Algorithmic game theory – Tutorial 8*

December 2, 2024

1 Games in extensive form

The *sequence form* of an imperfect-information game G is a 4-tuple (P, S, u, C) where P is a set of n players, $S = (S_1, \ldots, S_n)$, where S_i is a set of sequences of player i, $u = (u_1, \ldots, u_n)$, where $u_i \colon S \to \mathbb{R}$ is the payoff function of player i, and $C = (C_1, \ldots, C_n)$ is a set of linear constraints on the realization probabilities of player i.

The following result states how to use the sequence form to find NE in zero-sum extensive games.

Theorem 1. *The equilibria of a two-player zero-sum game in an extensive form of perfect recall are the solutions to the following linear program:*

$$\min_{u,y} e^\top u \text{ subject to } Fy = f, E^\top u - Ay \geq \mathbf{0}, y \geq \mathbf{0}.$$

And the following result states how to use the sequence form to find NE in 2-player extensive games.

Theorem 2. *A pair (x, y) of realization plans in a 2-player game in the extensive form of perfect recall is equilibrium if and only if there are vectors u and v such that the following conditions are satisfied:*

$$\begin{aligned} x^\top(E^\top u - Ay) = 0, \quad & y^\top(F^\top v - B^\top x) = 0, \\ Ex = e, x \geq \mathbf{0}, \quad & Fy = f, y \geq \mathbf{0}, \\ E^\top u - Ay \geq \mathbf{0}, \quad & F^\top v - B^\top x \geq \mathbf{0}. \end{aligned}$$

Exercise 1. *Assume that G is an extensive game of two players with perfect information represented by a d-regular tree of height $2m + 1$ where $d \geq 2$ and $m \geq 1$ are integers and where players alternate in moves. Let G' be a normal-form game obtained from G where the set of actions of each player consists of a set of all pure strategies in G'. How many (asymptotically) action profiles are there in G'?*

Exercise 2. *Consider the following simple Poker game of two players:*

1. *Player one is dealt either Ace or King.*
2. *Player two is dealt either Ace or King.*
3. *Pot (money on the table players are playing for) is two dollars.*
4. *Player one acts and can either bet one dollar or check.*
5. *Player two acts and can bet or check (if player one checked), or if player one bet, he can either call (even up the bet) or fold (give up).*
6. *Player one acts again only if he checked first and second player bet. He can either call or fold.*
7. *Player who folded or had worse card loses one dollar.*

Formulate this game as an imperfect information game of two players in extensive form with a chance player.

Exercise 3. *Construct an extensive form of the game Matching Pennies from Table 1 and write its sequence form and the linear program for finding Nash equilibria in this game.*

Exercise 4. *Construct an extensive form of the Game of chicken from Table 2 and write its sequence form and the linear complementarity problem for finding Nash equilibria in this game.*

*Information about the course can be found at http://kam.mff.cuni.cz/~balko/

	Head	Tails
Head	(1,-1)	(-1,1)
Tails	(-1,1)	(1,-1)

Table 1: A normal form of the game Matching Pennies.

	Turn	Straight
Turn	(0,0)	(-1,1)
Straight	(1,-1)	(-10,-10)

Table 2: A normal form of the Game of chicken.