Algorithmic game theory — Homework 2[1]
Nash equilibria

assigned 4.11.2024, deadline 18.11.2024

Homework 1. *Draw the best response polyhedron and normalized best response polytope for the Battle for Gotham's soul. Then, find all completely labeled pairs of vertices that correspond to Nash equilibria.* **[4]**

	Cooperate (3)	Detonate (4)
Cooperate (1)	$(1, 1)$	$(1, 2)$
Detonate (2)	$(2, 1)$	$(1, 1)$

Table 1: Battle for Gotham's soul.

Homework 2. *Use the Lemke–Howson algorithm and compute a Nash equilibrium of the following bimatrix game:* **[3]**

$$M = \begin{pmatrix} 0 & 12 \\ 1 & 4 \\ 3 & 0 \end{pmatrix} \quad a \quad N = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 3 \end{pmatrix}.$$

Start by choosing the label 1.

Homework 3. *[Sperner's Lemma] Let S be a given subdivision of a triangle T in the plane. A* legal coloring *the vertices of S assigns one of three colors (red, blue, and green) to each vertex of S such that all the three colors are used on the vertices of T. Moreover, a vertex of S lying on an edge of T must have one of the two colors of the endpoints of this edge.*

Prove that, in every legal coloring of S, there is is a triangular face of S whose vertices are colored with all three colors.

Hint: Use a reduction to the END-OF-THE-LINE problem. **[3]**

[1]Information about the course can be found at http://kam.mff.cuni.cz/~balko/