

Algorithmic game theory

Martin Balko

10th lecture

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Mechanism design basics

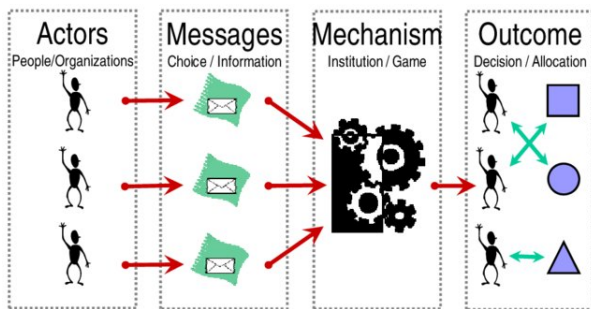
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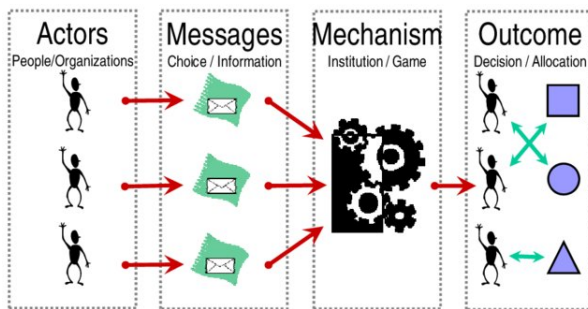
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Source: Innovations in Defense Acquisition: Asymmetric Information and Incentive Contract Design

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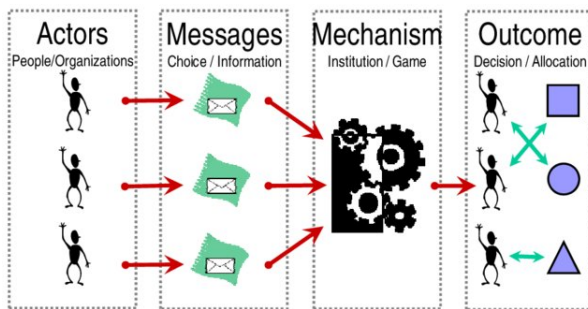


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- We start with **single item auctions**.
- We then extend these desired properties to a more general setting of **single-parameter environments** using so-called **Myerson's lemma**.

Single item auctions



Source: <https://www.widewalls.ch>

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- To do so, **we need to appropriately implement the rules for the seller** how to decide the winner and the selling price.

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 - **Strong performance guarantees**: If all bidders bid truthfully then the auction maximizes the social surplus.
 - **Computational efficiency**: The auction can be implemented in polynomial time.

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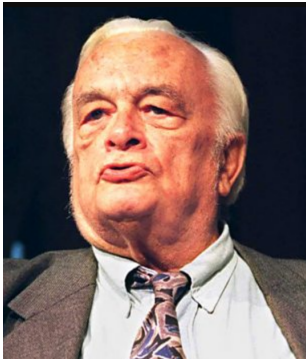


Figure: William Vickrey (1914–1996).

Sources: <https://en.wikipedia.org> and <https://ichef.bbc.co.uk/>

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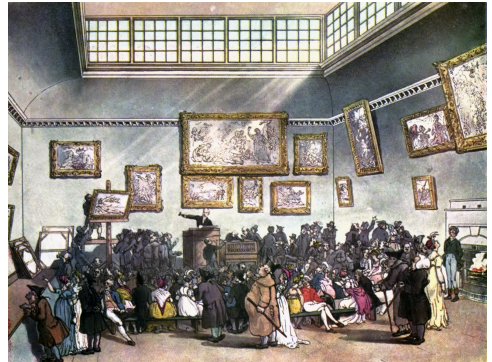
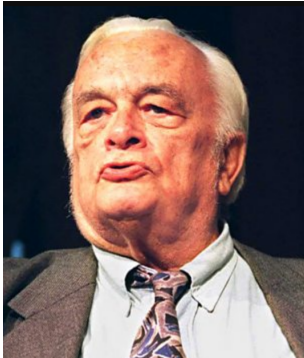


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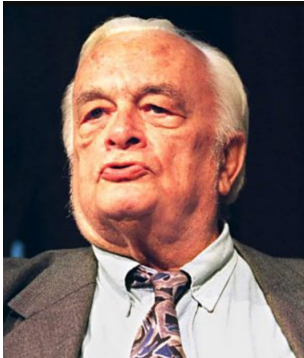


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- Vickrey posthumously received a **Nobel prize** in Economic Sciences.

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- We now illustrate single-parameter environments with a few specific examples.

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Source: <https://www.widewalls.ch>

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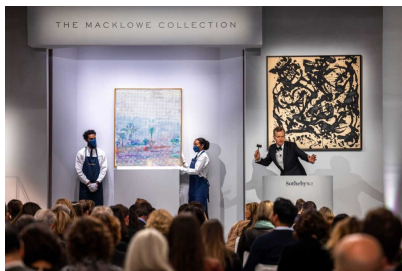


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We have **two assumptions**: first, the more the slot is on the top, the higher the probability α_j that the slot is clicked on, and, second, the click-through rates do not depend on the occupant of the slot.

Sponsored search

Sponsored search

web images videos maps news shopping local more

Google [Advanced Search](#)

Results 1 - 10 of about 9,420,000 for **water restoration**. (0.28 seconds)

Water Restoration [Sponsored Links](#)
[www.ChicagoWaterAndFire.com](#) Fast, Free Estimates 24/7 On **Water** Removal. Call Us Now @ 630.629.9000

Fire & Water Restoration
[www.PDRestoration.com](#) Fast 24/7 Fire & **Water** Remediation Service in Chicago - 888-898-3891

Local Flood Restoration
[SpectrumRestoration.com](#) Flood & **water restoration** services since 1995. Call us at 800-730-3656

SERVPRO - Fire & Water Cleanup & Restoration
When fire and **water** take control of your life, we help you take it back - Like it never even happened. SERVPRO specializes in the cleanup and **restoration** of ...
[Locate a Franchise - Careers - Contact/Directions](#)
[www.servpro.com/](#) - [Cached](#) - [Similar](#)

ICRC
According to the ICRC Standard and Reference Guide for Professional **Water** Damage **Restoration** (ICRC S500), there are three categories of **water** that cause ...
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RestorationSOS® - Water Damage & Fire Damage Restoration, Sewage ...
The Most Helpful **Restoration** Service in America - Affordable Services in Your Area - 24/7 Hotline.
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Arizona Water Restoration Fire Damage Mold Remediation Crime Scene ...
Helping Hand Fire and **Water restoration** emergency flood damage and smoke clean-up contractors offer mold remediation, sewage intrusion, crime scene cleanup ...
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Arizona Fire and Water Restoration Services: Mold and Mildew ...
Arizona Fire and **Water Restoration**, Inc. is a full-service contractor, specializing in insurance restoration. Their services encompass the assessment and ...
[www.azfirewater.com/](#) - [Cached](#) - [Similar](#)

Water Damage Local® - 24/7 Emergency Restoration Services
WaterDamageLocal.com offers 24/7 emergency services for **water** damage, flood damage, and sewage emergencies. All of our providers are pre-qualified, ...
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1-800 Water Damage Home
Locate a 1-800-WATER DAMAGE™ **restoration** service provider in your area. ... Our dispatchers and **water** damage **restoration** specialists are on-call 24 hours a ...
[www.1800waterdamage.com/](#) - [Cached](#) - [Similar](#)

Emergency Water Damage Restoration
Flood loss prevention, specially drying and **water** damage **restoration** ... **Water** Damage **Restoration** Emergency Service Complete Basement Clean up & Druud

Water Damage Repair Pros
Chicago Water Restoration Experts
Free Estimates, Call 312-970-0985.
[ChicagoWaterDamageRestoration.com](#)

Water Damage
Fast & Cost Effective Recovery
Serving Alsip IL. Call 800-589-8845
[bbsteamatic.com](#)

24/7 Water Damage Cleanup
Licensed, Insured & Trained Prof.
We Are Here To Help. Chicagoland.
[ServiceMaster-smgrs.com/WaterDamage](#)
Hoffman Estates, IL

Water Damage
Expert water damage **restoration**.
Call today for more information.
[servicemasterrestorationby.com](#)
Hoffman Estates, IL

Water Damage Cleanup
Professional **Water** Damage Company
Emergency **Water** Damage Help
[A1R-Restoration](#) Ad2Zoo.com

Water Damage
24-Hour **Water** & Fire **Restoration**.
Reliable Certified Techns. Call Now.
[www.AllFireFire.com](#)

Water Restoration
24-Hour Service. **Water**. Fire. Mold
Chicagoland Specialist Since 1986.
[www.ACRRestores.com](#)
Chicago, IL

Paul Davis Restoration
Water Fire Flood Clean up Repair
1-630-499-8002
[www.foodFireCleanup.com](#)
2740 Beverly Drive Unit C, Aurora, IL

Source: <https://proceedinnovative.com>

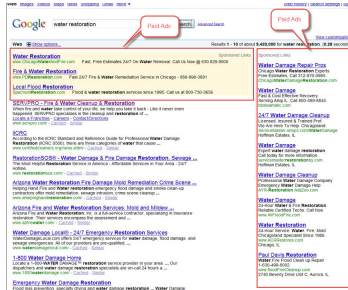
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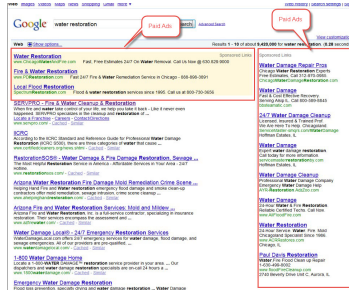


The image shows a screenshot of a Google search results page for the query "water restoration". The search bar at the top contains the text "water restoration" and "Paid Ads" labels are visible above the search bar and on the right side of the page. The search results are displayed in a grid format. The first result is "Water Restoration" from Chicago Water Restoration, Inc., with a phone number 312-353-3030. The second result is "Fire & Water Restoration" from Fire & Water Restoration Service in Chicago, with a phone number 800-800-3091. The third result is "Local Flood Restoration" from Flood & Water Restoration Services since 1995, with a phone number 800-730-3626. The fourth result is "SERVPRO - Fire & Water Cleanup & Restoration" from SERVPRO, with a phone number 800-475-6283. The fifth result is "ICRC" from International Council of Professional Water Damage Restorers (ICRC), with a phone number 800-475-6283. The sixth result is "Restoration320 - Water Damage & Fire Damage Restoration, Storage..." from Restoration320, with a phone number 800-475-6283. The seventh result is "Arizona Water Restoration, Fire Damage Mold Remediation Crime Scene..." from Arizona Water Restoration, with a phone number 800-475-6283. The eighth result is "Arizona Fire and Water Restoration Services, Mold and Mildew..." from Arizona Fire and Water Restoration Services, with a phone number 800-475-6283. The ninth result is "Water Damage Leads - 24/7 Emergency Restoration Services" from Water Damage Leads, with a phone number 800-475-6283. The tenth result is "1-800 Water Damage Home" from 1-800 Water Damage Home, with a phone number 800-475-6283. The eleventh result is "Emergency Water Damage Restoration" from Emergency Water Damage Restoration, with a phone number 800-475-6283. The twelfth result is "Water Damage Repair Pros" from Chicago Water Restoration, with a phone number 312-353-3030. The thirteenth result is "Water Damage" from Water Damage, with a phone number 800-475-6283. The fourteenth result is "24/7 Water Damage Cleanup" from 24/7 Water Damage Cleanup, with a phone number 800-475-6283. The fifteenth result is "Water Damage" from Water Damage, with a phone number 800-475-6283. The sixteenth result is "Water Damage Cleanup" from Professional Water Damage Company, with a phone number 800-475-6283. The seventeenth result is "Water Damage" from Water Damage, with a phone number 800-475-6283. The eighteenth result is "Water Restoration" from Water Restoration, with a phone number 800-475-6283. The nineteenth result is "Paul Davis Restoration" from Paul Davis Restoration, with a phone number 800-475-6283.

Source: <https://proceedinnovative.com>

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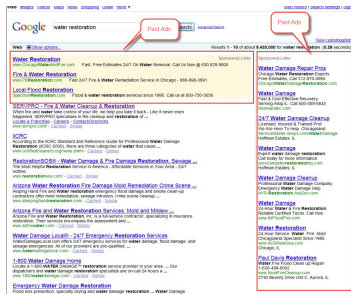


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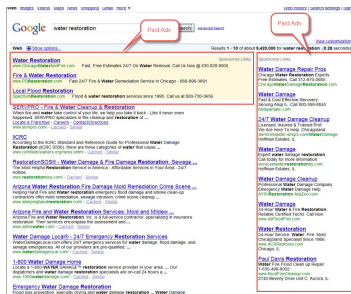
The image shows a Google search results page for the query 'water restoration'. At the top, the search bar contains 'water restoration' and the search button is labeled 'Paid Ads'. Below the search bar, the results are displayed. A red box highlights the 'Paid Ads' label above the first search result. Another red box highlights the search results themselves, which include several sponsored listings for water restoration services. The results are arranged in two columns, with the left column containing more results than the right. The sponsored results are clearly marked with 'Sponsored Links' and include titles, URLs, and snippets of text.

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Figure: **Roger Myerson** (born 1951) receiving a Nobel prize in economics.

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- We will see the proof next week, now we show some applications.

Applications of Myerson's lemma I

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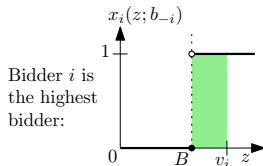
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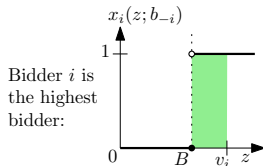
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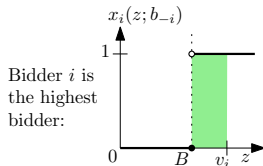
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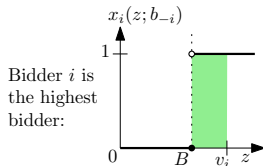


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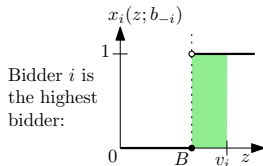


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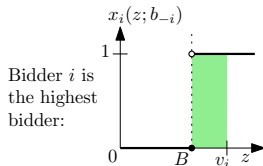


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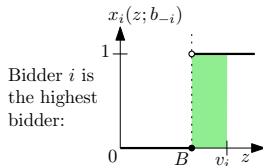


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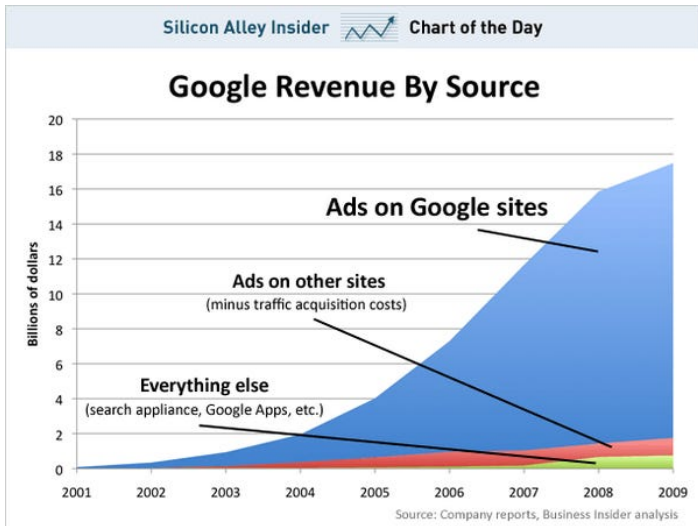
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- In general, for i th highest bidder, Myerson's lemma gives the payment formula (for $\alpha_{k+1} = 0$)

$$p_i(b) = \sum_{j=i}^k b_{j+1}(\alpha_j - \alpha_{j+1}).$$



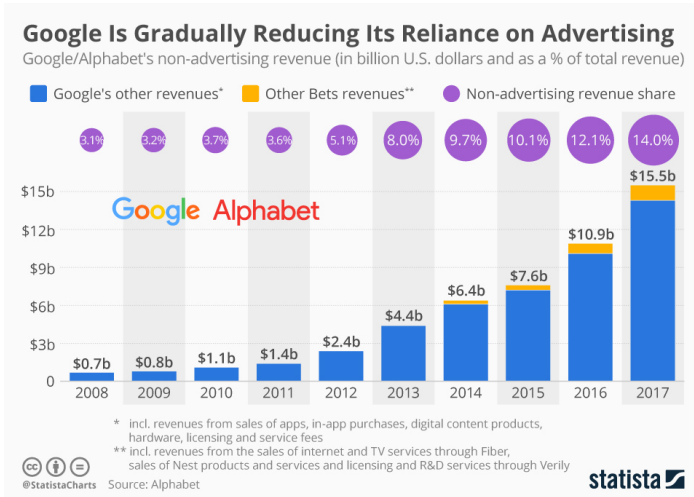
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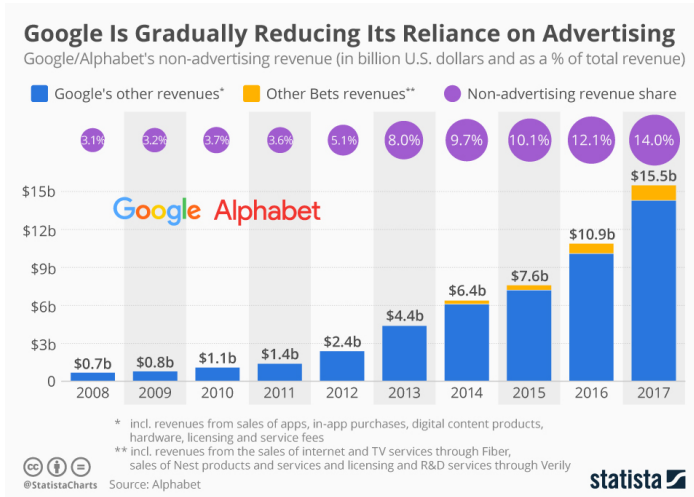
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Thank you for your attention.