Algorithmic game theory

Martin Balko

9th lecture

December 7th 2023

Games in extensive form

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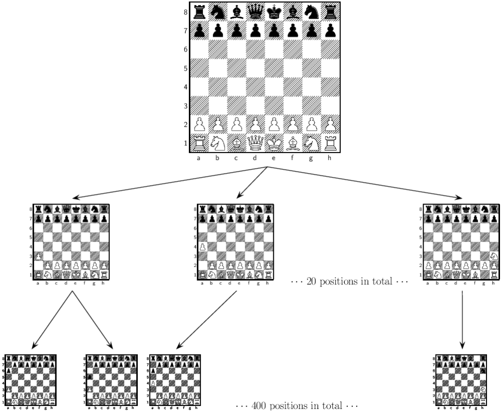

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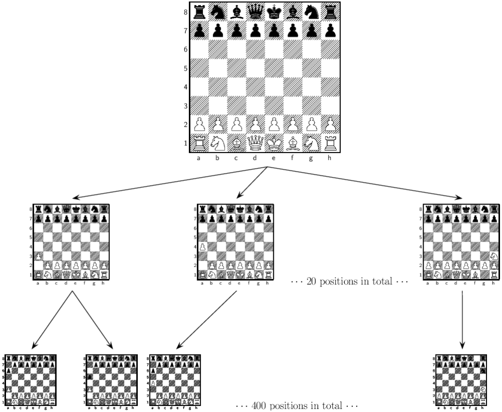

- Today, we describe strategies for such games and how to compute Nash equilibria.

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- In perfect-information games all information sets are singletons.

Example

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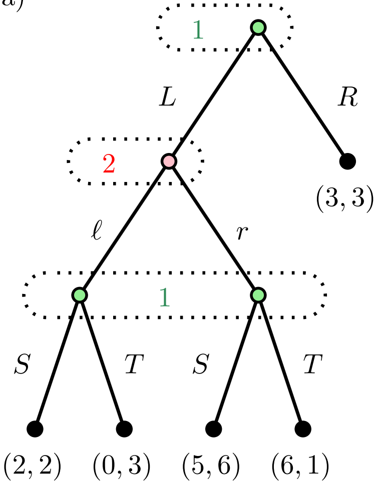
- An example of an imperfect-information game in extensive form (part (a)) and its normal-form (part (b)).

(a)



(b)

	(ℓ)	(r)
(L,S)	(2,2)	(5,6)
(L,T)	(0,3)	(6,1)
(R,S)	(3,3)	(3,3)
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Example: Prisoner's dilemma

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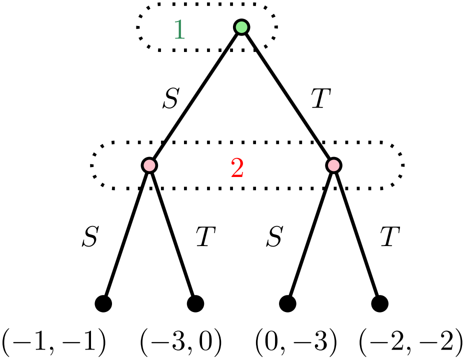

(b)

	T	S
T	(-2,-2)	(0,-3)
S	(-3,0)	(-1,-1)

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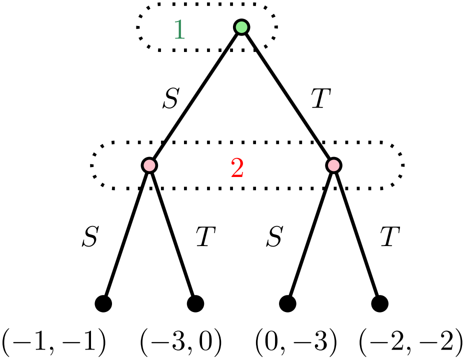

(b)

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- Every normal-form game can be expressed as an imperfect-information extensive game.

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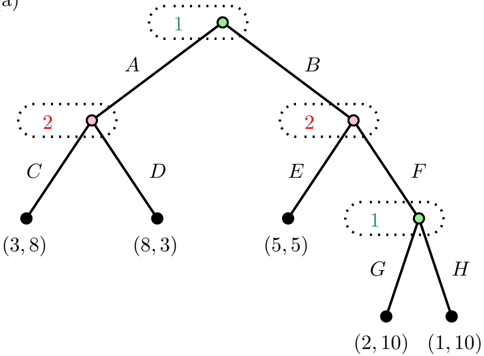

(b)

	(C,E)	(C,F)	(D,E)	(D,F)
(A,G)	(3,8)	(3,8)	(8,3)	(8,3)
(A,H)	(3,8)	(3,8)	(8,3)	(8,3)
(B,G)	(5,5)	(2,10)	(5,5)	(2,10)
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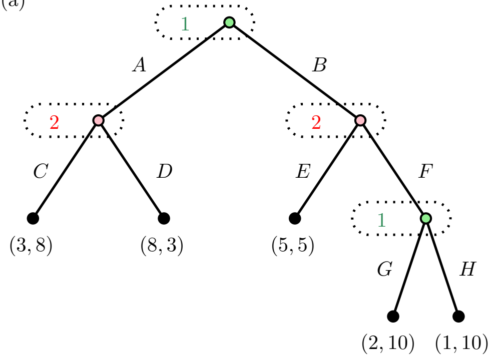

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- A strategy of player 1 that selects A with probability $\frac{1}{2}$ and G with probability $\frac{1}{3}$ is a behavioral strategy.

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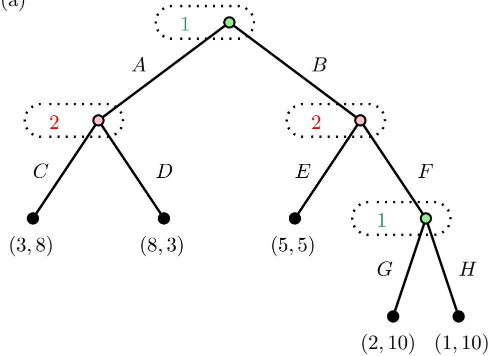

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- The mixed strategy $(\frac{3}{5}(A, G), \frac{2}{5}(B, H))$ is not a behavioral strategy for 1 as the choices made by him at the two nodes are not independent.

Kuhn's Theorem

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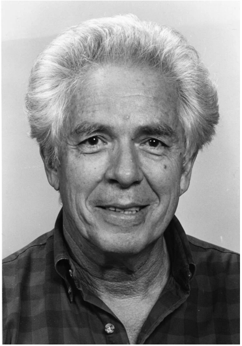

Figure: Harold William Kuhn (1925–2014).

Sources: https://alchetron.com/Harold-W-Kuhn and https://www.cantorsparadise.com/

Example: sequence form constraints

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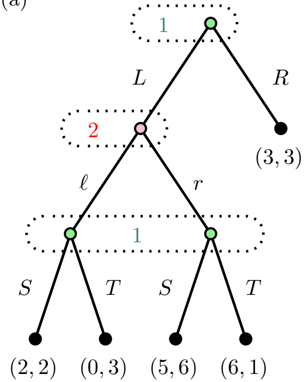
- An example of an imperfect-information game in extensive form (part (a)) and linear constraints in its sequence form (part (b)).

(a)



(b)

$$E = \begin{pmatrix} 1 & & & & \\ -1 & 1 & 1 & & \\ & -1 & & 1 & 1 \end{pmatrix}, \quad e = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},$$

$$F = \begin{pmatrix} 1 & & \\ -1 & 1 & 1 \end{pmatrix}, \quad f = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

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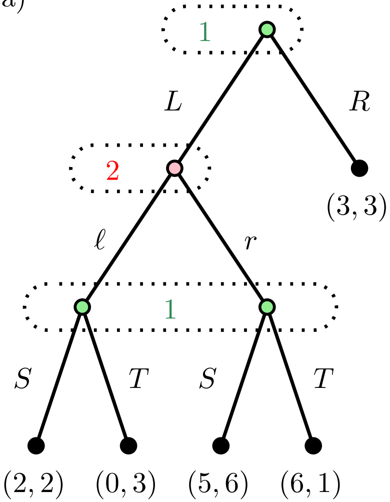

(b)

$$A = \begin{array}{c} \\ \end{array}\begin{array}{c|ccc|c} & \emptyset & \ell & r & \\ \hline & & & & \emptyset \\ & & & & L \\ & 3 & & & R \\ & & 2 & 5 & LS \\ & & 0 & 6 & LT \end{array}$$

$$B = \begin{array}{c|ccc|c} & \emptyset & \ell & r & \\ \hline & & & & \emptyset \\ & & & & L \\ & 3 & & & R \\ & & 2 & 6 & LS \\ & & 3 & 1 & LT \end{array}$$

- More about games in extensive form + implementation of the algorithms is taught in a new lecture by Martin Schmid.

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Vědci z Matematicko-fyzikální fakulty Univerzity Karlovy a Fakulty elektrotechnické ČVUT v Praze pracovali několik posledních měsíců na vývoji umělé inteligence, jejímž hlavním úkolem bude stát se špičkou v karetní hře Poker Texas Hold'em. A to se skutečně podařilo, program porazil hned několik profesionálních hráčů.

Pokerový software DeepStack vytvořili Češi Novinky

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Thank you for your attention.