Algorithmic game theory

Martin Balko

5th lecture

November 9th 2023

Nash equilibria in bimatrix games

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- Is there a chance to get an efficient algorithm?
- NASH = the problem of finding NE in bimatrix games.
- Today, we discuss the computational complexity of NASH.

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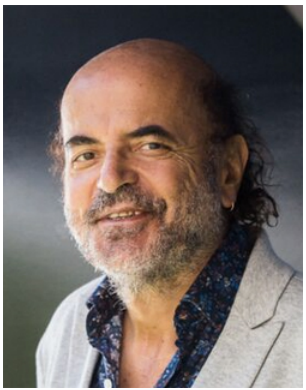

Figure: Christos Papadimitriou (born 1949).

Source: https://cs.columbia.edu

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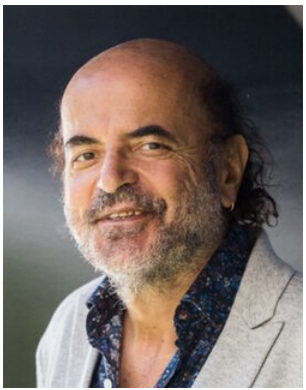

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- Abbreviation for "Polynomial Parity Arguments on Directed graphs".

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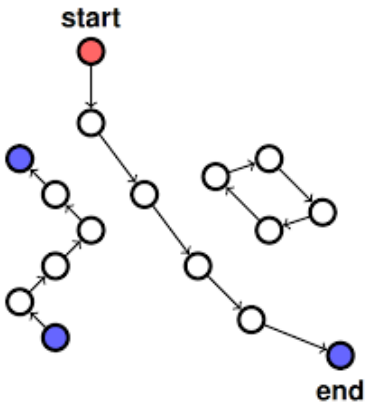

Source: R. Savani "Polymatrix Games" Tutorial at WINE 2015

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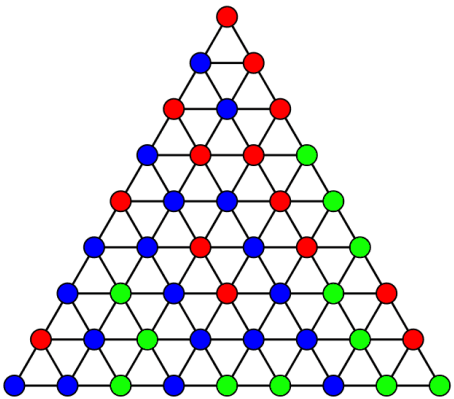

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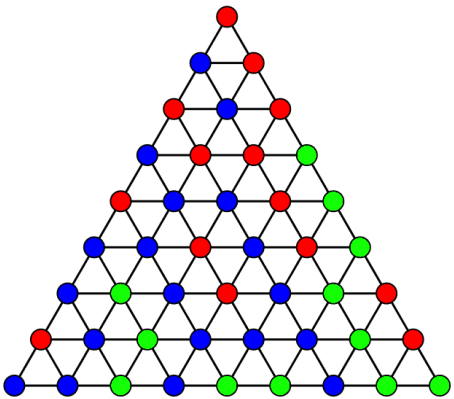

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- Discrete version of the Brouwer's fixed point theorem.

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- An approximate version of the following theorem is in PPAD: For each $d \in \mathbb{N}$, a non-empty compact convex set K in $\mathbb{R}^d$, and a continuous mapping $f : K \to K$, there exists $x_0 \in K$ such that $f(x_0) = x_0$.

Figure: L. E. J. Brouwer (1881–1966).

Source: https://arxiv.org/pdf/1612.06820.pdf

Problems from PPAD: Ham sandwich theorem

Source: https://www.seekpng.com/

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- Given n sets of $2n$ points in $\mathbb{R}^n$, find a hyperplane H that contains exactly n points from each of the sets in each open halfspace determined by H.

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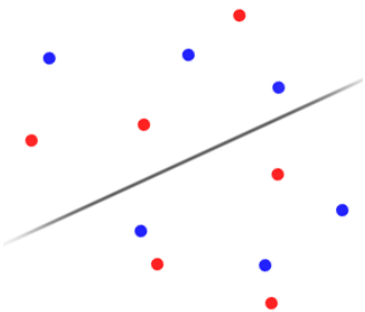

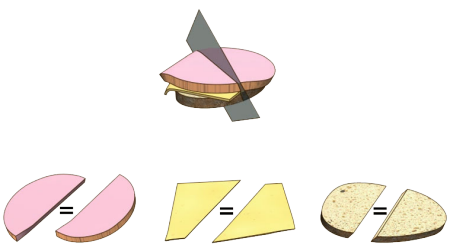

Sources: https://ejarzo.github.io and https://curiosamathematica.tumblr.com

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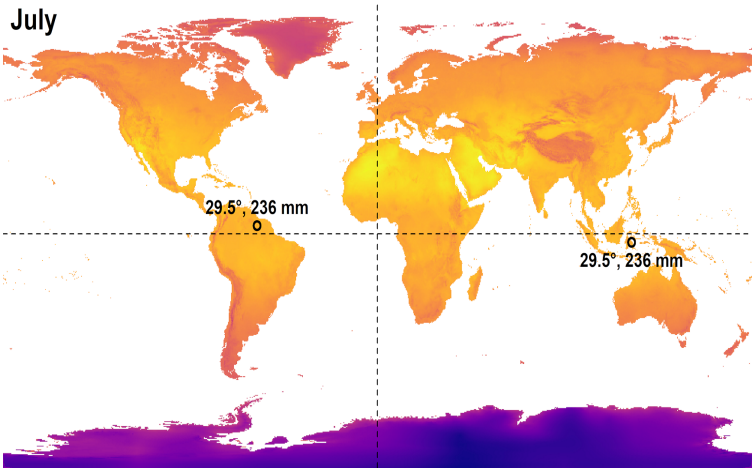

Source: https://scientificgems.wordpress.com/

Other notions of equilibria

- The concept of correlated equilibria was introduced by Robert Aumann, who received a Nobel prize in economics for his work in game theory.

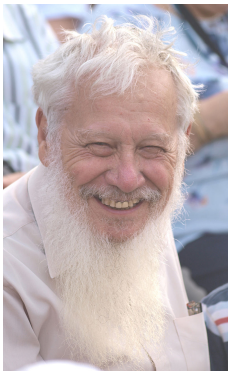

Figure: Robert Aumann (born 1930).

Sources: https://en.wikipedia.org and https://slideslive.com/38910863/strategic-information-theory

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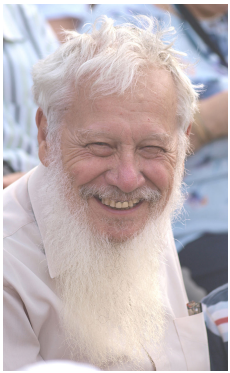

Figure: Robert Aumann (born 1930).

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- In 2018, Robert Aumann visited Prague and gave a lecture at Prague mathematical colloquium. You can see the lecture here: https://slideslive.com/38910863/strategic-information-theory.

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Thank you for your attention.