Bounding the pseudolinear crossing number of K_n via simulated annealing

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Charles University in Prague,
Czech Republic

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Graph drawings

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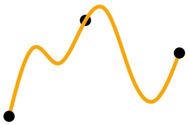

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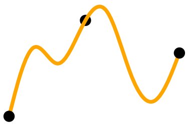
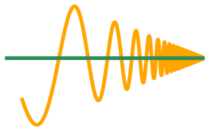


Passing through vertices



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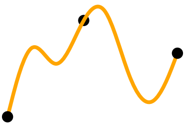

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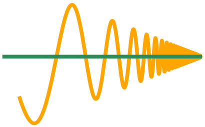

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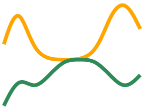

Edges touching

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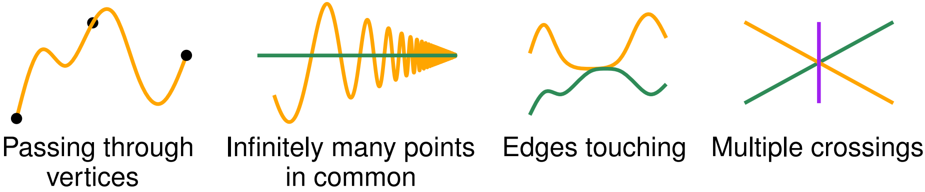

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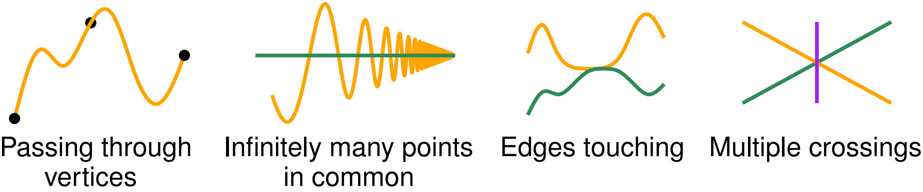

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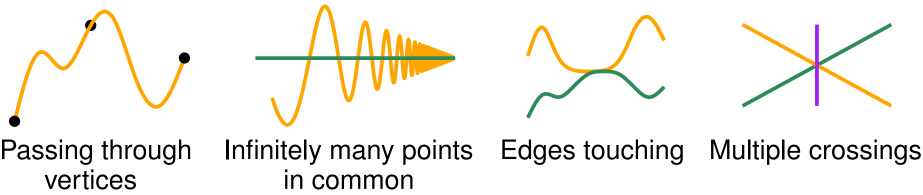

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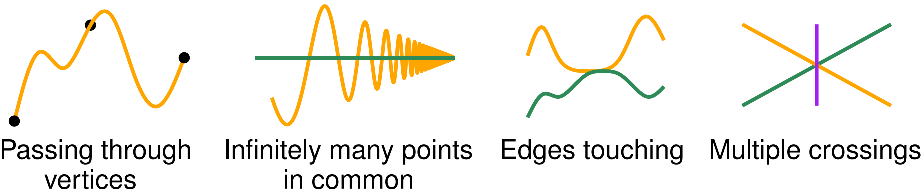

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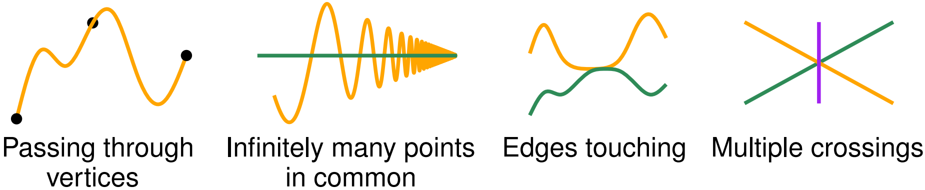

- A drawing is pseudolinear if its edges can be extended to form an arrangement of pseudolines.
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- We assume that all pseudolinear drawings are x-monotone.

Crossing numbers

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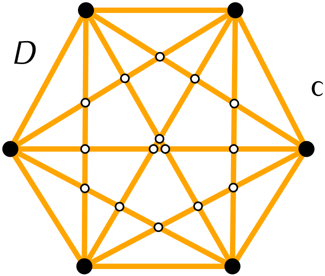

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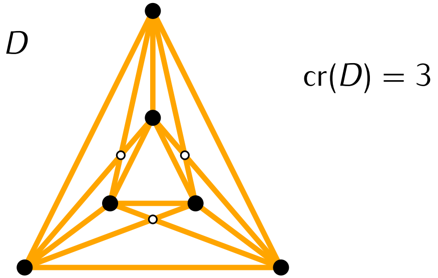

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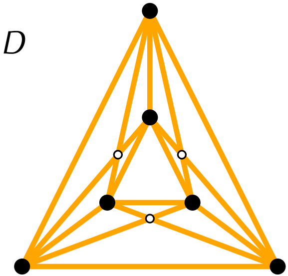

$\mathrm{cr}(D) = 3$

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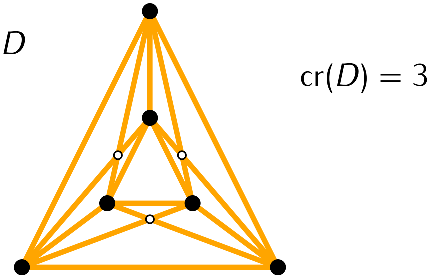

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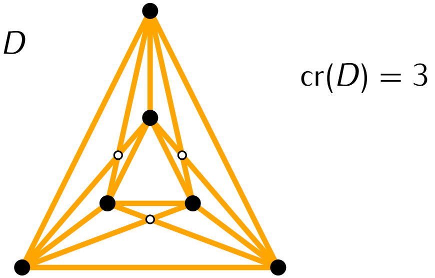

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- Rectilinear crossing number $\overline{\mathrm{cr}}(G)$ is $\min \mathrm{cr}(D)$ over rectilinear D.
- We have $\widetilde{\mathrm{cr}}(G) \leq \overline{\mathrm{cr}}(G)$ for every G.

Bounds for $\overline{\mathrm{cr}}(K_n)$ and $\widetilde{\mathrm{cr}}(K_n)$

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- The current best lower bound: $\overline{\text{cr}}(K_n) \geq \widetilde{\text{cr}}(K_n) > 0.379972\binom{n}{4} - O(n^3)$ [Ábrego, Cetina, Fernández-Merchant, Leaños, Salazar (2012)]

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  - $\overline{\text{cr}}(K_n) < 0.380559\binom{n}{4} + O(n^3)$ [Ábrego, Fernández-Merchant (2007)]

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- All upper bounds on $\widetilde{\text{cr}}(K_n)$ follow from upper bounds on $\overline{\text{cr}}(K_n)$.

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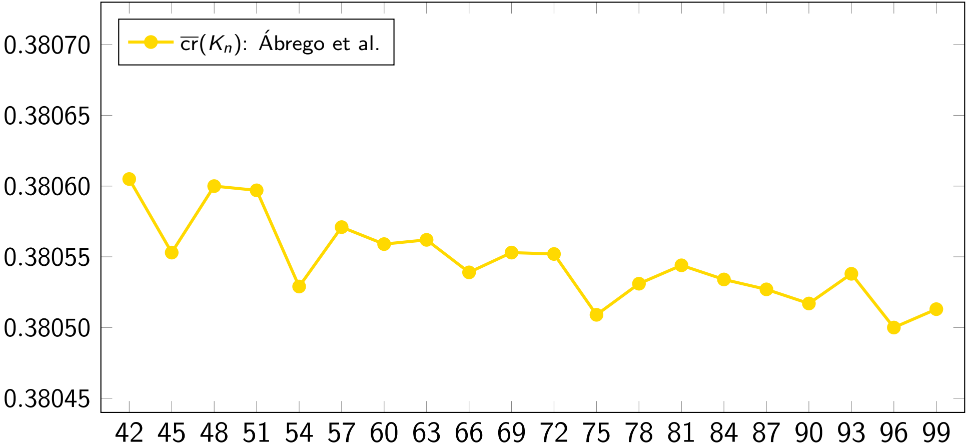

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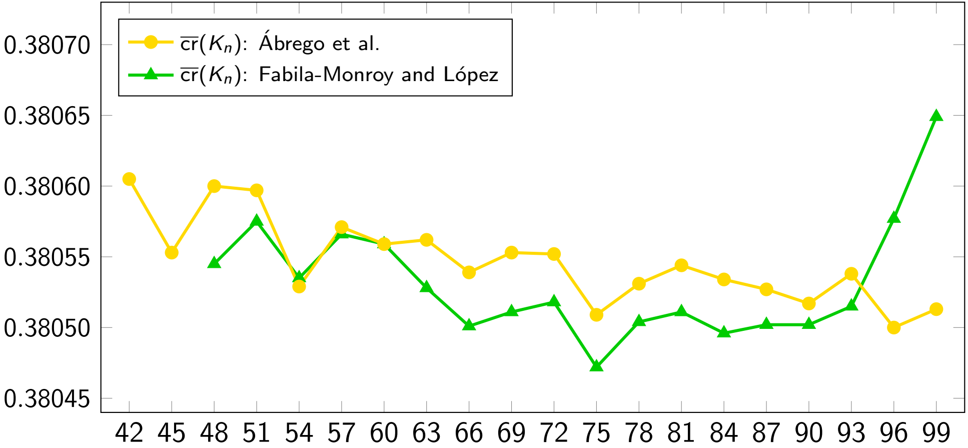

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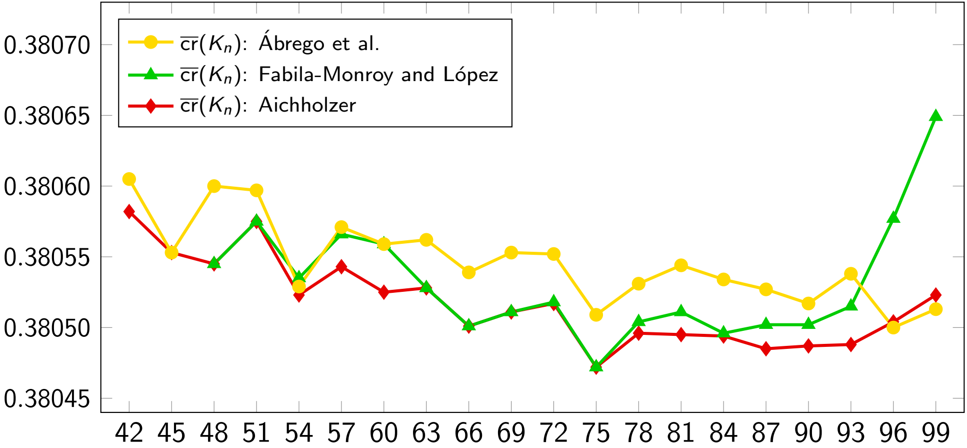

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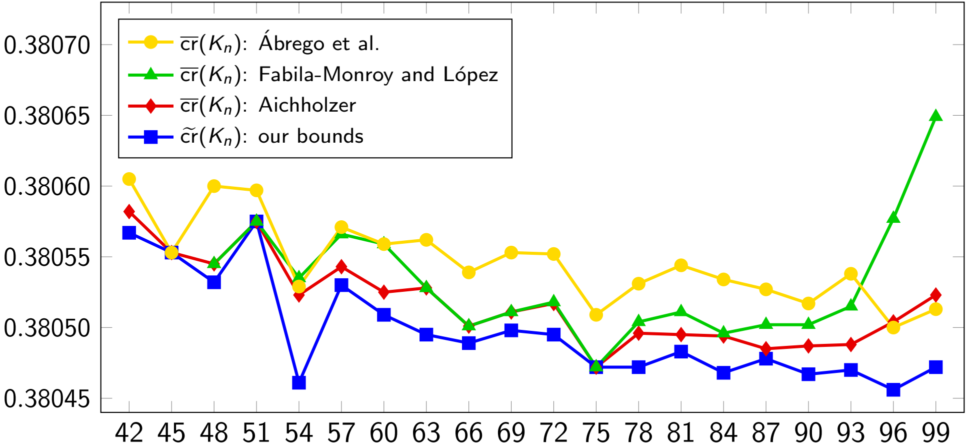

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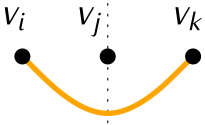

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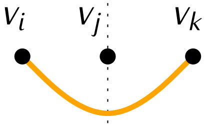

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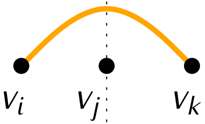

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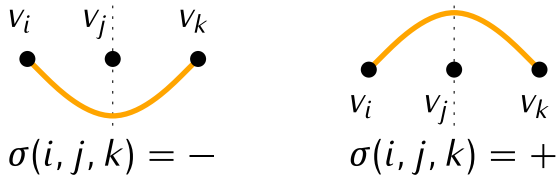

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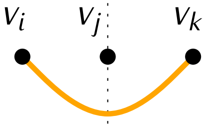

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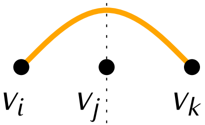

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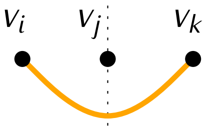

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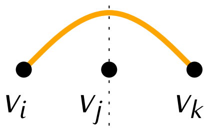

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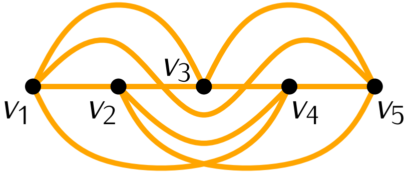

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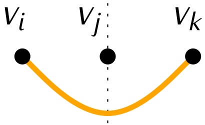

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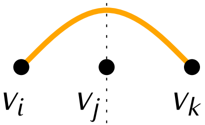

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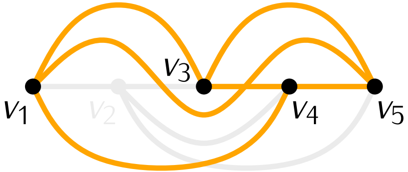

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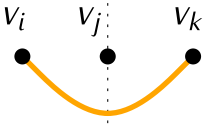

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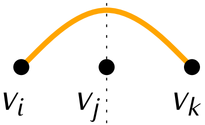

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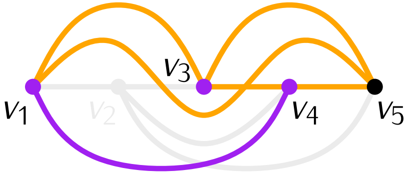

$-$

$\sigma(1,3,4) = -$

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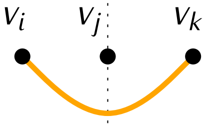

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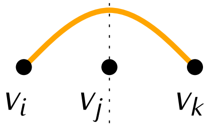

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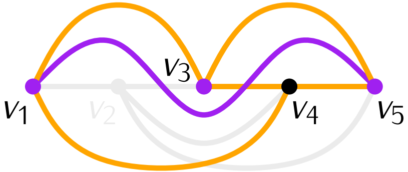

$- -$

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Representation of pseudolinear drawings of K_n I

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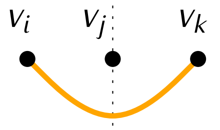

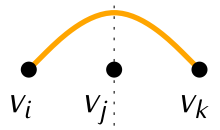

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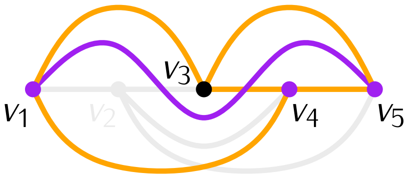

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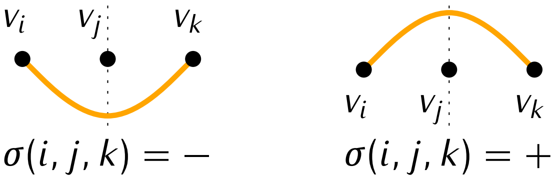

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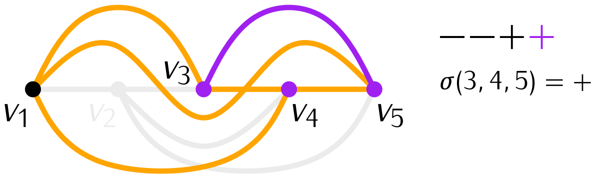

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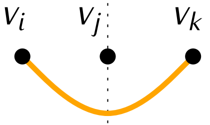

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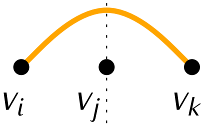

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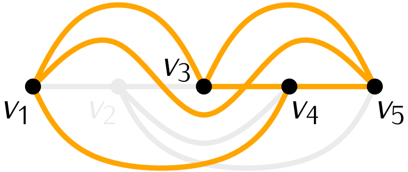

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An n-signature σ is realizable if and only if every 4-tuple from $\binom{[n]}{4}$ is of one of the forms $++++$, $+++-$, $++--$, $+---$, $----$, $---+$, $--++$, $-+++$ in σ.

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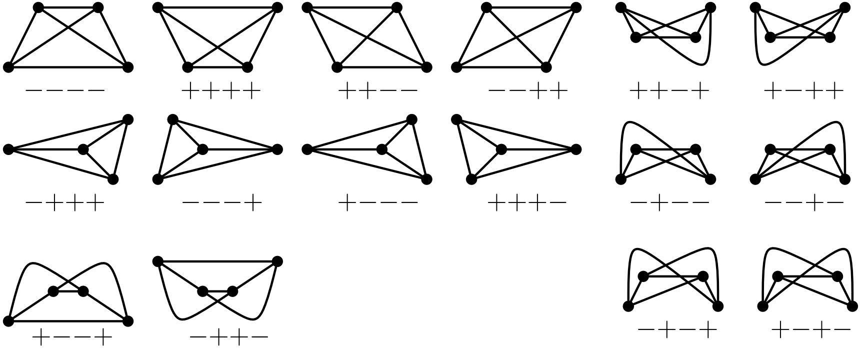

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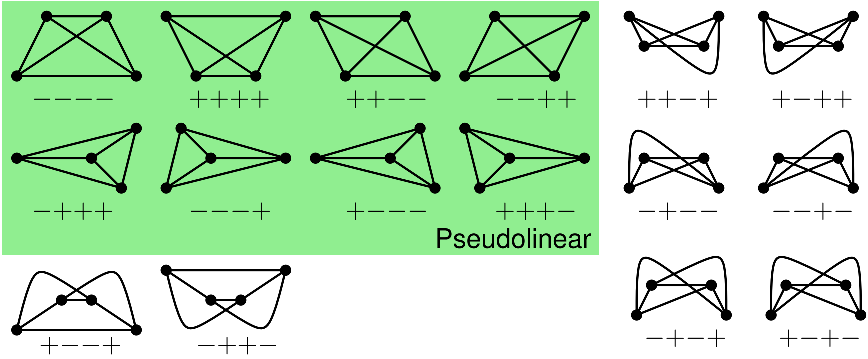

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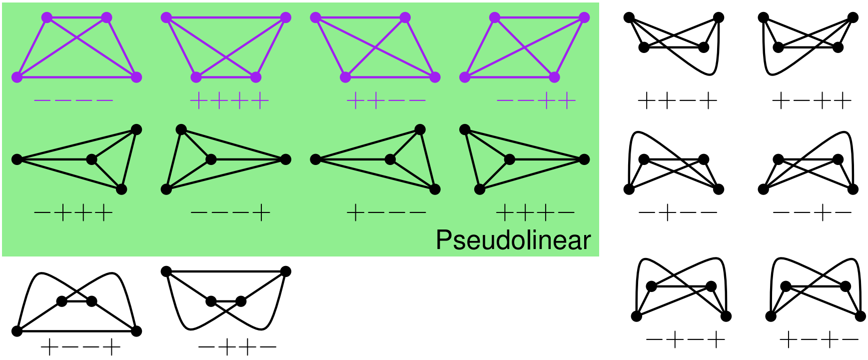

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n	Previously best	Currently best
42	40 590	40 588
44	49 370	49 366
46	59 463	59 459
48	71 010	71 007
50	84 223	84 219
52	99 161	99 158
54	115 975	115 953
56	134 917	134 901
57	145 164	145 158
58	156 042	156 040
59	167 506	167 490
60	179 523	179 514
63	219 659	219 637
64	234 447	234 441
65	249 962	249 938
66	266 151	266 142
67	283 238	283 230
68	301 057	301 043
69	319 691	319 679
70	339 252	339 241
71	359 645	359 635
72	380 925	380 900

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73	403 180	403 166
74	426 398	426 391
76	475 773	475 758
77	502 011	501 997
78	529 278	529 242
79	557 741	557 723
80	587 280	587 251
81	617 930	617 908
83	682 976	682 958
84	717 276	717 222
85	752 971	752 963
86	789 911	789 892
87	828 125	828 107
88	867 887	867 862
89	908 940	908 914
90	951 379	951 323
91	995 478	995 430
92	1 040 946	1 040 897
93	1 087 899	1 087 843
94	1 136 586	1 136 565
96	1 238 646	1 238 490
99	1 404 552	1 404 386

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$$\mathrm{cr}(D') = 16\,\mathrm{cr}(D) + 2n_0\left(\left\lceil \frac{n_0}{2} \right\rceil^2 + \left\lfloor \frac{n_0}{2} \right\rfloor^2\right) - \frac{7n_0^2}{2} + \frac{5n_0}{2}.$$

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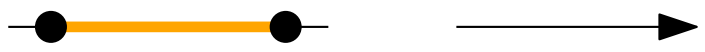
a) 

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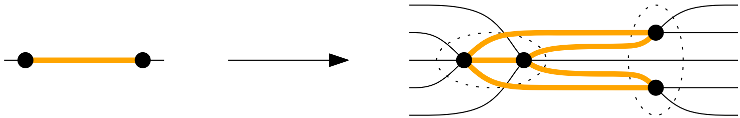

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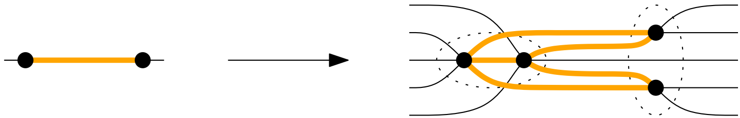
a) 

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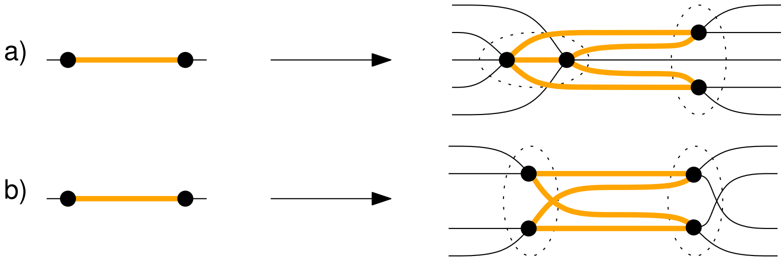

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