LAK tutorials 2

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October 14, 2022

To pass the tutorials, you have to attend all tutorials and submit (solve) at least 50% of homework assignments. There are two types of homework assignments:

1. A-type: You have to finish this assignment if you do not attend the tutorials,
2. Star-type: Throughout the semester, you have to submit (solve) at least 50% of these homework assignments.

Please, submit your solutions in PDF format to cerny@kam.mff.cuni.cz.

A-type assignment

Exercise 1

Show that the number of spanning trees of a complete graph K_n is $n^{(n-2)}$.

Exercise 2

Suppose $A_1, \dots, A_k$ are subsets of $\{1, \dots, n\}$ such that

- $|A_i| \equiv 0 \mod 2$ for every i,
- $|A_i \cap A_j| \equiv 1 \mod 2$ for every $i \neq j$.

Show that

1. $k \leq n + 1$,
2. if n is odd, then $k \leq n$.

Hint. Use the results on EvenOddtons.

Star-type assignment

Suppose $A_1, \dots, A_k$ are subsets of $\{1, \dots, n\}$ such that

- $|A_i| \equiv 0 \mod 2$ for every i,
- $|A_i \cap A_j| \equiv 1 \mod 2$ for every $i \neq j$.

Show that if n is even, then $k \leq n - 1$.
Hint. Use rank of $J_n - I_n$, where J_n is matrix of all 1s.