LAK tutorials 3

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To pass the tutorials, you have to attend all tutorials and submit (solve) at least 50% of homework assignments. There are two types of homework assignments:

1. A-type: You have to finish this assignment if you do not attend the tutorials,
2. Star-type: Throughout the semester, you have to submit (solve) at least 50% of these homework assignments.

Please, submit your solutions in PDF format to cerny@kam.mff.cuni.cz.

A-type assignment

Exercise 1

Suppose $A_1, \dots, A_k$ are subsets of $\{1, \dots, n\}$ such that

- $|A_i| \equiv 1 \mod 2$ for every i,
- $|A_i \cap A_j| \equiv 1 \mod 2$ for every $i \neq j$.

Show that

1. $k \leq 2^{\lfloor (n-1)/2 \rfloor}$,
2. show the bound is tight.

Hint. Use the results on Even-Even towns from the lecture.

Star-type assignment

Exercise 2

Let $m_s(n)$ be the maximal s-distance set in n-dimensional Euclidean space. Show that

$$\binom{n+1}{s} \leq m_s(n) \leq \binom{n+2s-1}{s}.$$

Hint. Follow the proof for bounds on $m_2(n)$.