Algorithmic game theory

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8th lecture

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Applications of regret minimization

Concluding the story of NE

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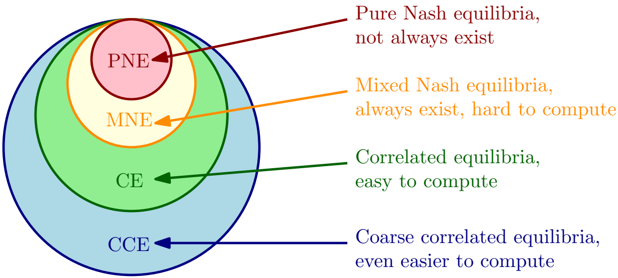

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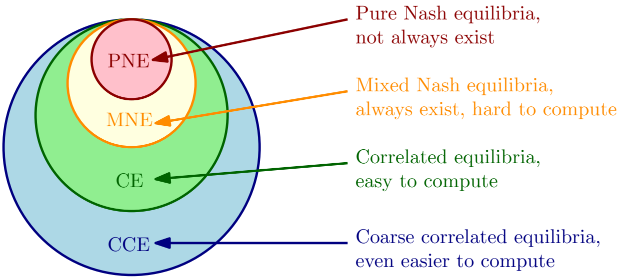

- Today, we introduce a new notion of regret that will converge to CE.

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- Given a comparison class $\mathcal{A}_X$ of agents A_i that select a single action i in all steps, we let $L_{min}^T = \min_{i \in X}\{L_{A_i}^T\}$ be the minimum cumulative loss of an agent from $\mathcal{A}_X$.

Weather					**Loss**
Algorithm					**1**
Umbrella					**1**
Sunscreen					**3**

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- Our goal is to minimize the external regret $R_A^T = L_A^T - L_{min}^T$.

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Algorithm 0.3: NO-SWAP-REGRET DYNAMICS(G, T, ε)

Input : A normal-form game $G = (P, A, C)$ of n players, $T \in \mathbb{N}$, and $\varepsilon > 0$.
Output : A prob. distribution p_i^t on A_i for each $i \in P$ and $t \in \{1, \dots, T\}$.
for every step $t = 1, \dots, T$
do Each player $i \in P$ independently chooses a mixed strategy p_i^t using an algorithm with average swap regret at most ε, with actions corresponding to pure strategies.
Each player $i \in P$ receives a loss vector $\ell_i^t = (\ell_i^t(a_i))_{a_i \in A_i}$, where $\ell_i^t(a_i) \leftarrow \mathbb{E}_{a_{-i}^t \sim p_{-i}^t}[C_i(a_i; a_{-i}^t)]$ for the product distribution $p_{-i}^t = \prod_{j \neq i} p_j^t$.
Output $\{p^t : t \in \{1, \dots, T\}\}$.

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Algorithm 0.4: NO-SWAP-REGRET DYNAMICS(G, T, ε)

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}
Output $\{p^t : t \in \{1, \dots, T\}\}$.

- No-swap-regret dynamics then converges to a correlated equilibrium.

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Zdroj: https://cz.pinterest.com

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- For some of these games, we show how to compute NE.

Example: normal-form of chess

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Source: https://edition.cnn.com/

- Chess as a normal-form game: Each action of player $i \in \{\text{black}, \text{white}\}$ is a list of all possible situations that can happen on the board together with the move player i would make in that situation. Then we can simulate the whole game of chess in one round.

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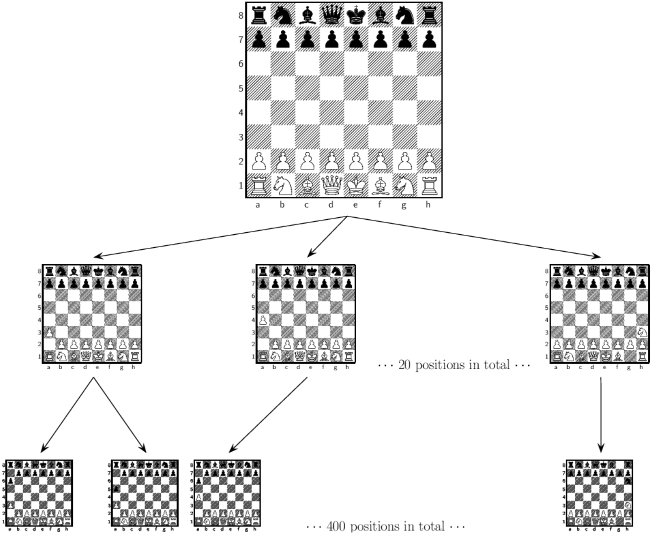
- Root corresponds to the initial position of the chessboard. Each decision node represents a position on the chessboard and its outgoing edges correspond to possible moves in such a position.

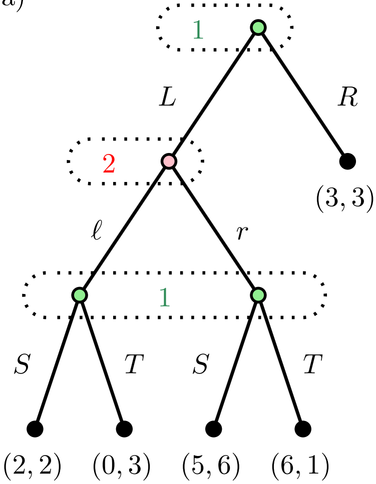


Example

Example

- An example of an imperfect-information game in extensive form (part (a)) and its normal-form (part (b)).

(a)



(b)

	(ℓ)	(r)
(L, S)	(2,2)	(5,6)
(L, T)	(0,3)	(6,1)
(R, S)	(3,3)	(3,3)
(R, T)	(3,3)	(3,3)

Example: Prisoner's dilemma

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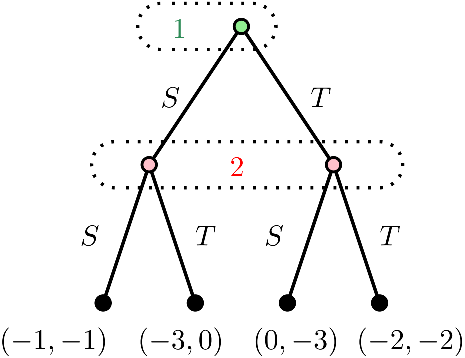
- Prisoner's dilemma in extensive form (part (a)) and its normal-form (part (b)).

(a)



(b)

	T	S
T	(-2,-2)	(0,-3)
S	(-3,0)	(-1,-1)

Example: Russian roulette

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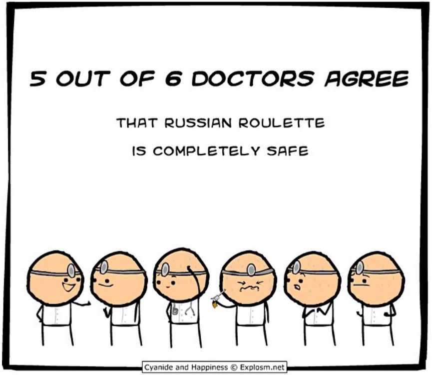
- We have two players with a six-shot revolver containing a single bullet. Each player has two moves: shoot or give up. If player gives up, he loses the game immediately. If he shoots, then he either dies or survives, in which case the other player is on turn.



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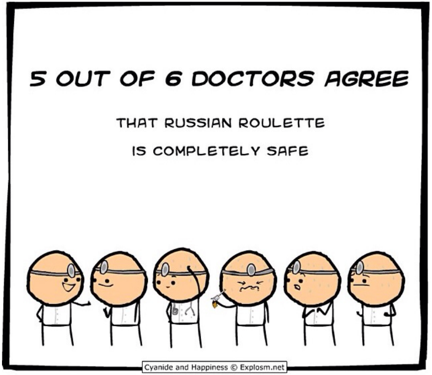

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- Consider that player 1 has payoffs $(10, 2, 1)$ for (Win, Loss, Death) and that player 2 has payoffs $(10, 0, 0)$.

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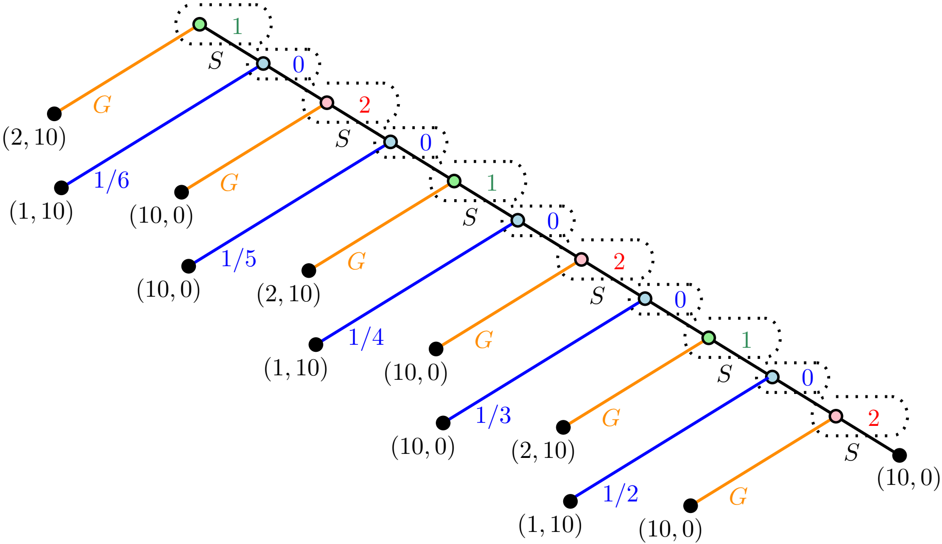
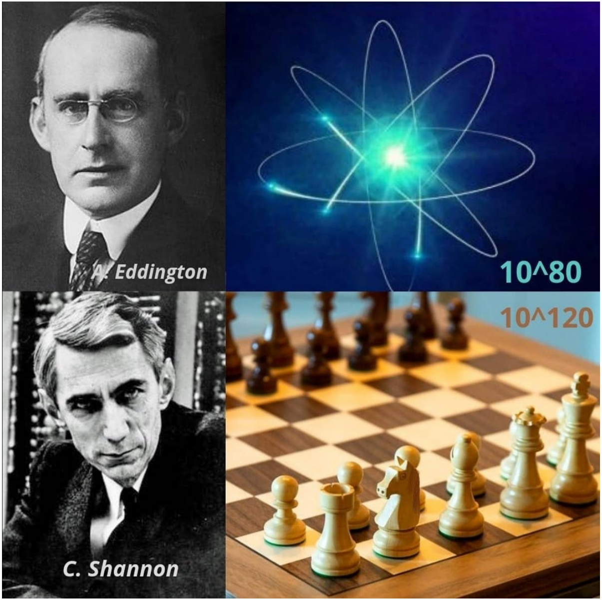
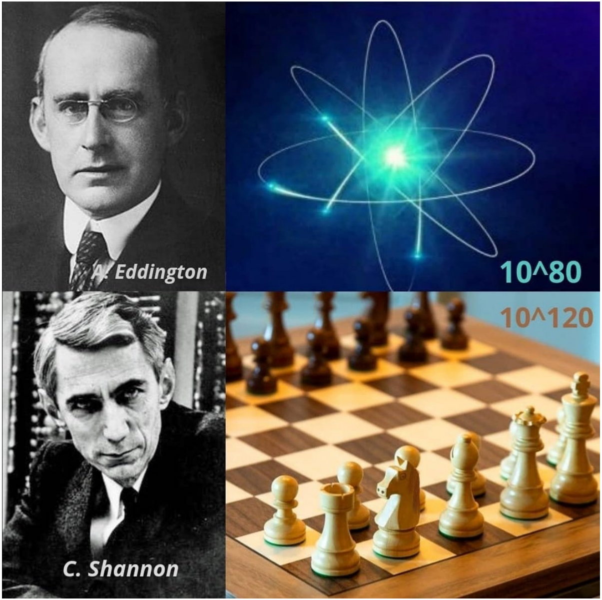
- The Russian roulette in the extensive form using the random player.





Source: https://twitter.com/curiosite12



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Thank you for your attention.