LAK tutorials 1

Martin Černý

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To pass the tutorials, you have to attend all tutorials and submit (solve) at least 50% of homework assignments. There are two types of homework assignments:

1. A-type: You have to finish this assignment if you do not attend the tutorials,
2. Star-type: Throughout the semester, you have to submit (solve) at least 50% of these homework assignments.

Please, submit your solutions in PDF format to cerny@kam.mff.cuni.cz.

A-type assignment

Since these were the first tutorials, the assignment is non-mandatory even for those, who did not attend the tutorials.

Exercise 1

For $A \in \mathbb{R}^{m\times n}, B \in \mathbb{R}^{n\times p}$ show the following:

1. $rank(AB) \leq \min\{rank(A), rank(B)\}$,
2. $rank(A+B) \leq rank(A) + rank(B)$.

Exercise 2

Let $x_1, \ldots, x_k \in \{0,1\}^n$. Show that they are linearly independent in $\mathbb{R}^n$ over $\mathbb{R}$ if and only if they are linearly independent in $\mathbb{R}^n$ over $\mathbb{Q}$.

Star-type assignment

Exercise 3

Let $x_1, \ldots, x_k \in \{0,1\}^n$. Prove or disprove the following:

- Linear independence of $x_1, \ldots, x_k$ in $\mathbb{Z}_p^n$ over $\mathbb{Z}_p \implies$ linear indepedence of $x_1, \ldots, x_k$ in $\mathbb{R}^n$ over $\mathbb{R}$.
- Linear independence of $x_1, \ldots, x_k$ in $\mathbb{R}^n$ over $\mathbb{R} \implies$ linear indepedence of $x_1, \ldots, x_k$ in $\mathbb{Z}_p^n$ over $\mathbb{Z}_p$.