NDMI107 • 2024 • Solutions to homework 1

1.

$$\sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}.$$

The left-hand side counts n-point subsets S of $\{1, 2, \ldots 2n\}$ as follows: First choose the size i of the intersection $S \cap \{1, 2, \ldots n\}$, then choose this intersection, and finally choose the i-point set $\{n+1, n+2, \ldots 2n\} - S$.

2.

$$\sum_{i=1}^{n} \binom{i}{k} = \binom{n+1}{k+1}.$$

The left-hand side counts $(k+1)$-point subsets S of $\{1, 2, \ldots n+1\}$ as follows: First choose the largest element $i+1$ of S and then choose the k-point set $S \cap \{1, 2, \ldots i\}$.