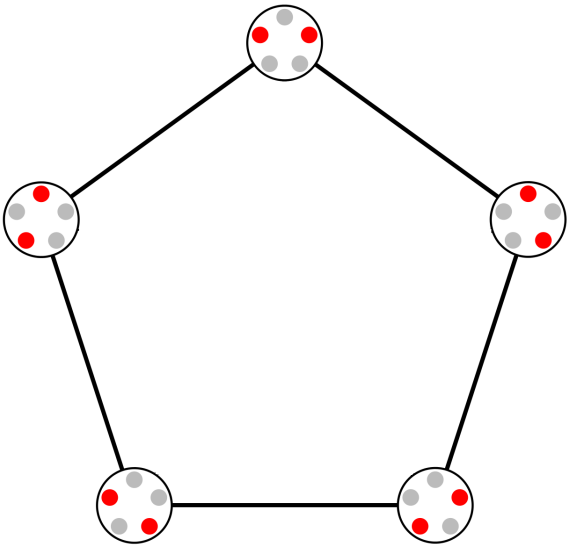
Topological methods in combinatorics

Martin Balko

5th lecture

March 25th 2022

Schrijver's graphs



Source: https://en.wikipedia.org (modified)

Schrijver's graphs

Schrijver's graphs

- We prove a strengthening of the Lovász–Kneser theorem for graphs that are vertex critical and yet have the same chromatic number as Kneser's graphs.

Figure: Alexander Schrijver (born 1948) and Imre Bárány (born 1947).

Sources: https://en.wikipedia.org and https://commons.wikimedia.org/

Schrijver's graphs

- We prove a strengthening of the Lovász–Kneser theorem for graphs that are vertex critical and yet have the same chromatic number as Kneser's graphs.

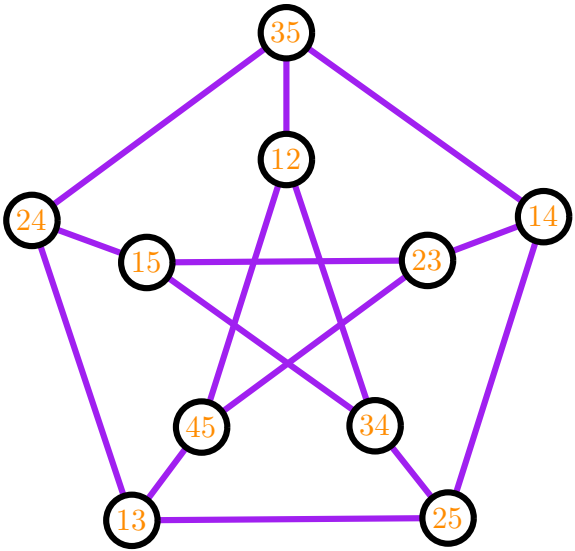
Figure: Alexander Schrijver (born 1948) and Imre Bárány (born 1947).

Sources: https://en.wikipedia.org and https://commons.wikimedia.org/

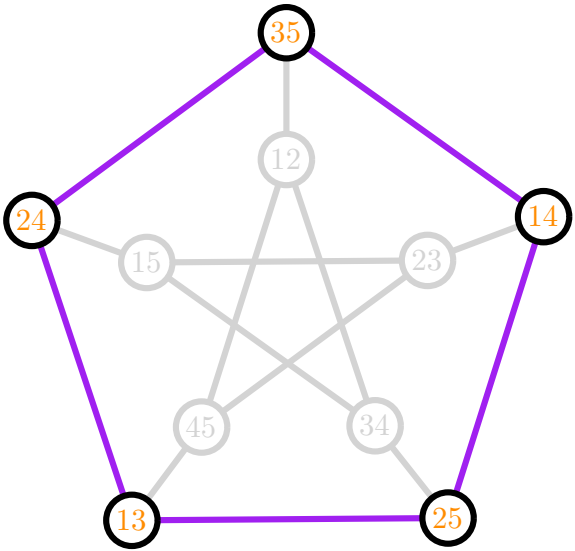
- We employ a third proof of the Lovász–Kneser theorem discovered by Bárány.

Example: Schrijver's graph $SG_{5,2} \subseteq KG_{5,2}$

Example: Schrijver's graph $SG_{5,2} \subseteq KG_{5,2}$

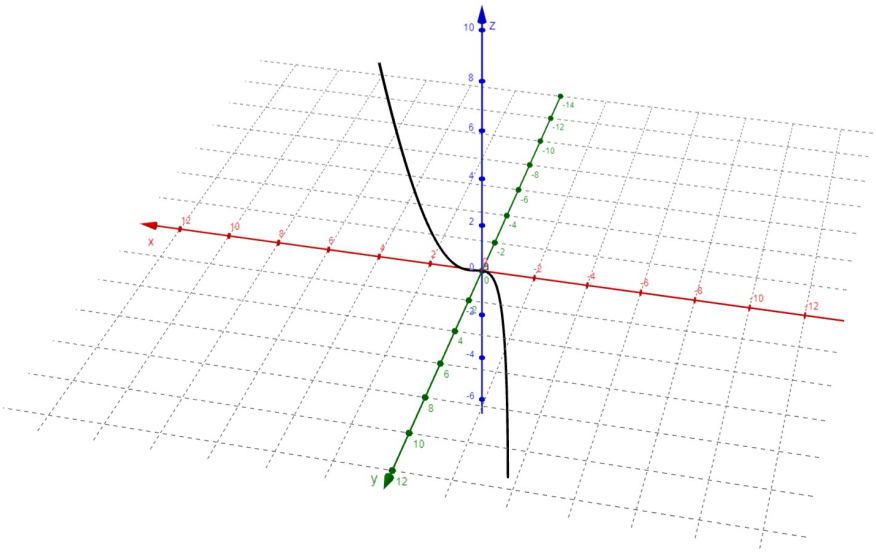


Example: Schrijver's graph $SG_{5,2} \subseteq KG_{5,2}$



The moment curve in $\mathbb{R}^3$

The moment curve in $\mathbb{R}^3$



Source: https://www.geogebra.org/m/H4HPtX2P

Gale's lemma

Gale's lemma

- For all $d \geq 1$ and $k \geq 1$, there exists a set $X \subset S^d$ of $2k + d$ points such that every open hemisphere of S^d contains at least k points of X.

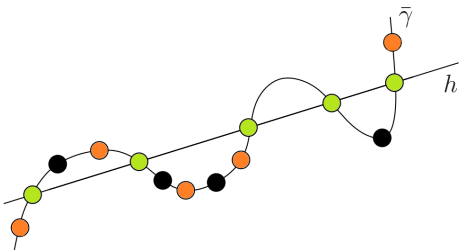


Figure: David Gale (1921–2008)

Sources: https://en.wikipedia.org and Matoušek: Using the Bosuk–Ulam theorem (colored)

The Ham sandwich theorem

Source: https://www.seekpng.com/

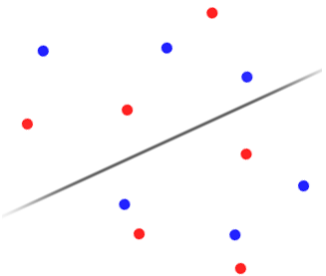
The Ham sandwich theorem

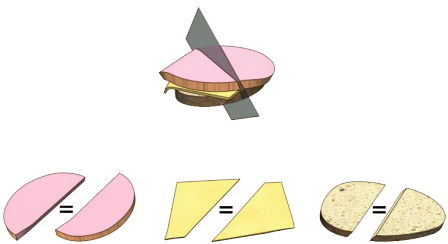
The Ham sandwich theorem

- Given finite sets $A_1, \ldots, A_d$ of points in $\mathbb{R}^d$, there is a hyperplane H that contains at most $\lfloor |A_i|/2 \rfloor$ points from each set A_i in each open halfspace determined by H.

The Ham sandwich theorem

- Given finite sets $A_1, \ldots, A_d$ of points in $\mathbb{R}^d$, there is a hyperplane H that contains at most $\lfloor |A_i|/2 \rfloor$ points from each set A_i in each open halfspace determined by H.

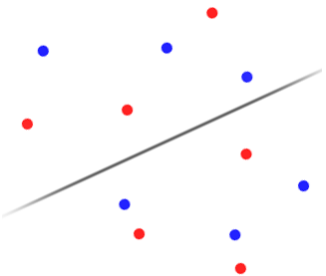


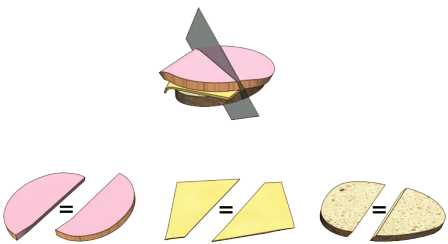


Sources: https://ejarzo.github.io and https://curiosamathematica.tumblr.com

The Ham sandwich theorem

- Given finite sets $A_1, \ldots, A_d$ of points in $\mathbb{R}^d$, there is a hyperplane H that contains at most $\lfloor |A_i|/2 \rfloor$ points from each set A_i in each open halfspace determined by H.

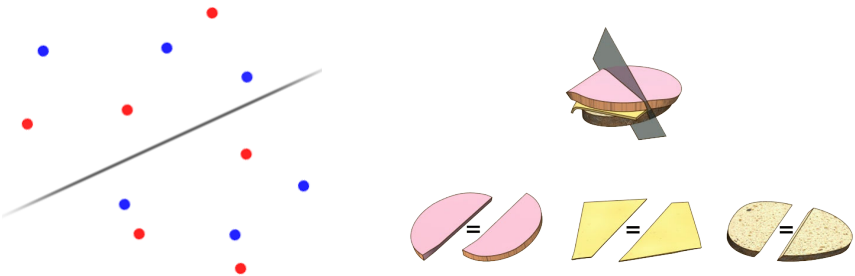




Sources: https://ejarzo.github.io and https://curiosamathematica.tumblr.com

The Ham sandwich theorem

- Given finite sets $A_1, \ldots, A_d$ of points in $\mathbb{R}^d$, there is a hyperplane H that contains at most $\lfloor |A_i|/2 \rfloor$ points from each set A_i in each open halfspace determined by H.



Sources: https://ejarzo.github.io and https://curiosamathematica.tumblr.com

- We first state it for finite Borel measures.

Humble ham Sandwich...

source: https://www.pinterest.pt/

Humble ham Sandwich...

source: https://www.pinterest.pt/

Thank you for your attention.