Algorithmic game theory

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2nd lecture

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Proof of Nash's Theorem

Nash's Theorem

- Every normal-form game has a Nash equilibrium.

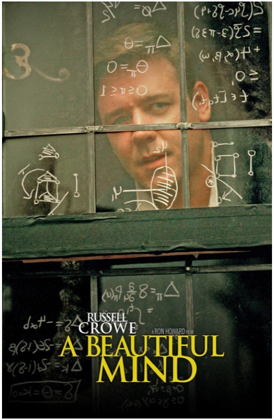

Figure: John Forbes Nash Jr. (1928–2015) and his depiction in the movie A Beautiful mind.

Sources: https://britannica.com and https://medium.com

Brouwer's Fixed Point Theorem

Brouwer's Fixed Point Theorem

- For each $d \in \mathbb{N}$, let K be a non-empty compact convex set in $\mathbb{R}^d$ and $f : K \to K$ be a continuous mapping. Then, there exists a fixed point $x_0 \in K$ for f, that is, $f(x_0) = x_0$.

Figure: L. E. J. Brouwer (1881–1966).

Source: https://arxiv.org/pdf/1612.06820.pdf

- `https://www.youtube.com/watch?v=csInNn6pfT4&t=268s&ab_`

Pareto optimality

Vilfredo Pareto

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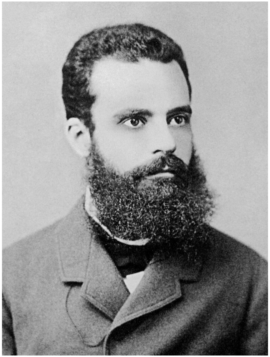

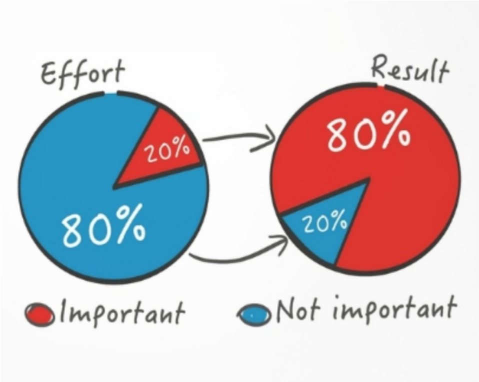

Figure: Vilfredo Pareto (1848–1923).

Sources: https://en.wikipedia.org and https://medium.com/

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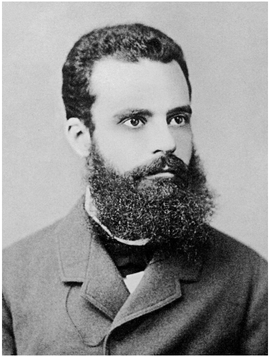

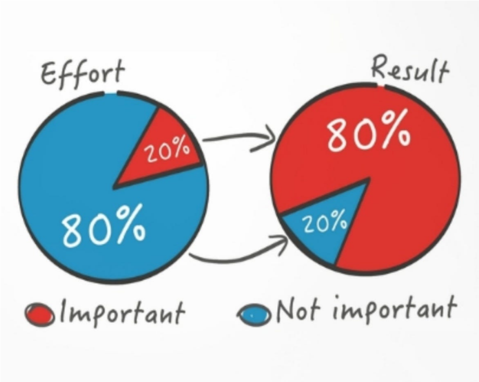

Figure: Vilfredo Pareto (1848–1923).

Sources: https://en.wikipedia.org and https://medium.com/

- Pareto principle: for many outcomes roughly 80% of consequences come from 20% of the causes.

Finding Nash equilibria in zero-sum games

Zero-sum games

Zero-sum games

- Two-player games (P, A, u) where $u_1(a) = -u_2(a)$ for every $a \in A$.

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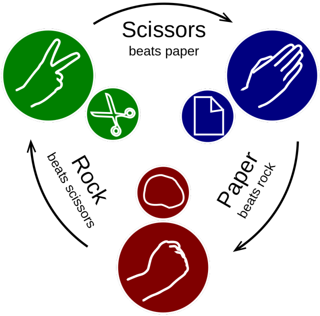
	Rock	Paper	Scissors
Rock	(0,0)	(-1,1)	(1,-1)
Paper	(1,-1)	(0,0)	(-1,1)
Scissors	(-1,1)	(1,-1)	(0,0)



Sources: https://en.wikipedia.org/

Zero-sum games examples: chess

Source: https://edition.cnn.com/

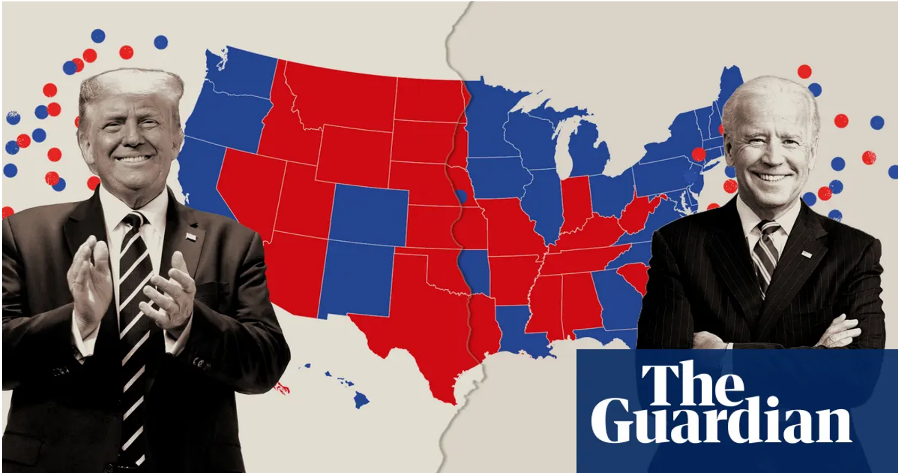
Zero-sum games examples: table tennis

Source: https://www.reddit.com/

Zero-sum games examples: derivative trading

Source: https://www.linkedin.com/

Zero-sum games examples: elections



Source: https://www.theguardian.com/

Zero-sum games examples: many more

Source: https://lhongtortai.com/collection/what-is-a-non-zero-sum-game

The Minimax Theorem

The Minimax Theorem

- For every zero-sum game, worst-case optimal strategies for both players exist and can be efficiently computed. There is a number v such that, for any worst-case optimal strategies x^* and y^*, the strategy profile (x^*, y^*) is a Nash equilibrium and $\beta(x^*) = (x^*)^\top M y^* = \alpha(y^*) = v$.

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Figure: John von Neumann (1903–1957) and Oskar Morgenstern (1902–1977).

Sources: https://en.wikiquote.org and https://austriainusa.org

Duality of linear programming

	Primal linear program	Dual linear program
Variables	$x_1, \ldots, x_m$	$y_1, \ldots, y_n$
Matrix	$A \in \mathbb{R}^{n \times m}$	$A^\top \in \mathbb{R}^{m \times n}$
Right-hand side	$b \in \mathbb{R}^n$	$c \in \mathbb{R}^m$
Objective function	$\max c^\top x$	$\min b^\top y$
Constraints	ith constraint has $\leq$	$y_i \geq 0$
	$\geq$	$y_i \leq 0$
	$=$	$y_i \in \mathbb{R}$
	$x_j \geq 0$	jth constraint has $\geq$
	$x_j \leq 0$	$\leq$
	$x_j \in \mathbb{R}$	$=$

Table: A recipe for making dual programs.

Source: https://czthomas.files.wordpress.com

Thank you for your attention.