LAK tutorials 4

Martin Černý

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To pass the tutorials, you have to attend all tutorials and submit (solve) at least 50% of homework assignments. There are two types of homework assignments:

1. A-type: You have to finish this assignment if you do not attend the tutorials,
2. Star-type: Throughout the semester, you have to submit (solve) at least 50% of these homework assignments.

Please, submit your solutions in PDF format to cerny@kam.mff.cuni.cz.

A-type assignment

Exercise 1

Suppose $A_1, \dots, A_m$ are subsets of $\{1, \dots, n\}$ such that

- $|A_i| \not\equiv 0 \mod q$ for every i,
- $|A_i \cap A_j| \equiv 0 \mod q$ for every $i \neq j$.

and let $q = p_1 \cdot p_2 \cdot \ \dots \ \cdot p_k$, where $p_1, \dots, p_k$ are primes. Show that there is $c(q) \in \mathbb{N}$ such that $m \leq c(q) \cdot n$.

Star-type assignment

Exercise 2

Suppose $A_1, \dots, A_m$ are subsets of $\{1, \dots, n\}$ such that

- $|A_i| \not\equiv 0 \mod q$ for every i,
- $|A_i \cap A_j| \equiv 0 \mod q$ for every $i \neq j$.

Show that for every q, there is $c(q) \in \mathbb{N}$ such that $m \leq c(q) \cdot n$.