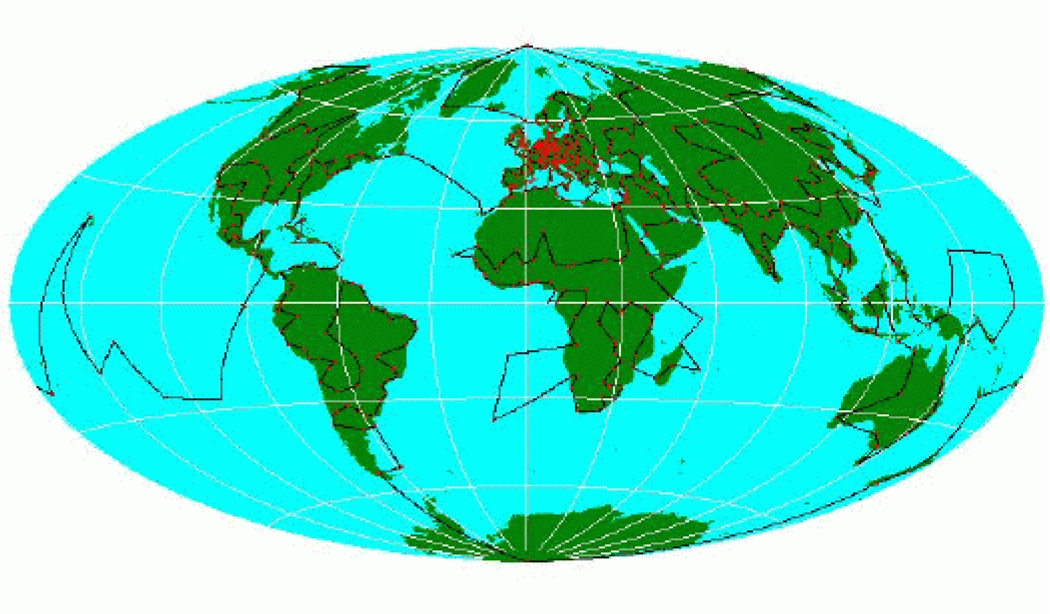
1977 Martin Grötschel: a 120-city instance



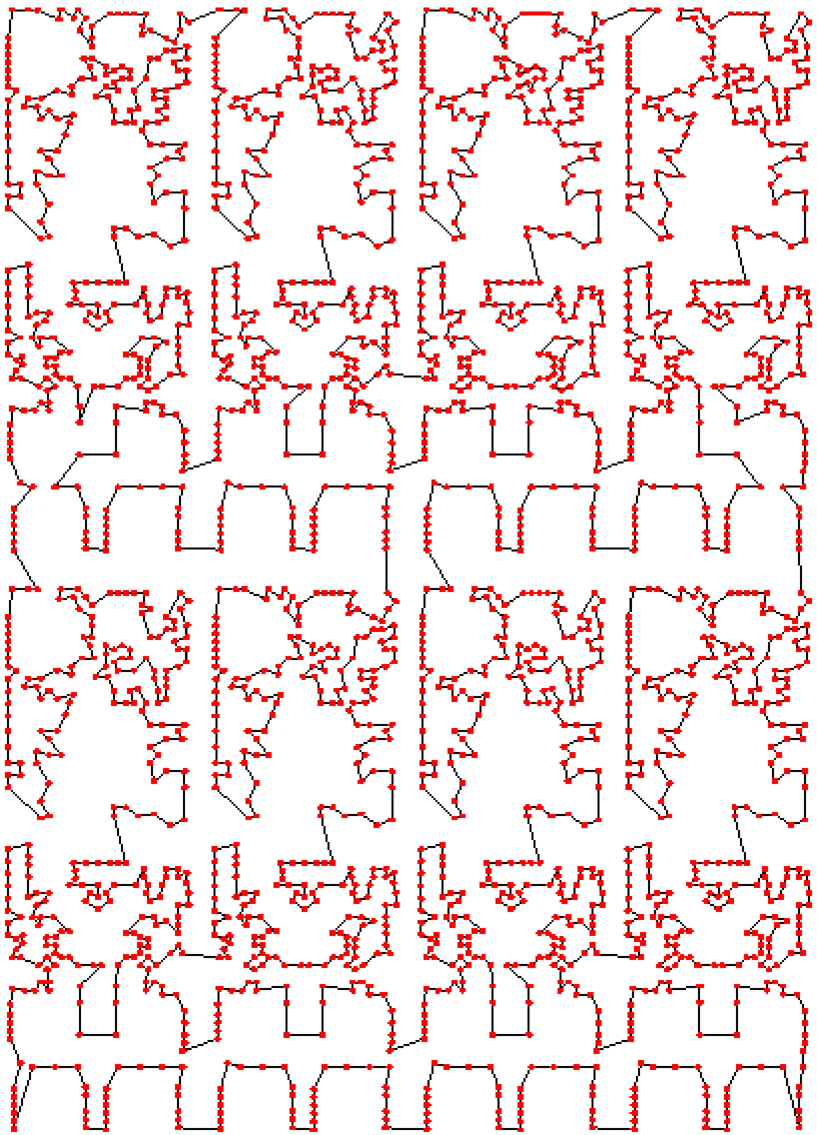
1986 Manfred Padberg and Giovanni Rinaldi: a 532-city instance

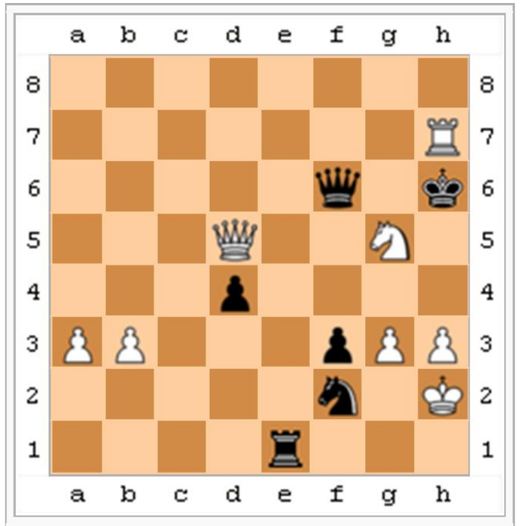


1987 Martin Grötschel and Olaf Holland: a 666-city instance

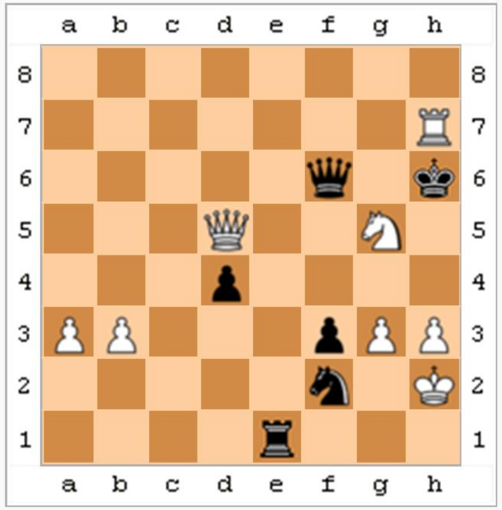


1987 Manfred Padberg and Giovanni Rinaldi: a 2,392-city instance



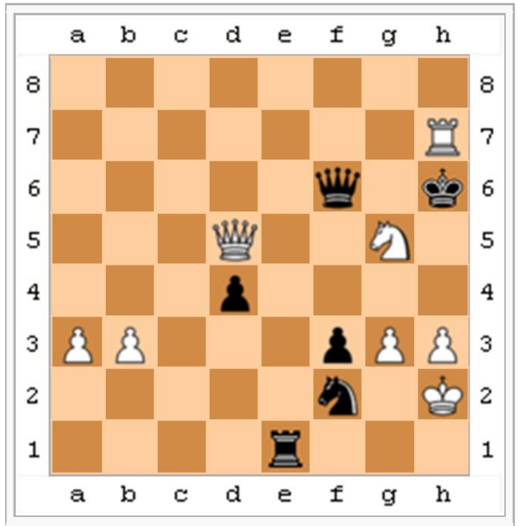


After 37. Rxh7+



After 37. Rxh7+

February 10, 1996



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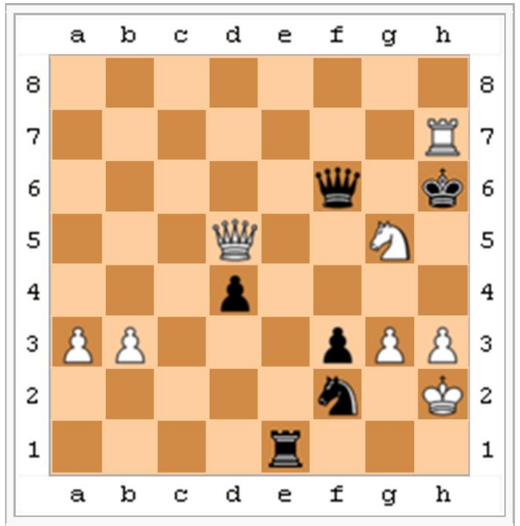
IBM’s “supercomputer” *Deep Blue* beats Garry Kasparov



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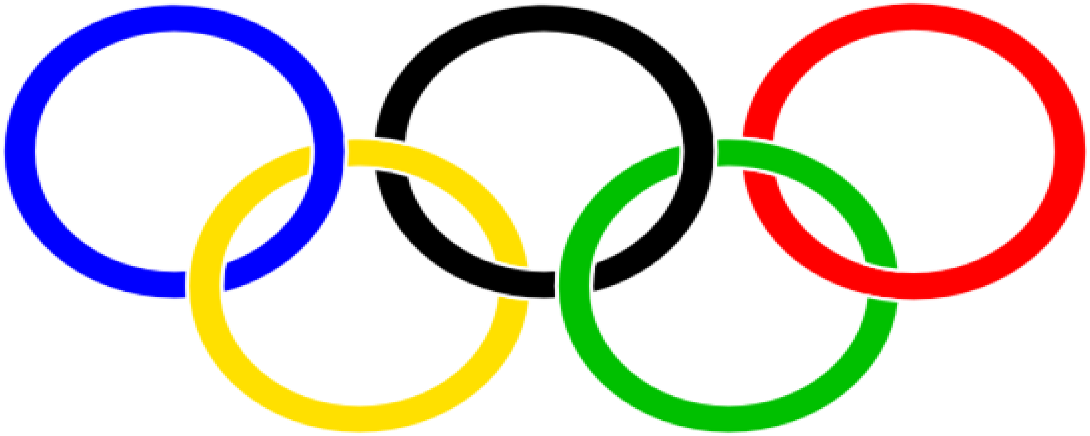
IBM's "supercomputer" *Deep Blue* beats Garry Kasparov

From an interview with V.C. published in 1996:

The traveling salesman problem is to mathematical programming what chess is to artificial intelligence: thoroughly useless and fiercely competitive sport that serves as a testing ground of your techniques.

The sport of solving the TSP



The sport of solving the TSP

My daddy can beat up your daddy

APPLICATION: Genome sequencing

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In which order are markers arranged on a genome?

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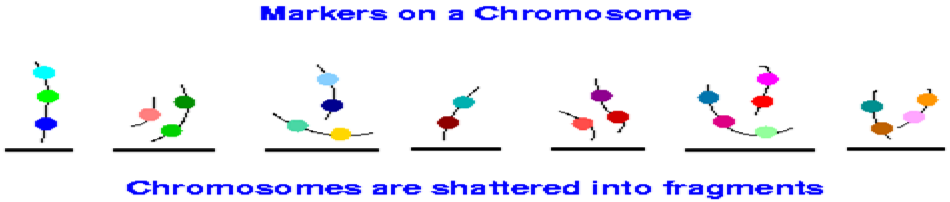
Radiation hybrid technique: Break the DNA into pieces by X-rays and grow hybrid cells from these pieces



The closer two markers are to each other on the DNA, the more often they appear together in the hybrid cells.

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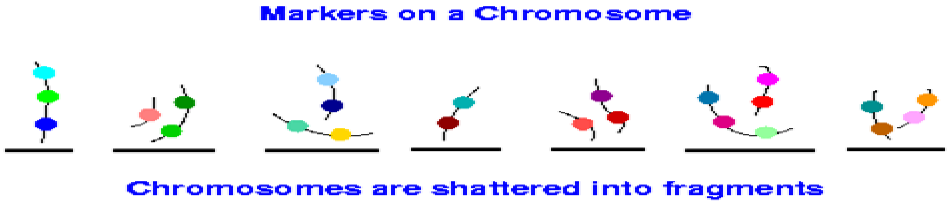

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R. Karp, W. Ruzzo, and M. Tompa (1996):
To find a sequence that best fits the radiation hybrid data, solve a TSP

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The two StarLight mission spacecraft will orbit the sun, sort of tagging along behind Earth. Both spacecraft will carry telescope mirrors. Both telescope mirrors will be turned to look at the same star. Star light from the two mirrors will be combined to create a very good image, which is then sent back to Earth. Laser beams are used to keep the two spacecraft very precisely aligned.

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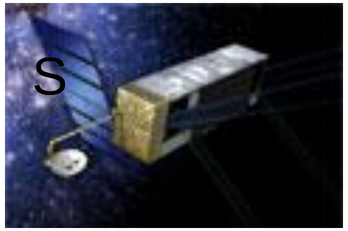
StarLight cancelled in 2002



OTHER APPLICATIONS

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- postal deliveries
- meals on wheels
- inspection tours
- school bus routing

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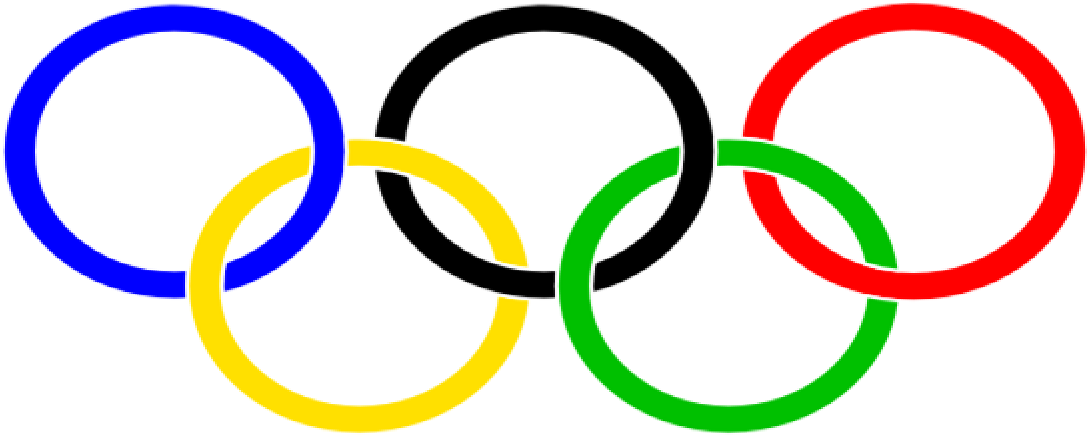
- etc. etc. etc.

The sport of solving the TSP



The sport of solving the TSP

TSPLIB:

a library of 111 instances
collected by Gerhard Reinelt (Heidelberg) in 1991

TSPLIB

a280
ali535
att48
att532
bayg29
bays29
berlin52
bier127
brazil58
brd14051
brg180
burma14
ch130
ch150
d198
d493
d657
d1291
d1655
d2103
d15112
d18512
dantzig42
dsj1000
eil51
eil76
eil101
fl417
fl1400
fl1577
fl3795
fnl4461
fri26
gil262
gr17
gr21
gr24
gr48
gr96
gr120
gr137
gr202
gr229
gr431
gr666
hk48
kroA100
kroB100
kroC100
kroD100
kroE100
kroA150
kroB150
kroA200
kroB200
lin105
lin318
linhp318
nrw1379
p654
pa561
pcb442
pcb1173
pcb3038
pla7397
pla33810
pla85900
pr76
pr107
pr124
pr136
pr144
pr152
pr226
pr264
pr299
pr439
pr1002
pr2392
rat99
rat195
rat575
rat783
rd100
rd400
rl1304
rl1323
rl1889
rl5915
rl5934
rl11849
si175
si535
si1032
st70
swiss42
ts225
tsp225
u159
u574
u724
u1060
u1432
u1817
u2152
u2319
ulysses16
ulysses22
usa13509
vm1084
vm1748

TSPLIB

a280
ali535
att48
att532
bayg29
bays29
berlin52
bier127
brazil58
brd14051
brg180
burma14
ch130
ch150
d198
d493
d657
d1291
d1655
d2103
d15112
d18512
dantzig42
dsj1000
eil51
eil76
eil101
fl417
fl1400
fl1577
fl3795
fnl4461
fri26
gil262
gr17
gr21
gr24
gr48
gr96
gr120
gr137
gr202
gr229
gr431
gr666
hk48
kroA100
kroB100
kroC100
kroD100
kroE100
kroA150
kroB150
kroA200
kroB200
lin105
lin318
linhp318
nrw1379
p654
pa561
pcb442
pcb1173
pcb3038
pla7397
pla33810
pla85900
pr76
pr107
pr124
pr136
pr144
pr152
pr226
pr264
pr299
pr439
pr1002
pr2392
rat99
rat195
rat575
rat783
rd100
rd400
rl1304
rl1323
rl1889
rl5915
rl5934
rl11849
si175
si535
si1032
st70
swiss42
ts225
tsp225
u159
u574
u724
u1060
u1432
u1817
u2152
u2319
ulysses16
ulysses22
usa13509
vm1084
vm1748

The red instances were unsolved in 1991

MISCONCEPTION #8: SIZE IS IMPORTANT

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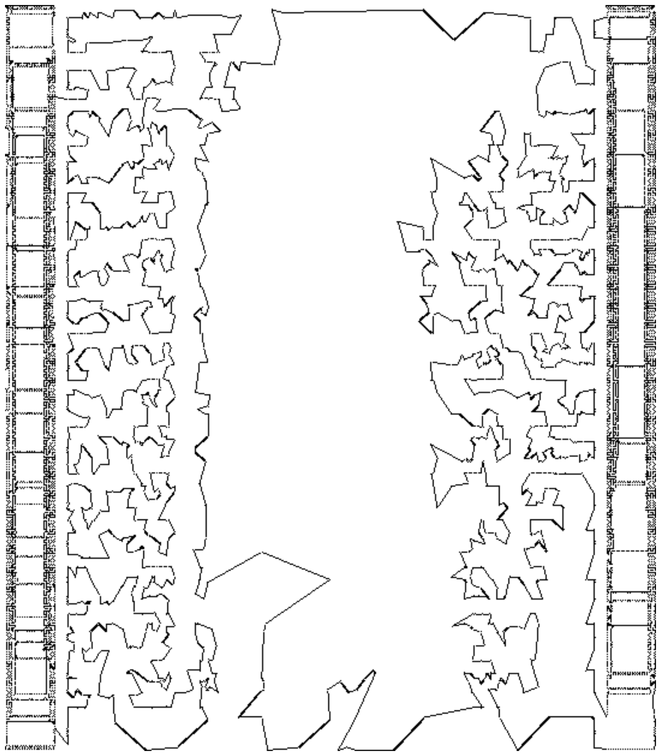

pla7397 solved
in October 1994

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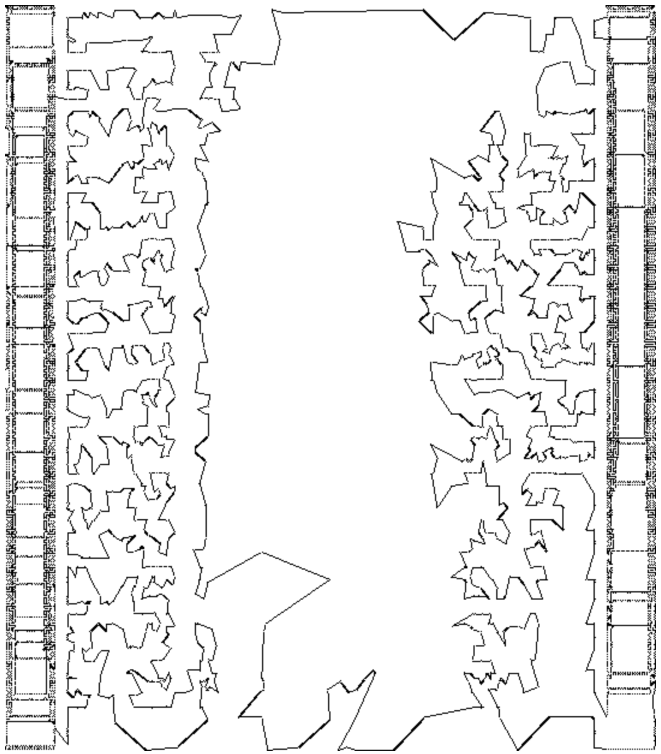
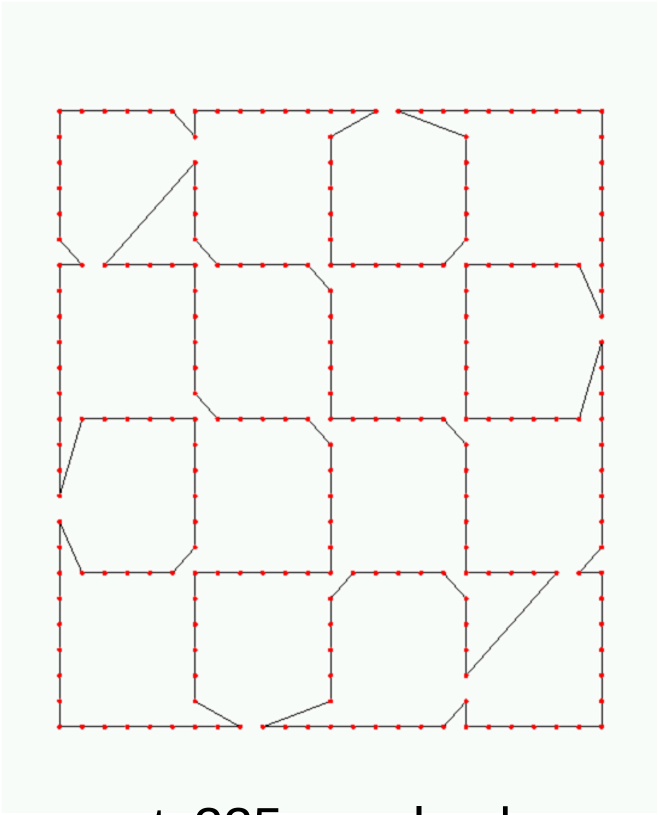
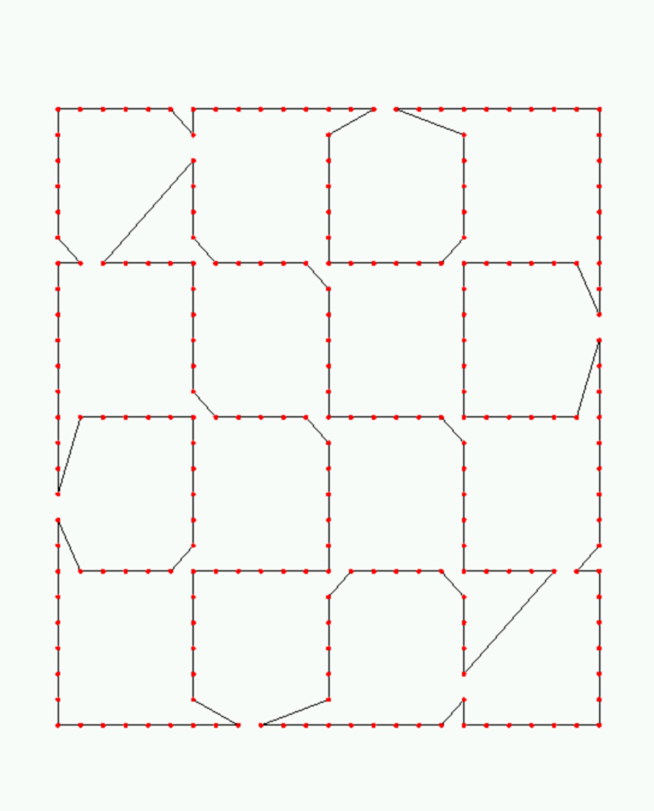


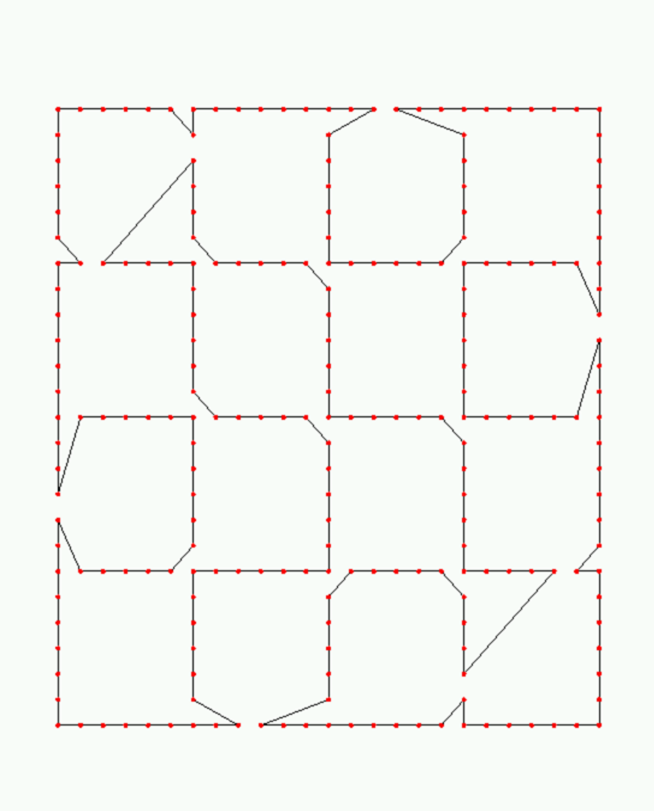
pla7397 solved
in October 1994



ts225 unsolved
in October 1994

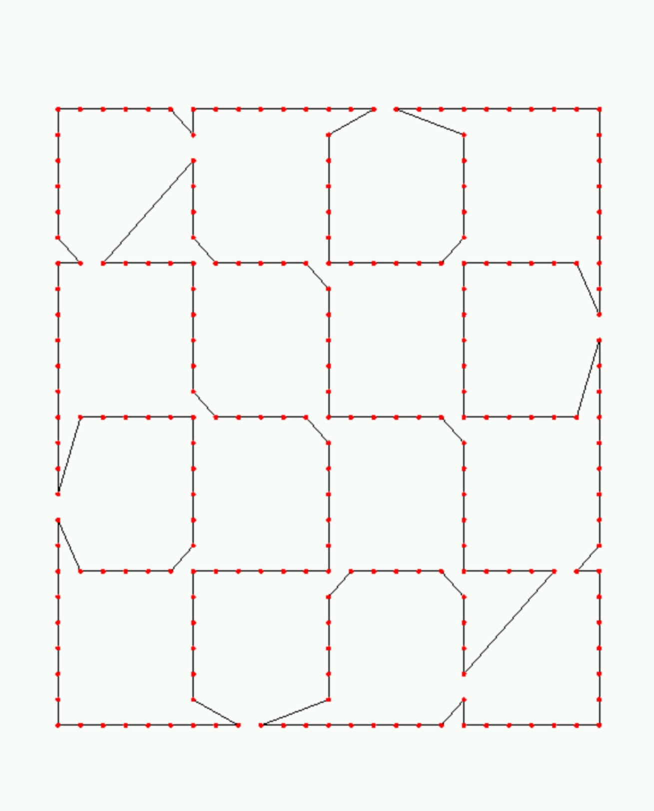


ts225 constructed by Stefan Tschöke



ts225 constructed by Stefan Tschöke

with malice aforethought



ts225 constructed by Stefan Tschöke

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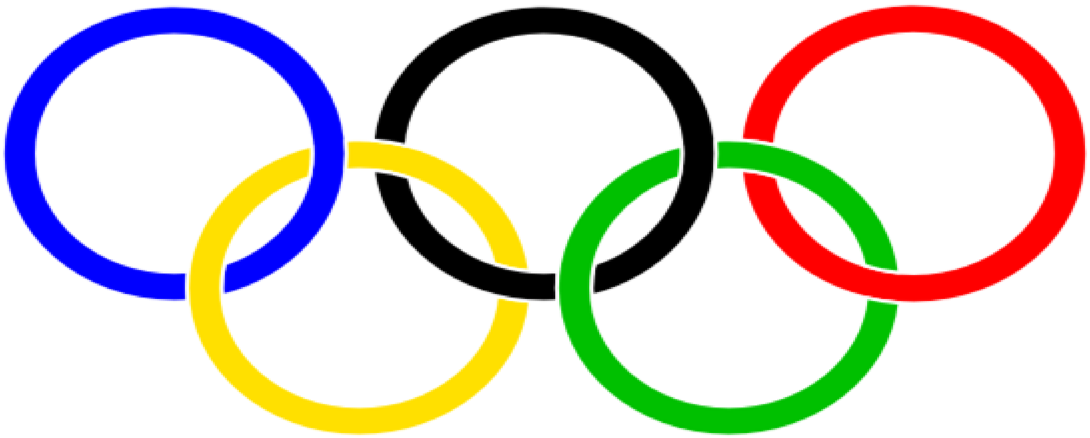
LESSON #1:
MISCHIEF IS A POWERFUL ENGINE OF DISCOVERY

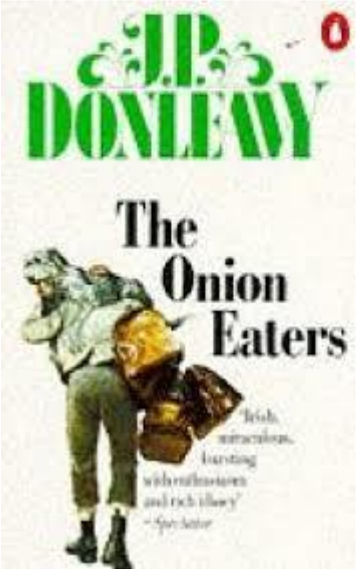
The sport of solving the TSP

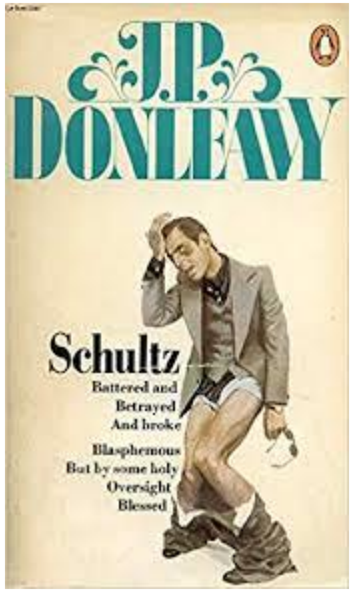


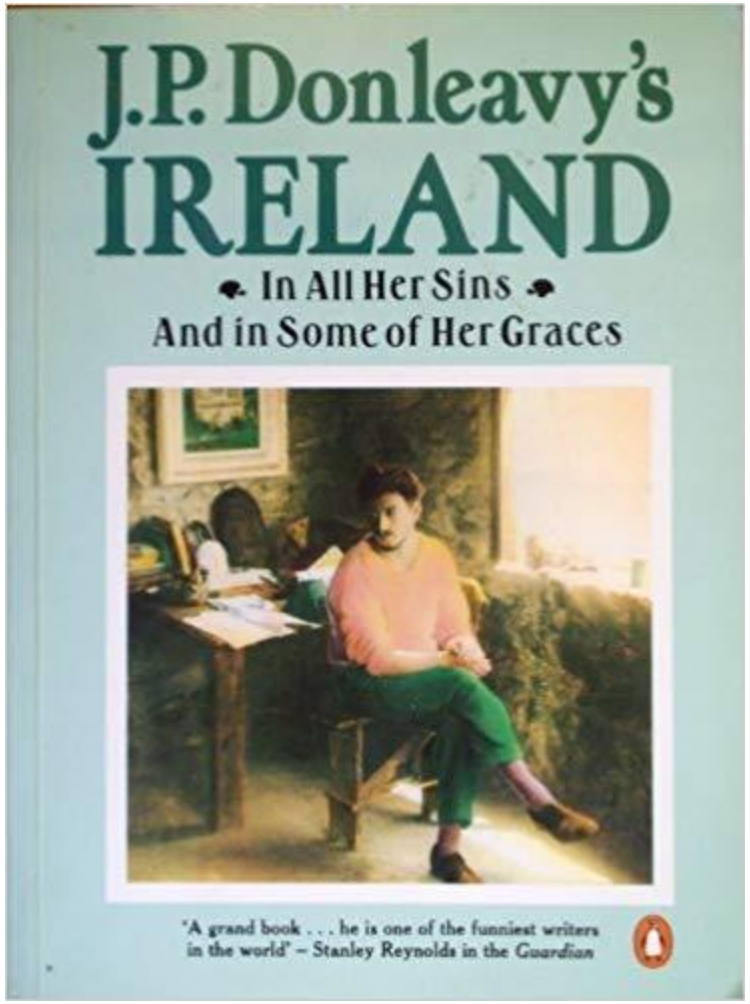
My daddy can beat up your daddy

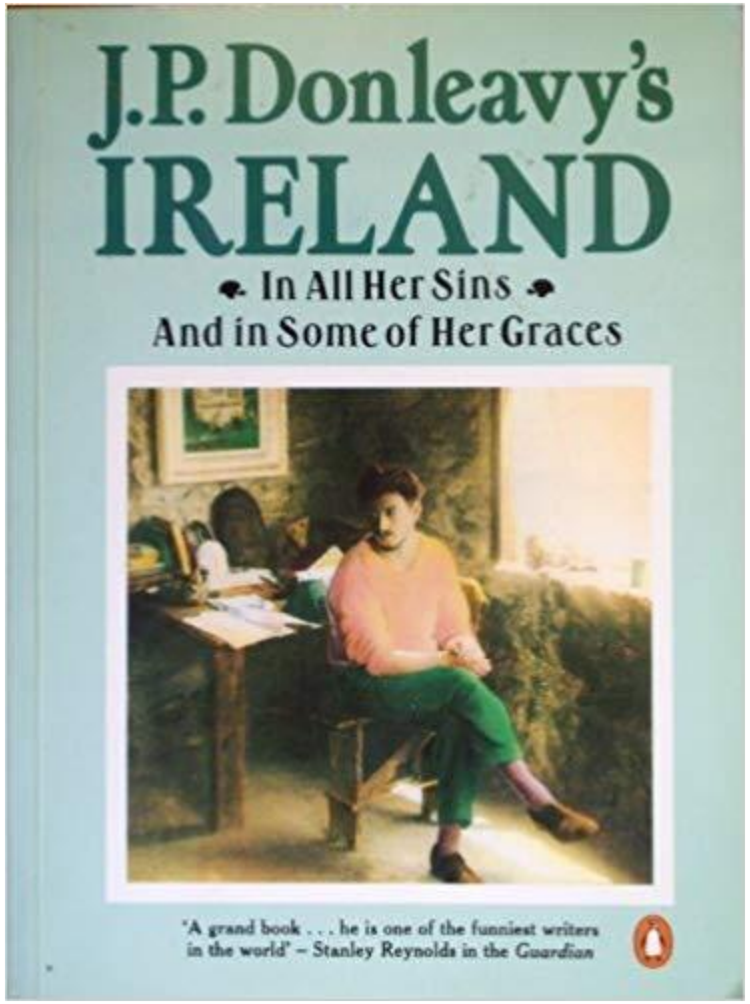
Bill Cook

With my friend and co-author Najiba Sbihi

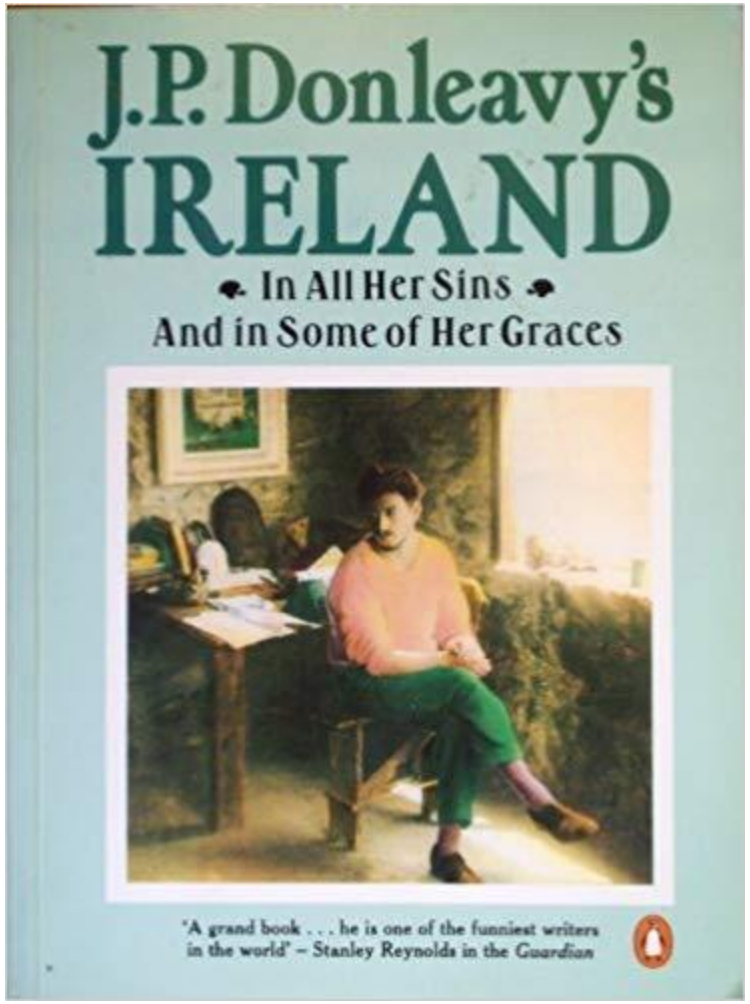

J.P.
DONLEAVY
The Onion Eaters


J.P.
DONLEAVY
Schultz
Battered and
Betrayed
And broke
Blasphemous
But by some holy
Oversight
Blessed


J.P. Donleavy's
IRELAND
In All Her Sins
And in Some of Her Graces
'A grand book . . . he is one of the funniest writers in the world' – Stanley Reynolds in the *Guardian*



There is also the oft-repeated recollection in Donleavy's autobiographical writing that his Irish parents grew up "without a pot to piss in".



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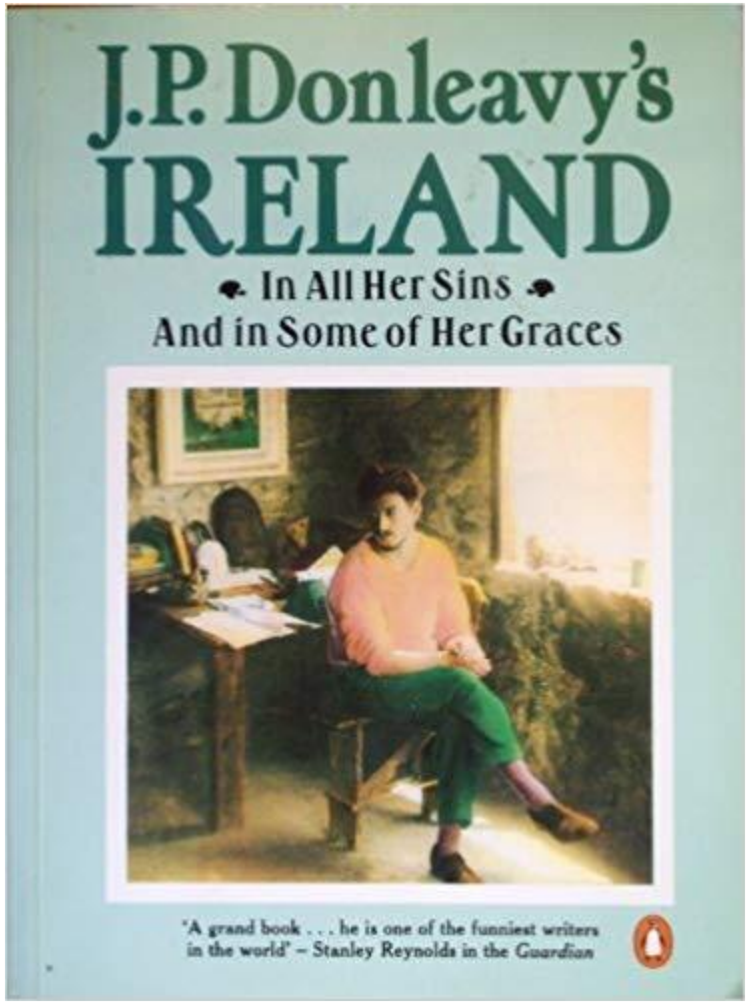

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… the author's rise to artistic glory, when he finally found in his home at Levington Park, with eleven toilets, a pot to piss in.

McGill 1987: Chính Hoàng’s Ph.D. defense

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Professor Cook first from the left (in the doorway)

Discrete Mathematics 86 (1990) 191–198
North-Holland

THE DISCIPLINE NUMBER OF A GRAPH

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Graduate School of Business, Columbia University, New York, NY 10027, USA

Received 2 December 1988

1. Introduction

The *domination number* $\gamma(G)$ of a graph G is the size of a smallest set D of vertices such that every vertex outside D has at least one neighbour in D; Fink, Jacobson, Kinch, and Roberts [4] defined the *bondage number* $b(G)$ of a graph G as the least number of edges whose deletion makes $\gamma(G)$ increase. As we are about to point out, computing $b(G)$ amounts to solving an integer linear program.

Define a *whip* in a graph G as any spanning subgraph F of G such that each component of F is a star and F has precisely $\gamma(G)$ components; let $E(G)$ denote the set of edges of G and let $W(G)$ denote the set of all whips in G. Obviously, $b(G)$ is the optimal value of the problem

$$\begin{aligned} &\text{minimize} \quad \sum \{x_e : e \in E(G)\} \\ &\text{subject to} \sum \{x_e : e \in E(F)\} \geq 1 \quad \text{for all } F \text{ in } W(G), \\ &\qquad\qquad x_e \geq 0 \qquad\qquad\qquad\quad \text{for all } e \text{ in } E(G). \\ &\qquad\qquad x_e = \text{integer} \qquad\qquad\quad\; \text{for all } e \text{ in } E(G). \end{aligned} \tag{1}$$

By the *fractional bondage number* $b^*(G)$ we shall mean the optimal value of the 'linear programming relaxation' of (1),

$$\begin{aligned} &\text{minimize} \quad \sum \{x_e : e \in E(G)\} \\ &\text{subject to} \sum \{x_e : e \in E(F)\} \geq 1 \quad \text{for all } F \text{ in } W(G), \\ &\qquad\qquad x_e \geq 0 \qquad\qquad\qquad\quad \text{for all } e \text{ in } E(G). \end{aligned} \tag{2}$$

By the duality theorem of linear programming, $b^*(G)$ equals the optimal value of the dual of (2),

$$\begin{array}{lll} \text{maximize } \sum \{y_F: F \in W(G)\} & & \\ \text{subject to } \sum \{y_F: e \in E(F)\} \leqslant 1 & \text{for all } e \text{ in } E(G), & \\ \qquad y_F \geqslant 0 & \text{for all } F \text{ in } W(G). & \end{array} \tag{3}$$

Since (3) can be seen as the linear programming relaxation of

$$\begin{array}{lll} \text{maximize } \sum \{y_F: F \in W(G)\} & & \\ \text{subject to } \sum \{y_F: e \in E(F)\} \leqslant 1 & \text{for all } e \text{ in } E(G), & \\ \qquad y_F \geqslant 0 & \text{for all } F \text{ in } W(G), & \\ \qquad y_F = \text{integer} & \text{for all } F \text{ in } W(G), & \end{array} \tag{4}$$

problems (1) and (4) are in a sense dual. Therefore we refer to the optimal value of (4) as the *discipline number* $\operatorname{dis}(G)$ of G.

We have

$$1 \leqslant \operatorname{dis}(G) \leqslant b^*(G) \leqslant b(G) \tag{5}$$

for all graphs G. Apart from establishing upper bounds on $b(G)$, Fink et al. computed the bondage number of cycles, paths, and complete multipartite graphs and studied the bondage number of trees (several of these results can also be found in Bauer, Harary, Nieminen, and Suffel [1]). The purpose of this paper is to provide ties with analogous results for the fractional bondage number and for the discipline number.

2. The fractional bondage number

The principle restraining device of this section goes as follows.

Theorem 1. *If G has n vertices and m edges then $b^*(G) \leqslant m/(n-\gamma(G))$.*

Proof. Observe that the constraints of (2) are satisfied by $x_e = 1/(n-\gamma(G))$ for all e. □

As usual, let $\Delta(G)$ denote the largest degree of a vertex in G. Fink et al. conjectured that $b(G) \leqslant \Delta(G)+1$.

Theorem 2. $b^*(G) \leqslant \Delta(G)$.

Claim 5. *If $k \geqslant 3$, then $|Q| \geqslant 4$.*

Proof of Claim 5. Assume the contrary: $k \geqslant 3$ but $|Q| \leqslant 3$. Since G has at least four vertices, some vertex w lies outside Q; since $F_1, F_2, \ldots, F_k$ are edge-disjoint, w is adjacent to at least k distinct vertices in Q. Hence $|Q| = k = 3$. Now no S_j can include a vertex from Q and a vertex w outside Q (w has to be adjacent to at least three distinct vertices in Q); since $|S_j| \geqslant 2$ for all j, it follows that $Q = S_j$ for some j. Finally, this S_j includes some vertex w distinct from u_1 and v_1, a contradiction: w must be adjacent to at least one of u_1 and v_1. □

Claim 6. *If $k \geqslant 4$, then $k \leqslant a + \lfloor b/2 \rfloor$.*

Proof of Claim 6. By virtue of Claim 4, we only need show that $|Q| = 2k$. For this purpose, assume the contrary: without loss of generality $u_1 = u_2$. Write

$$Q_0 = \{u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4\}$$

and consider the graph H_0 whose set of vertices is Q_0, two vertices being adjacent in H_0 if and only if they are adjacent in some F_i with $1 \leqslant i \leqslant 4$. Since each F_i with $1 \leqslant i \leqslant 4$ contributes $|Q_0| - 2$ edges to H_0, we have

$$4(|Q_0| - 2) \leqslant \binom{|Q_0|}{2};$$

observing that $|Q_0| \leqslant 7$ (since $u_1 = u_2$) and $|Q_0| \geqslant 4$ (by Claim 5), we conclude that $|Q_0| = 7$. Now H_0 has twenty edges, which is a contradiction: $\binom{7}{2} = 21$ and no u_i with $2 \leqslant i \leqslant 4$ is adjacent to v_i in H_0. □

Claim 7. *If $k = 3$, then $k \leqslant a + \lfloor b/2 \rfloor$ or $a + b \geqslant 4$.*

Proof of Claim 7. Claim 4 allows us to assume that $|Q| \leqslant 5$; Claim 5 guarantees that $|Q| \geqslant 4$. Defining H as in the proof of Claim 4, observe that H has $3(|Q| - 2)$ edges. It follows that H (and hence also G) contains four pairwise adjacent vertices. □

Claim 8. *If $a = 0$ and $b = 2$, then $k \neq 2$.*

Proof of Claim 8. Assume the contrary: $k = 2$ but $a = 0$ and $b = 2$. Claim 4 implies that $|Q| \leqslant 3$ and so, without loss of generality, $u_1 = u_2 \in S_1$. Since S_1 includes a vertex distinct from both u_1, v_1 but adjacent to at least one of them, we must have $v_1 \in S_2$; a symmetric argument shows that $v_2 \in S_2$. But then S_2 includes a vertex outside Q and adjacent to only one vertex in Q, a contradiction. □

This ties down the proof of Theorem 9. □

The reader interested in additional results in a similar vein is directed to [2, Chapter 5].

Acknowledgement

We thank the two referees for their thoughtful comments which helped to improve the presentation of our results.

References

[1] D. Bauer, F. Harary, J. Nieminen and C.L. Suffel, Domination alteration sets in graphs, Discrete Math. 47 (1983) 153–161.
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[6] M. Padberg and G. Rinaldi, Optimization of a 532-city symmetric traveling salesman problem by branch and cut, Oper. Res. Lett. 6 (1987) 1–8.
[7] D.A.F. de Sade, Oeuvres Completes, 2–3 (J.-J. Pauvert, Paris, 1955).

December 1987

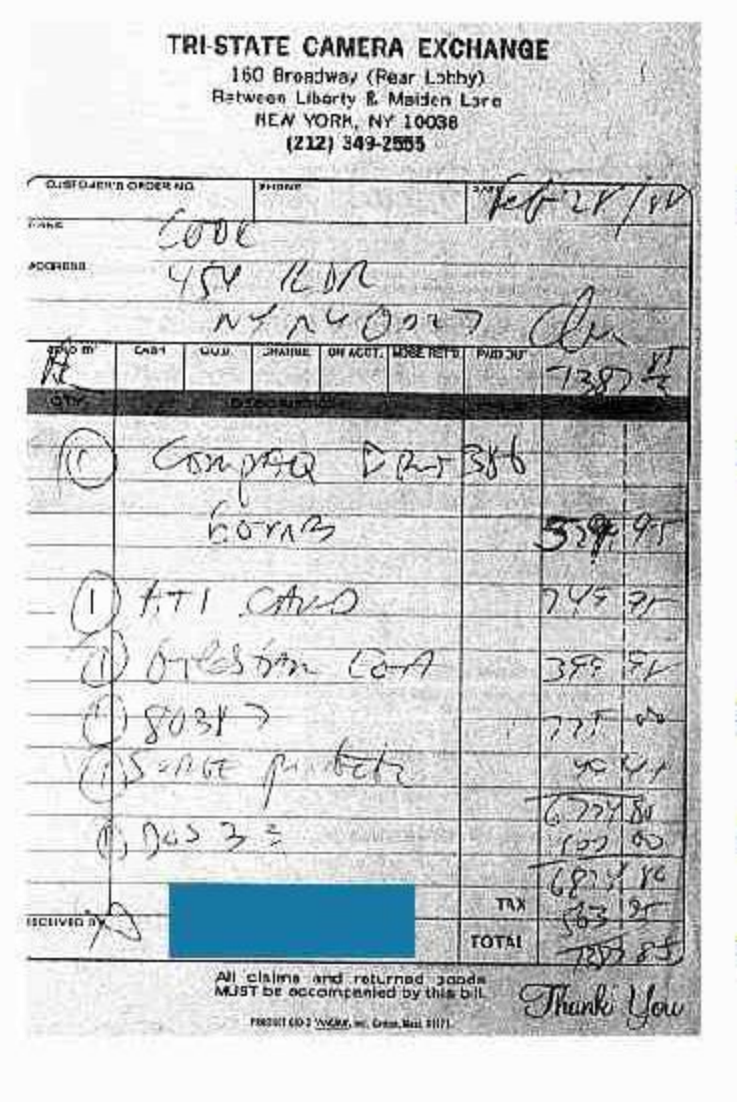
LESSON #1:
MISCHIEF IS A POWERFUL ENGINE OF DISCOVERY

January 1988

David Applegate

Don't leave home without it!

Don't leave home without it!



TRI-STATE CAMERA EXCHANGE
160 Broadway (Rear Lobby)
Between Liberty & Maiden Lane
NEW YORK, NY 10038
(212) 349-2555

CUSTOMER'S ORDER NO. | PHONE | DATE Feb 28/88

NAME COOK

ADDRESS 45N 12 DR

NY NY 10027

SOLD BY | CASH | C.O.D. | CHARGE | ON ACCT. | MDSE RET'D | PAID OUT

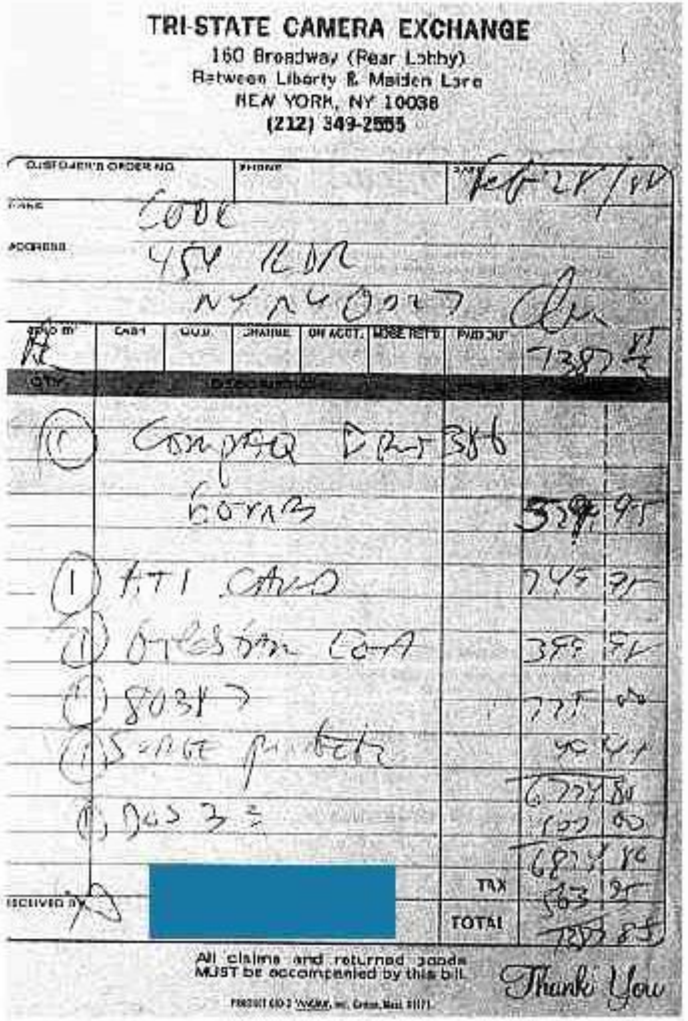
QTY	DESCRIPTION		
1	Compaq Desk 386		
	60 MB	3599	95
1	ATI Card	745	95
1	... Com	395	95
1	80387	775	00
1	Single printer	99	95
		6,229	80
1	Dos 3.3	100	00
		6,829	80
	TAX	563	95
	TOTAL	7393	85

RECEIVED BY [redacted]

All claims and returned goods MUST be accompanied by this bill.

Thank You

Don’t leave home without it!



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NAME COOK
ADDRESS 45 W BDW
NY NY 10027

SOLD BY | CASH | C.O.D. | CHARGE | ON ACCT. | MDSE RET'D | PAID OUT 7387 85

QTY | DESCRIPTION

1 Compaq DPro 386 60MB 5299 95
1 ATI CARD 245 95
1 Everstrom CoA 395 95
1 80387 775 00
1 Surge protector 59 95
1 DOS 3.3
6773 80
100 00
6823 80
TAX 563 05
TOTAL 7387 85

RECEIVED BY [redacted]

All claims and returned goods MUST be accompanied by this bill.

Thank You

February 28, 1988

Compaq DeskPro 386

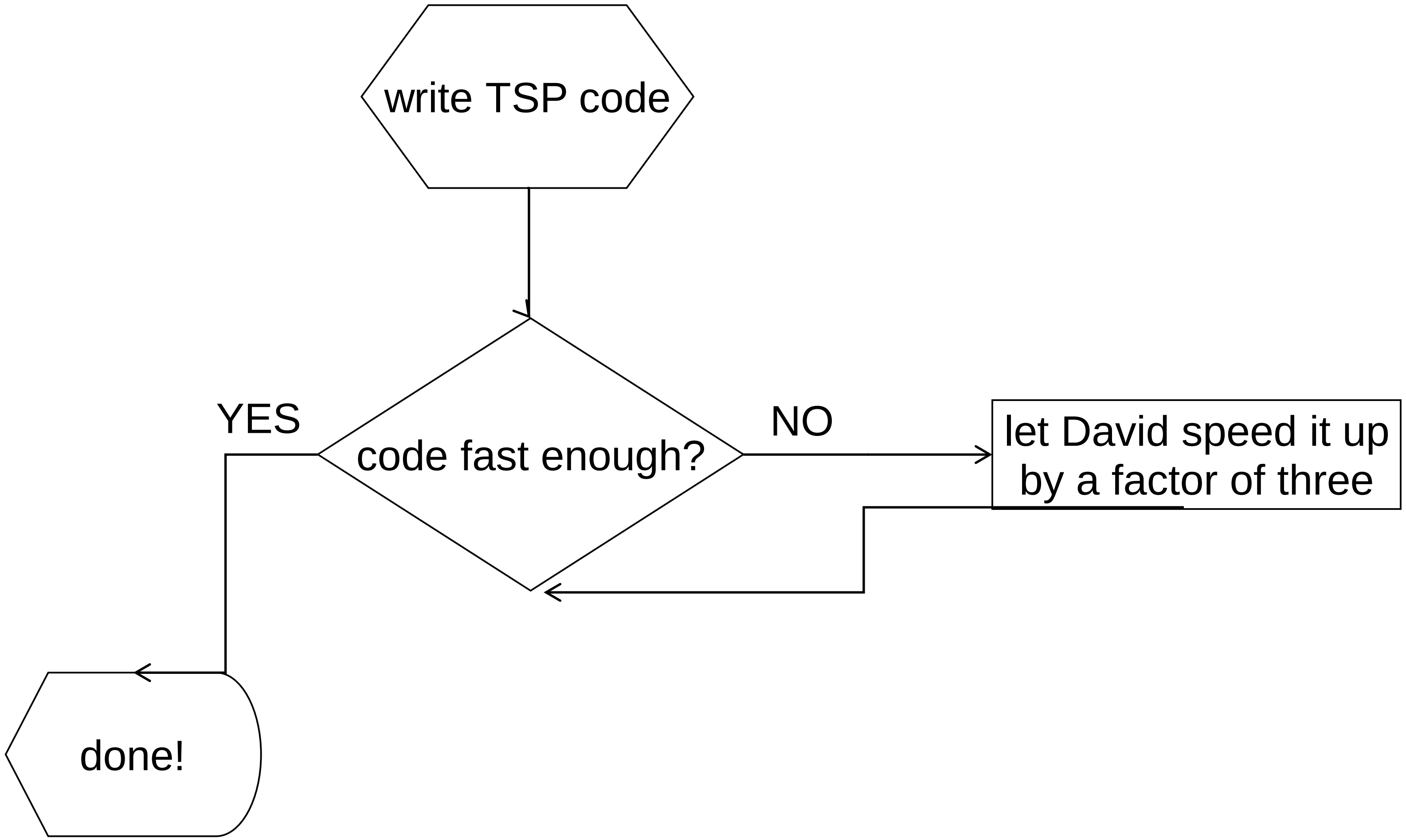
80387

DOS 3.3

$7387.85

Bill Cook’s original TSP algorithm

Bill Cook's original TSP algorithm



1988 – 1989: The Age of Innocence

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Battle cry:

If everyone does it in a certain way, we will do it differently!!

OUT WITH THE OLD, IN WITH THE NEW

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SOMETIMES THE OLD IS USED FOR A GOOD REASON

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OUT WITH THE OLD, IN WITH THE NEW

LESSON #3:

SOMETIMES THE OLD IS USED FOR A GOOD REASON

Case in point:

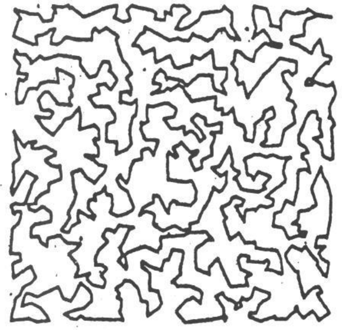
The simplex method of linear programming

The New York Times, March 12, 1991

Math Problem, Long Baffling, Slowly Yields

The traveling salesman problem still isn't solved, but computers can now get most answers.

By GINA KOLATA



A. T.&T.

Approximation of best route for 1,000 cities.

A CENTURY-OLD math problem of notorious difficulty has started to crumble. Even though an exact solution still defies mathematicians, researchers can now obtain answers that are good enough for almost all practical applications.

The traveling salesman problem, as it is known, crops up in many practical applications, from the design of computer chips to the designation of work orders in factories.

Brute number-crunching by computers can now produce answers to most such problems, even though not an immaculate solution. "Everybody likes to point to the

The New York Times, March 12, 1991

Contests Among Investigators

Solving traveling salesman problems has become something of an obsession for aficionados. Last year, for example, Dr. Robert Bixby of Rice University invited about 50 groups of investigators to a meeting that focused only on traveling salesman problems. Investigators brought their computer programs to the meeting and Dr. William Pulleybank of I.B.M.'s Thomas J. Watson Research Center in Yorktown Heights, N.Y., challenged them with a 783-city problem that he had devised, he thought, to be especially difficult.

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The Texas Shootout: April 22-24, 1990

[Facsimile of a transparency used by V.C. in his talk in Texas]

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SURVEY OF PROGRESS IN TSP SOLVING

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1990	Applegate, Chvátal, and Cook	17 cities

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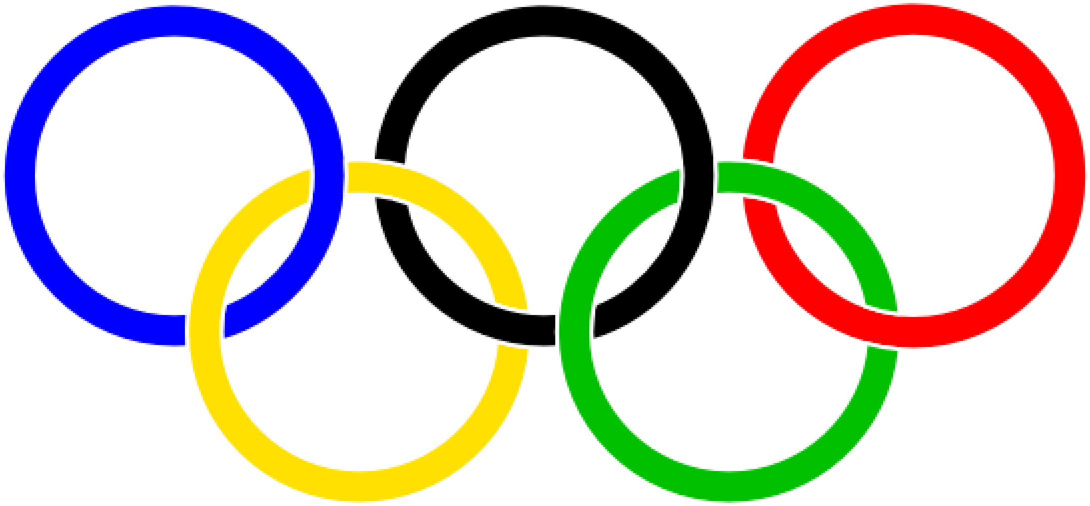
The winning team, made up of Dr. William Cook of Bell Communications Research, and Dr. David Applegate and Dr. Vasek Chvatal of Rutgers University, solved the problem in half an hour. "It turned out to be easy," Dr. Cook said.

Bob Bixby

Bob Bixby

CPLEX

The sport of solving the TSP



My daddy can beat up your daddy

The sport of solving the TSP

The goal: pcb3038

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Development
Environment

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January 1992: The Great Dismay

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Desperate last-ditch efforts:

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- cut tightening (Section 10.1)
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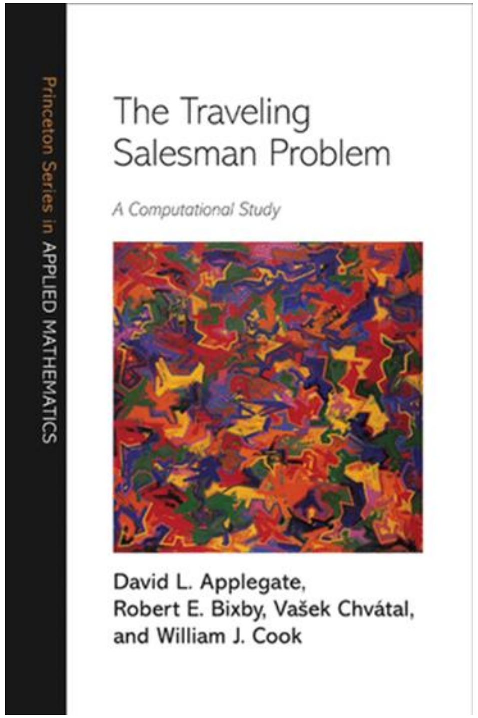

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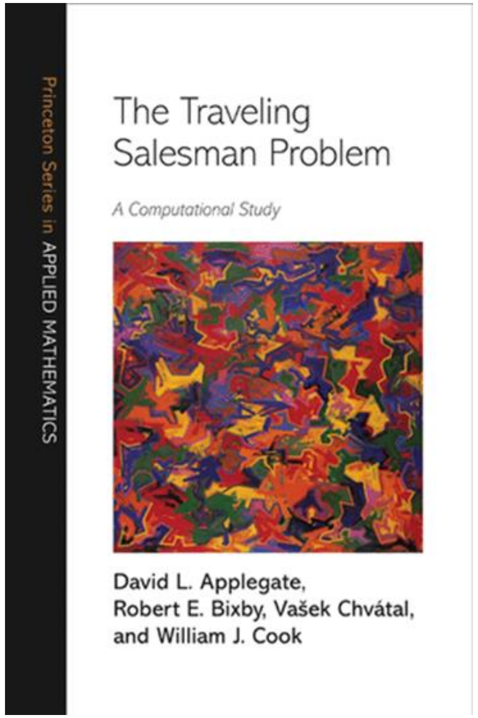

April 1992: pcb3038 falls

January 1992: The Great Dismay

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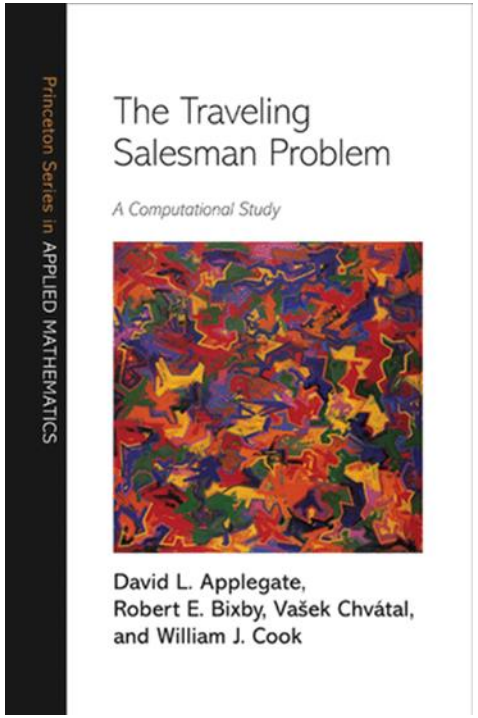
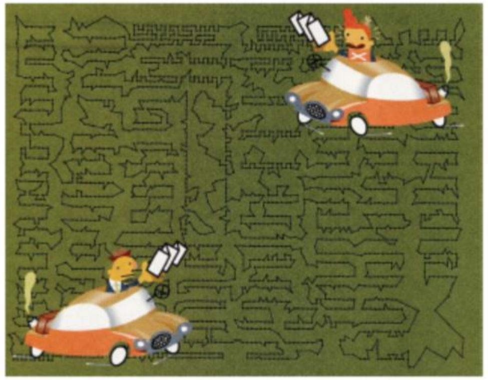


April 1992: pcb3038 falls

The Discover Magazine, January 1993
“A new TSP record, 3,038 cities”



June 1993: fnl4461

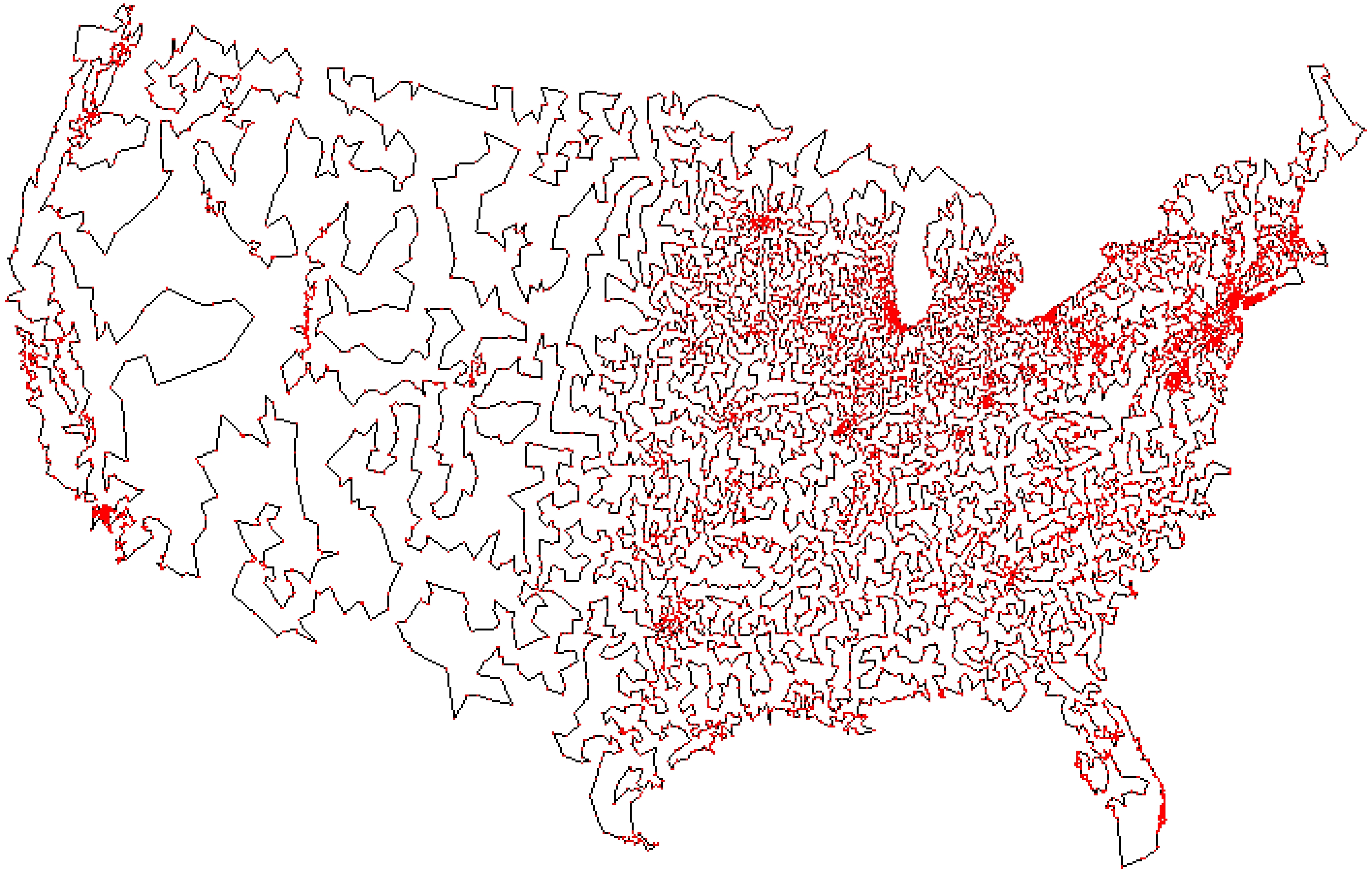
January 1996

Another breakthrough: local cuts (Chapter 11)

January 1996

Another breakthrough: local cuts (Chapter 11)

1998: usa13509



TSPLIB

a280
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bays29
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brazil58
brd14051
brg180
burma14
ch130
ch150
d198
d493
d657
d1291
d1655
d2103
d15112
d18512
dantzig42
dsj1000
eil51
eil76
eil101
fl417
fl1400
fl1577
fl3795
fnl4461
fri26
gil262
gr17
gr21
gr24
gr48
gr96
gr120
gr137
gr202
gr229
gr431
gr666
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kroB100
kroC100
kroD100
kroE100
kroA150
kroB150
kroA200
kroB200
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linhp318
nrw1379
p654
pa561
pcb442
pcb1173
pcb3038
pla7397
pla33810
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pr1002
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rl1304
rl1323
rl1889
rl5915
rl5934
rl11849
si175
si535
si1032
st70
swiss42
ts225
tsp225
u159
u574
u724
u1060
u1432
u1817
u2152
u2319
ulysses16
ulysses22
usa13509
vm1084
vm1748

The red instances were unsolved in 1991

TSPLIB

a280
ali535
att48
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bays29
berlin52
bier127
brazil58
brd14051
brg180
burma14
ch130
ch150
d198
d493
d657
d1291
d1655

d2103
d15112
d18512
dantzig42
dsj1000
eil51
eil76
eil101
fl417
fl1400
fl1577
fl3795
fnl4461
fri26
gil262
gr17
gr21
gr24

gr48
gr96
gr120
gr137
gr202
gr229
gr431
gr666
hk48
kroA100
kroB100
kroC100
kroD100
kroE100
kroA150
kroB150
kroA200
kroB200
lin105

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linhp318
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pa561
pcb442
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pcb3038
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pla33810
pla85900
pr76
pr107
pr124
pr136
pr144
pr152
pr226

pr264
pr299
pr439
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rl5934
rl11849
si175
si535

si1032
st70
swiss42
ts225
tsp225
u159
u574
u724
u1060
u1432
u1817
u2152
u2319
ulysses16
ulysses22
usa13509
vm1084
vm1748

The red instances were unsolved in 2006

Large TSP instances solved in the new millennium

April 2001	d15112	34 CPU years
December 2002	it16862	11 CPU years
March 2004	brd14051	5 CPU years
May 2004	sw24978	85 CPU years
October 2004	pla33810	16 CPU years
March 2005	d18512	57 CPU years
April 2006	pla85900	136 CPU years

年の9月に日本オペレーションズリサーチ学会の会員に中央大学の伊理正夫を通して
Chvátal からの協力依頼の電子メイルが流れたんだ．一部割愛して引用しよう．

Date: Tue, 28 Sep 93 18:00:00
Subject: ohisashiburi desu ne

Dear Professor Iri,

you may be surprised to get this message out of the blue. I wonder if you would do me a favor. David Applegate, Bob Bixby, Bill Cook, and I have written a computer code to solve travelling salesman problems. Last year we have solved one with 3038 cities (from a database called TSPLIB), which was the world record at that time; a couple of months ago we have broken the record by solving a problem with 4461 cities, and now we would like to try 7397 cities. We solve those problems in parallel on different workstations; currently we are using about 65 of them. The way it works is that people open accounts for us on their machines and we run parts of our program on them whenever no one else is using them (we have a clever program that kills our computations as soon as somebody touches the keyboard or logs into the machine from home). The 65 workstations are sitting around here in New Jersey, but with our sights on the 7397 cities we would like to recruit as many new machines as we can. David will try to get us some at McGill, we are asking a few other friends in North America to do the same, and thought it would be nice if we had a few helpers in Japan as well. If you know anybody who would be willing to lend his or her machine to contribute to breaking the record again, would you, please, pass this message on to him or her? (This is not terribly important, so please, don't go to too much trouble to help us.)

Thank you very much!

Ogenki de,

Vasek Chvatal

何台ものワークステーションをつなげて世界記録を目指しているが，日本でも協力し

CONCORDE'S YOUNGER HELPERS

- An implementation of Adam Letchford's *domino parity constraints* developed by Bill Cook, Daniel Espinoza, and Marcos Goycoolea.
Used in the solution of d18512, pla33810, and pla85900.

- Keld Helsgaun's refinement of the Lin-Kernighan heuristic.
Used in the solution of sw24978 and pla85900.

Bill Cook’s Mona Lisa Challenge: $1000 Prize for a tour shorter than 5,757,191 the 100,000-city problem created by Robert Bosch.

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Techniques for developing such point sets have evolved over through work of Bosch and Craig Kaplan.

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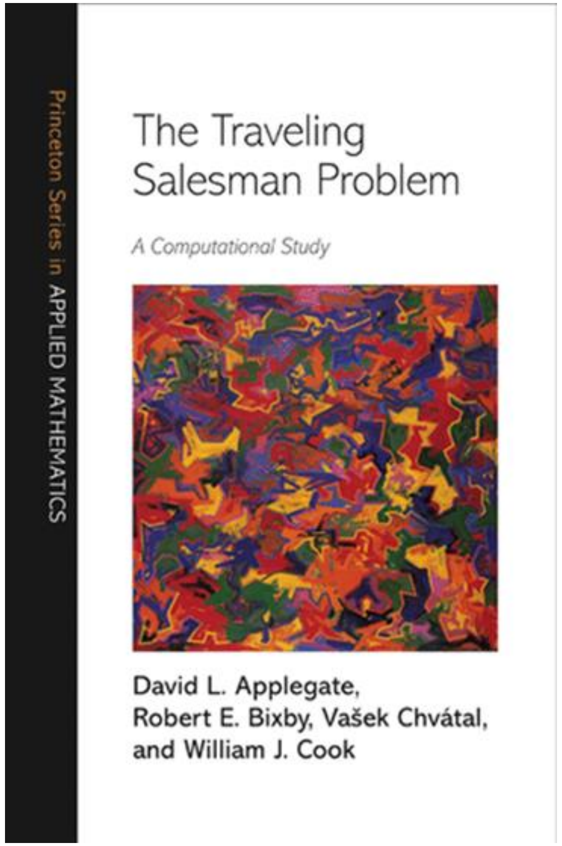
Techniques for developing such point sets have evolved over through work of Bosch and Craig Kaplan.

A tour of length 5,757,191 was found on March 17, 2009, by Yuichi Nagata

It is never too early to think of Christmas gifts

It is never too early to think of Christmas gifts



Hardcover $80.81
at amazon.com

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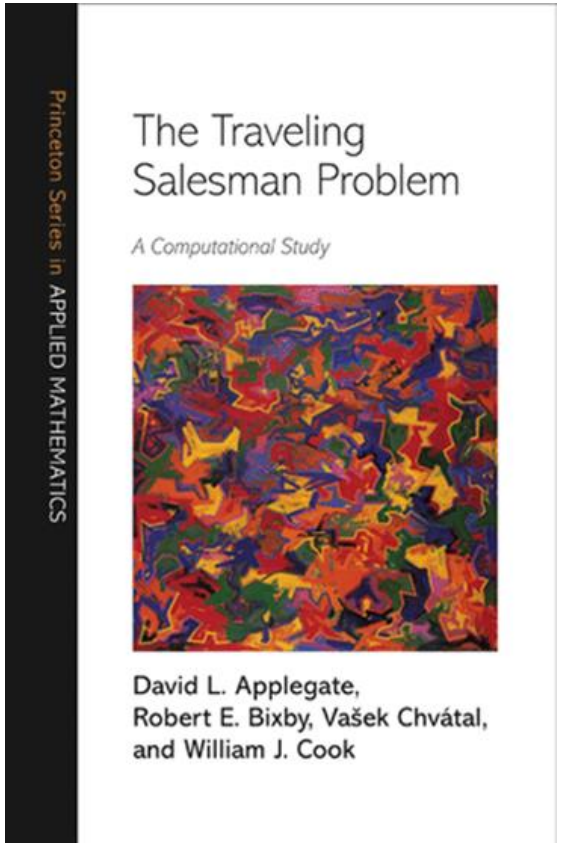

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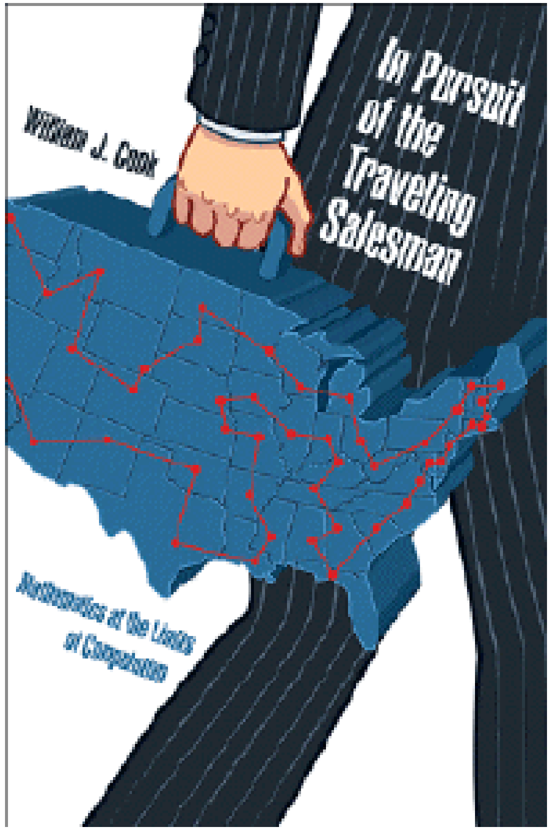

Paperback $13.78
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