Positivity of Incomplete Cooperative Games Revisited

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Abstract

We consider incomplete cooperative games, where only some coalitions' values are specified and others remain indeterminate. Focusing on *positive extensions*—fully defined cooperative games that agree with the partial data and have nonnegative dividends—we introduce a novel, two-stage dividend-assignment procedure that fully characterizes all such extensions. Our method offers a general criterion for positivity-extendability, introduces an explicit lower bound game, and provides an understanding of the structure of extreme points in the extension set. These contributions significantly expand the toolkit for theoretical analyses and practical computations of incomplete cooperative games, and also shed new light on properties of classical cooperative games.

Keywords: Cooperative games, Incomplete information, Positive games, Extensions
JEL: C71, D81, D85

1. Introduction

Incomplete or partially defined cooperative games [1, 2, 3, 4] have recently emerged as a complement to the theory of cooperative games with restricted cooperation [5, 6]. In classical cooperative games, every subset of players, called coalition, is assigned a real value representing their joint payoff or cost. In contrast, in incomplete or restricted cooperative games, some coalition values are missing. In the restricted case, certain coalitions are disallowed due to incompatibilities among agents or external constraints (e.g., geo-political reasons). In the incomplete case, data may be absent due to costs of acquisition or corruption, yet these missing values are just as important as those provided.

A fundamental tool in analyzing incomplete games and in formulating their solution concepts is the set of *extensions*—that is, all fully specified games with additional properties that agree with the partial data. Among the most studied of these are the *positive extensions*. Positivity has received considerable attention because it yields powerful theoretical results for special incomplete-information structures [1], provides bounds for broader classes of extensions that are more difficult to analyze [7], and naturally arises when characterizing solution concepts [8].

We build on our previous work [4], which partially addressed positivity. Although we presented a characterization of the extreme points of positive extensions, in practice, it did not give much insight into the structure of these games. Here, we fully resolve this question by describing the set of positive extensions for general incomplete games by a two-stage dividend-assignment procedure. We then show how these results can be used to answer accompanying questions, namely the question of extendability, deriving the lower bound game and getting a structural result on the set of extreme points—illustrating broader opportunities opened by this new description.

In the realm of operations research, cooperative game theory has proven instrumental for addressing a variety of resource allocation and cost-sharing problems. Examples include minimum cost spanning tree settings where coalitional arrangements reduce infrastructure expenses [9], facility location and supply chain management, where shared investments or joint operations can lower overall costs [10], as well as scheduling and inventory management problems that benefit from fair allocation of joint setup times or holding costs. By providing new methods and insights for handling incomplete or partially defined games, our approach can thus support more robust and data-efficient decision-making in these and other OR applications.

2. Settings

A *(complete) cooperative game* (N, v) is represented by a set of players N and the characteristic function $v\colon 2^N \to \mathbb{R}$ with $v(\emptyset) = 0$. We call $S \subseteq N$ a *coalition* and $v(S) \in \mathbb{R}$ represents the *value of coalition* S. A cooperative game (N, v) is *positive*, if it has non-negative *dividends*, i.e., $d_v(S) \geq 0$ for every $S \subseteq N$, where dividends are defined recursively by $d_v(S) = v(S) - \sum_{T \subsetneq S} d_v(T)$. We denote the set of all positive games on n players by $\mathbb{P}^n$.

An *incomplete cooperative game* formally enhances a cooperative game with a *set of known coalitions* $\mathcal{K} \subseteq 2^N$. This set serves as a *mask* on the characteristic function, hiding information about values of coalitions outside $\mathcal{K}$.

Definition 1. *An* incomplete cooperative game $(N, \mathcal{K}, v)$ *is given by a set of agents* $N = \{1, \dots, n\}$*, a set of known coalitions* $\mathcal{K} \subseteq 2^N$ *and a characteristic function* $v\colon 2^N \to \mathbb{R}$*. Further,* $\emptyset \in \mathcal{K}$ *and* $v(\emptyset) = 0$*.*

When $\mathcal{K} = 2^N$, an incomplete cooperative game $(N, \mathcal{K}, v)$ coincides with a complete cooperative game (N, v).

Definition 2. *Let* $(N, \mathcal{K}, v)$ *be an incomplete cooperative game. Then* (N, w) *is a* $\mathbb{P}^n$-extension *of* $(N, \mathcal{K}, v)$ *if* $(N, w) \in \mathbb{P}^n$ *and*

$$v(S) = w(S), \qquad S \in \mathcal{K}. \tag{1}$$

We say $(N, \mathcal{K}, v)$ is $\mathbb{P}^n$-*extendable* if it has a $\mathbb{P}^n$-extension and we denote the set of $\mathbb{P}^n$-extensions of $(N, \mathcal{K}, v)$ by $\mathbb{P}^n_{\mathcal{K}}(v)$. Understanding sets of extensions is vital for analyzing the underlying game, understanding the importance of different subsets of values, and deriving solution concepts for incomplete games. The set of $\mathbb{P}^n$-extensions forms a convex set, which is bounded if and only if $N \in \mathcal{K}$ [4]. Bounded convex sets are uniquely defined by their extreme points, however, deriving explicit formulae for these can be a challenging task. Until now, the only result concerning the extreme points was employing *sets of negligable coalitions* $\mathcal{N}(w) = \{S \subseteq N \mid d_w(S) = 0\}$.

Theorem 1. *[4] For a* $\mathbb{P}^n$*-extendable incomplete game* $(N, \mathcal{K}, v)$*,* $\mathbb{P}^n$*-extension* (N, e) *is an extreme point if and only if there is no* $\mathbb{P}^n$*-extension* (N, w) *satisfying*

$$\mathcal{N}(e) \subsetneq \mathcal{N}(w). \tag{2}$$

Although this result was used to express the set of $\mathbb{P}^n$-extensions in cases with highly structured $\mathcal{K}$, for most of the scenarios, it does not give much insight into the structure of the set. In such scenarios, it is beneficial to derive at least tight bounds on the set, represented by *bound games*.

Definition 3. *Let* $(N, \mathcal{K}, v)$ *be a* $\mathbb{P}^n$*-extendable incomplete game. The* lower bound game $(N, \underline{v})$ *of the set of* $\mathbb{P}^n$*-extensions satisfies:*

1. *(boundedness)* $\forall S \subseteq N, \underline{v}(S) \leq w(S)$*,*
2. *(tightness)* $\forall S \notin \mathcal{K}$*,* $\exists (N, w) \in \mathbb{P}^n_{\mathcal{K}}(v)$ *such that* $w(S) = \underline{v}(S)$*.*

3. New description for positive extensions

The key component of our approach is the *closure of a coalition* $S \subseteq N$ *in* $\mathcal{K}$, defined as

$$c_{\mathcal{K}}(S) \coloneqq \bigcap_{X \in \mathcal{K}, S \subseteq X} X. \tag{3}$$

For every coalition $S \in \mathcal{K}$, it holds $c_{\mathcal{K}}(S) = S$, i.e. $\mathcal{K} \subseteq \mathcal{C}(\mathcal{K})$ and for $X \subseteq Y \subseteq N$, it holds $c_{\mathcal{K}}(X) \subseteq c_{\mathcal{K}}(Y)$. We denote the set of all closures in $\mathcal{K}$ by $\mathcal{C}(\mathcal{K})$ and for every $S \in \mathcal{K}$, we denote $\mathcal{C}(S) = \{T \subseteq N \mid c_{\mathcal{K}}(T) = S\}$. We note $\{\mathcal{C}(S)\}_{S \in \mathcal{C}(\mathcal{K})}$ forms a partition of 2^N.

Employing coalition closures, each $\mathbb{P}^n$-extension of an incomplete game $(N, \mathcal{K}, v)$ can be viewed through the following two-stage process. First, $v(N)$ is allocated among sets $\mathcal{C}(S)$, $S \in \mathcal{C}(\mathcal{K})$, each receiving a value denoted Δ_S. Second, each value Δ_S is allocated between all coalitions in $\mathcal{C}(S)$, specifically among dividends of the coalitions. This process always results in a $\mathbb{P}^n$-extension and every $\mathbb{P}^n$-extension can be achieved by this process.

Proposition 2. *For a $\mathbb{P}^n$-extendable incomplete game $(N, \mathcal{K}, v)$, any $\mathbb{P}^n$-extension (N, w) has the following form:*

- $\forall T \in \mathcal{C}(\mathcal{K}) : \sum_{X \in \mathcal{C}(T)} d_w(X) = \Delta_T$,
- $\forall S \in \mathcal{K} : \sum_{T \in \mathcal{C}(\mathcal{K}), T \subseteq S} \Delta_T = v(S)$,

for some $\Delta_S \in \mathbb{R}_+$.

Proof. If (N, w) is a $\mathbb{P}^n$-extension of $(N, \mathcal{K}, v)$, it must hold for every $S \in \mathcal{K}$,

$$v(S) = w(S) = \sum_{T \subseteq S} d_w(S). \tag{4}$$

From the fact that $T \subseteq S \implies c_{\mathcal{K}}(T) \subseteq c_{\mathcal{K}}(S)$, we can rewrite (4) as

$$\sum_{T \subseteq S} d_w(S) = \sum_{T \subseteq S, T \in \mathcal{C}(\mathcal{K})} \sum_{X \in \mathcal{C}(T)} d_w(X). \tag{5}$$

Setting $\Delta_T \coloneqq \sum_{X \in \mathcal{C}(T)} d_w(X)$, we get the conditions from the statement of the proposition. □

The two-stage process groups coalitions into disjoint sets and treats each group as a single entity in the first stage. For fixed values Δ_S, the corresponding $\mathbb{P}^n$-extensions form a family of regular simplices with a symmetrical structure. Specifically, each $S \in \mathcal{C}(\mathcal{K})$ corresponds to a regular simplex in which the coordinates (the surplus values) sum to $\Delta_v(S)$. Each vertex of this simplex represents the allocation of the full surplus $\Delta_v(S)$ to exactly one coalition $T \in \mathcal{C}(S)$, while assigning zero surplus to all other coalitions in $\mathcal{C}(S)$.

Proposition 2 simplifies additional questions regarding positivity in incomplete cooperative games. In what follows, we show how it can be used to derive important results, starting with $\mathbb{P}^n$-extendability.

Proposition 3. *Let $(N, \mathcal{K}, v)$ be an incomplete cooperative game. It is $\mathbb{P}^n$-extendable if and only if there are $\Delta_T \geq 0$ for every $T \in \mathcal{C}(\mathcal{K})$ satisfying*

$$\sum_{T \in \mathcal{C}(\mathcal{K}), T \subseteq S} \Delta_T = v(S), \quad \forall S \in \mathcal{K}. \tag{6}$$

Proof. For a $\mathbb{P}^n$-extension (N, w), by setting $\Delta_T = \sum_{X \in \mathcal{C}(T)} d_w(X)$ for every $T \in \mathcal{C}(\mathcal{K})$, we get a solution satisfying (6). If there is a solution for (6), we can define (N, w) as

$$d_w(T) = \begin{cases} \Delta_T & T \in \mathcal{C}(\mathcal{K}), \\ 0 & \textit{otherwise.} \end{cases}$$

Game (N, w) is clearly a $\mathbb{P}^n$-extension of $(N, \mathcal{K}, v)$. □

In case there is $S \in \mathcal{K}$ satisfying

$$\forall T \in \mathcal{C}(\mathcal{K}), T \subseteq S \implies T \in \mathcal{K},$$

one can recursively express $\Delta_T = v(T) - \sum_{X \in \mathcal{C}(\mathcal{K}), X \subsetneq T} \Delta_T$, thus further simplify system (6). In case $\mathcal{C}(\mathcal{K}) = \mathcal{K}$, the problem reduces to non-negativity of Δ_T for every $T \in \mathcal{K}$, where

$$\Delta_T = v(T) - \sum_{X \in \mathcal{C}(\mathcal{K}), X \subsetneq T} \Delta_T.$$

Proposition 2 can be further applied to derivation of the lower bound game. It reduces to a linear programming minimization problem with $|\mathcal{K}|$

constraints and $|\mathcal{C}(\mathcal{K})|$ variables. For the rest of this paper, let

$$\Delta_{\mathcal{K}}(v) = \left\{ \Delta \in \mathbb{R}_{\geq 0}^{|\mathcal{C}(\mathcal{K})|} \mid \forall S \in \mathcal{K}, \sum_{T \in \mathcal{C}(\mathcal{K}), T \subseteq S} \Delta_T = v(S) \right\}. \tag{7}$$

Proposition 4. *The lower bound game* $(N, \underline{v})$ *of a* $\mathbb{P}^n$*-extendable* $(N, \mathcal{K}, v)$ *is defined for* $S \subseteq N$ *as*

$$\underline{v}(S) = \min_{\Delta \in \Delta_{\mathcal{K}}(v)} \sum_{T \in \mathcal{C}(\mathcal{K}), T \subseteq S} \Delta_T. \tag{8}$$

Proof. Denote $\Delta \in \Delta_{\mathcal{K}}(v)$ for which the minimum attained and consider all $\mathbb{P}^n$-extensions (N, w) with $\sum_{X \in \mathcal{C}(T)} d_w(X) = \Delta_T$ for every $T \in \mathcal{C}(\mathcal{K})$. It follows

$$\min_w w(S) = \sum_{T \subseteq S} d_w(T) = \sum_{T \in \mathcal{C}(\mathcal{K}), T \subseteq S} \Delta_T + \sum_{T \subseteq S, c_{\mathcal{K}}(T) \not\subseteq S} d_w(T). \tag{9}$$

In other words, if $T \in \mathcal{C}(\mathcal{K})$ satisfies $T \subseteq S$, the distribution of Δ_T among $\mathcal{C}(T)$ does not affect the value of S. For the rest of the coalitions, since $c_{\mathcal{K}}(T) \not\subseteq S$, we can choose (N, w^*) satisfying $d_w^*(T) = 0$ and $d_w^*(c_{\mathcal{K}}(T)) = \Delta_T$, attaining the minimum. □

The main consequence of Proposition 2 that we present here is on the structure of extreme points of the set of $\mathbb{P}^n$-extensions. Recall $\Delta_{\mathcal{K}}(v)$ from (7) and denote $E(\Delta_{\mathcal{K}}(v))$ the set of its extreme points. We show that for every extreme point of $\Delta_{\mathcal{K}}(v)$, there is a set of extreme points of $\mathbb{P}^n_{\mathcal{K}}(v)$, corresponding to elements in

$$\mathcal{E} = \left\{ \{T_1, \dots, T_{|\mathcal{C}(\mathcal{K})|}\} \middle| c_{\mathcal{K}}(T_i) \neq c_{\mathcal{K}}(T_j) \text{ and } c_{\mathcal{K}}(T_i) \in \mathcal{C}(\mathcal{K}) \right\}. \tag{10}$$

Definition 4. *Let* $\Delta \in E(\Delta_{\mathcal{K}}(v))$ *and* $e \in \mathcal{E}$*.* Extreme game $(N, v_{\Delta,e})$ *is defined as*

$$d_{v_{\Delta,e}}(T) = \begin{cases} \Delta_{c_{\mathcal{K}}(T)} & T \in e, \\ 0 & \textit{otherwise}. \end{cases} \tag{11}$$

Proposition 5. *Let* $(N, \mathcal{K}, v)$ *be a* $\mathbb{P}^n$*-extendable incomplete game. The extreme points of* $\mathbb{P}^n_{\mathcal{K}}(v)$ *are extreme games* $(N, v_{\Delta,e})$ *for* $\Delta \in E(\Delta_{\mathcal{K}}(v))$ *and* $e \in \mathcal{E}$*.*

Proof. First, we show that any extreme point (N, w) corresponding to some $\Delta \in \Delta_{\mathcal{K}}(v)$ has to be $(N, v_{\Delta,e})$ for some $e \in \mathcal{E}$. This follows from Theorem 1, as $\mathcal{N}(w)$ excludes at least one coalition from every $\mathcal{C}(S)$, $S \in \mathcal{K}$ with $\Delta_S > 0$, however, until it excludes exactly one from every such $\mathcal{C}(S)$, there is $(N, v_{\Delta,e})$, which satisfies $\mathcal{N}(w) \subsetneq \mathcal{N}(v_{\Delta,e})$.

Second, assume there is an extreme point $(N, v_{\Delta,e})$ for which Δ is not an extreme point of $\Delta_{\mathcal{K}}(v)$, i.e. $\Delta = \alpha\Delta_1 + (1-\alpha)\Delta_2$ for some $\alpha \in [0, 1]$. However, this means $v_{\Delta,e} = \alpha v_{\Delta_1,e} + (1-\alpha)v_{\Delta_2,e}$ is not an extreme point of $\mathbb{P}^n_{\mathcal{K}}(v)$, a contradiction.

It remains to show $(N, v_{\Delta,e})$ for every $\Delta \in E(\Delta_{\mathcal{K}}(v))$, $e \in \mathcal{E}$ is an extreme point. Considering again Theorem 1, if $(N, v_{\Delta,e})$ is not an extreme point, it means there is (N, w) with $\mathcal{N}(v_{\Delta,e}) \subsetneq \mathcal{N}(w)$. This means, the corresponding Δ^w has to have one $S \in \mathcal{C}(\mathcal{K})$ for which $\Delta^w_S = 0$, while $\Delta_S \neq 0$. One can show, following the proof of Theorem 1, a similar result with sets of negligible coalitions of games in $\mathbb{P}^n_{\mathcal{K}}(v)$ holds for elements in $\Delta_{\mathcal{K}}(v)$. As a result, Δ^w would serve as a witness that $\Delta \notin E(\Delta_{\mathcal{K}}(v))$ is not an extreme point, which contradicts our assumption. Details of the last part of this proof can be found in Appendix A. □

4. Conclusions

Our two-stage dividend-assignment procedure for describing the set of positive extensions holds promise for exploration along multiple dimensions. From a theoretical standpoint, refining specific structures of $\mathcal{K}$ and applying our two-stage procedure can yield stronger outcomes than those presented here. Especially illustrative is the *player-centered* [4] or *intersection-closed* [8] structure of $\mathcal{K}$. The results obtained in those settings are immediate consequences of the general approach presented here.

From a computational perspective, the methods proposed can significantly accelerate algorithms whenever $|\mathcal{K}|$ and $|\mathcal{C}(\mathcal{K})|$ remain polynomial in size. Under these conditions, key tasks such as testing extendability or computing the lower bound can be performed in polynomial time, thus improving on previously best-known approaches running in exponential time in the number of players. This advantage may be especially relevant in areas like characteristic function learning [11], which has broad applications in submodular optimization [12], auctions [13], and machine learning [14].

Looking ahead, our insights can also help to derive values akin to the *R-value* [3] or the *UD-value* [8], as well as other variants of classical solution concepts. An interesting direction for future research would also be to adapt similar two-stage procedures to other classes of extensions—e.g., convex, superadditive, or more specialized OR extensions mentioned in the introduction.

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Appendix A. Proposition 5 – details of the proof

In the proof of this proposition, it was briefly mentioned that a result similar to Theorem 1 holds for the extreme points of the set $\Delta_{\mathcal{K}}(v)$. To formally show this, we need the following lemma.

Lemma 6. *[15] Let P be a polyhedron given by*

$$P := \left\{ x \in \mathbb{R}^k \,\middle|\, \sum_{j=1}^{k} a_j^t\, x_j \;\geq\; b^t \textit{ for } t = 1, \ldots, m \right\}.$$

For $x \in P$, define

$$S(x) := \big\{ t \in \{1, \ldots, m\} \;\big|\; \sum_{j=1}^{k} a_j^t\, x_j \;=\; b^t \big\}.$$

Then $x \in P$ is an extreme point of P if and only if the system of linear equations

$$\sum_{j=1}^{k} a_j^t\, y_j \;=\; b^t \quad \textit{for all } t \in S(x)$$

has x as its unique solution.

Applying Lemma 6 to

$$\Delta_{\mathcal{K}}(v) = \left\{ \Delta \in \mathbb{R}_{\geq 0}^{|\mathcal{C}(\mathcal{K})|} \mid \forall S \in \mathcal{K}, \sum_{T \in \mathcal{C}(\mathcal{K}), T \subseteq S} \Delta_T = v(S) \right\}.$$

yields $\Delta \in \Delta_{\mathcal{K}}(v)$ is an extreme point if and only if it is the only element of $\Delta_{\mathcal{K}}(v)$ with $\mathcal{N}(\Delta) = \{S \in \mathcal{C}(\mathcal{K}) \mid \Delta_S = 0\}$. Now the last step is to show that the uniqueness of $\mathcal{N}(\Delta)$ among all $\Delta \in \Delta_{\mathcal{K}}(v)$ is equivalent to the following lemma.

Lemma 7. *Vector $\Delta \in \Delta_{\mathcal{K}}(v)$ is its extreme point if and only if there is no $\Delta^w \in \Delta_{\mathcal{K}}(v)$ such that $\mathcal{N}(\Delta) \subsetneq \mathcal{N}(\Delta^w)$.*

Proof. First, assume Δ is an extreme point and Δ^w exists. Consider $\Delta^\alpha = \alpha\Delta + (1-\alpha)\Delta^w$ for any $\alpha \in [0,1]$. It is immediate to see that $\Delta^\alpha \in \Delta_{\mathcal{K}}(v)$ and $\mathcal{N}(\Delta^\alpha) = \mathcal{N}(\Delta)$, a contradiction.

Second, if Δ is not an extreme point, there is Δ^y with $\mathcal{N}(\Delta) = \mathcal{N}(\Delta^y)$. By selecting a specific combination of $\Delta^\beta = \Delta - \beta(\Delta^y - \Delta)$, we construct Δ^β with $\mathcal{N}(\Delta) \subsetneq \mathcal{N}(\Delta^\beta)$. It is sufficient to choose

$$\beta = \min_{\substack{T \notin \mathcal{N}(\Delta) \\ \Delta(T) \neq \Delta^y(T)}} \frac{\Delta(T)}{\Delta^y(T) - \Delta(T)}.$$

Now $T \in \mathcal{N}(\Delta^\beta)$, while $T \notin \mathcal{N}(\Delta)$. □

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