

# Algorithmic game theory – Tutorial 5\*

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## 1 $\varepsilon$ -Nash and correlated equilibria

Let  $G = (P, A, u)$  be a normal-form game of  $n$  players and let  $\varepsilon > 0$ . A strategy profile  $s = (s_1, \dots, s_n)$  is an  $\varepsilon$ -Nash equilibrium if, for every player  $i \in P$  and for every strategy  $s'_i \in S_i$ , we have  $u_i(s_i; s_{-i}) \geq u_i(s'_i; s_{-i}) - \varepsilon$ .

Let  $p$  be a probability distribution on  $A$ , that is,  $p(a) \geq 0$  for every  $a \in A$  and  $\sum_{a \in A} p(a) = 1$ . The distribution  $p$  is a *correlated equilibrium* in  $G$  if

$$\sum_{a_{-i} \in A_{-i}} u_i(a_i; a_{-i})p(a_i; a_{-i}) \geq \sum_{a_{-i} \in A_{-i}} u_i(a'_i; a_{-i})p(a_i; a_{-i})$$

for every player  $i \in P$  and all pure strategies  $a_i, a'_i \in A_i$ .

**Exercise 1.** Show that, in every normal-form game  $G = (P, A, u)$ , every convex combination of correlated equilibria is a correlated equilibrium.

**Exercise 2.** Prove that, in every normal-form game  $G = (P, A, u)$  of  $n$  players, for every Nash equilibrium there exists a corresponding correlated equilibrium.

**Exercise 3.** Let  $G = (P = \{1, 2\}, A, u)$  be a normal-form game of two players with  $A_1 = \{U, D\}$  and  $A_2 = \{L, R\}$  with payoff function  $u$  depicted in Table 1.

	L	R
U	(1,1)	(0,0)
D	$(1 + \frac{\varepsilon}{2}, 1)$	(500,500)

Table 1: The game from Exercise 3.

Show that there is an  $\varepsilon$ -Nash equilibrium  $s$  of  $G$  such that  $u_i(s') > 10u_i(s)$  for every  $i \in P$  and every Nash equilibrium  $s'$  of  $G$ . In other words, there might be games where some  $\varepsilon$ -Nash equilibria are far away from any Nash equilibrium.

**Exercise 4.** Compute all correlated equilibria in Prisoner's dilemma.

	T	S
T	(-2,-2)	(0,-3)
S	(-3,0)	(-1,-1)

Table 2: The game from Exercise 4

**Exercise 5.** Let  $G = (P = \{1, 2\}, A, u)$  be a normal-form game of two players with  $A_1 = \{U, D\}$  and  $A_2 = \{L, R\}$  with payoff function  $u$  depicted in Table 3.

	L	R
U	(6,6)	(2,7)
D	(7,2)	(0,0)

Table 3: A game from Exercise 5.

- Compute all Nash equilibria of  $G$  and draw the convex hull of Nash equilibrium payoffs.
- Is there any correlated equilibrium of  $G$  (for some distribution  $p$ ) that yields payoffs outside this convex hull?

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\*Information about the course can be found at <http://kam.mff.cuni.cz/~balko/>