Algorithmic game theory

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13th lecture

January 11th 2024

Revenue maximization: what we know

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- In DSIC mechanisms, maximizing the expected revenue is then the same as maximizing the expected virtual social surplus $\sum_{i=1}^{n} \varphi_i(v_i)x_i(v)$. (Theorem 3.14) where

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- What if the seller does not know the distributions $F_1, \ldots, F_n$?

The Bulow–Klemperer theorem

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- If $F = F_1 = \cdots = F_n$ is a regular probability distribution, then

$$\mathbb{E}_{v_1,\ldots,v_{n+1}\sim F}\left[\mathrm{Rev}(VA_{n+1})\right] \geq \mathbb{E}_{v_1,\ldots,v_n\sim F}\left[\mathrm{Rev}(OPT_{F,n})\right].$$

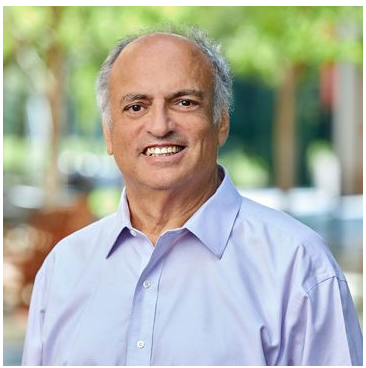

Figure: Jeremy Bulow and Paul Klemperer.

Sources: https://economics.stanford.edu/ and https://www.economics.ox.ac.uk/

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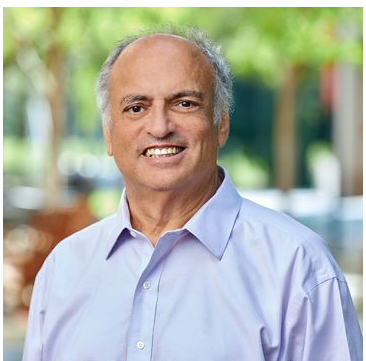

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- More competition is better than finding the right auction format.

Multi-parameter mechanism design

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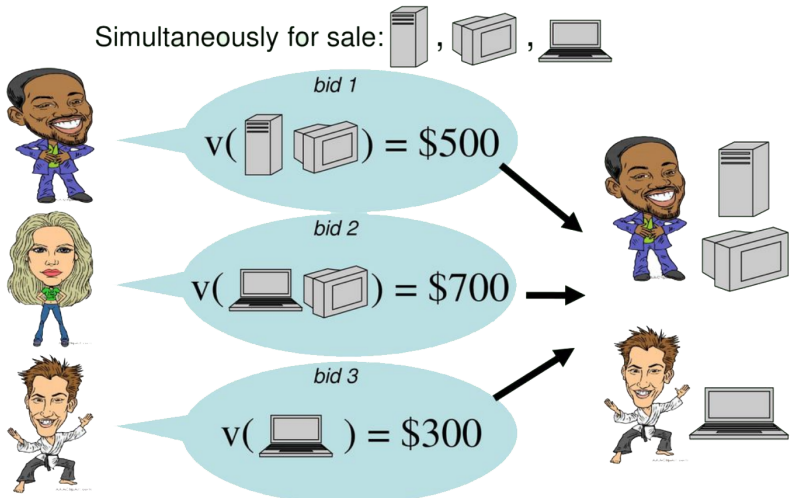

Sources: Auctions & Combinatorial auctions (Vincent Conitzer)

The Vickrey–Clarke–Groves (VCG) mechanism

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- In every multi-parameter mechanism design environment, there is a DSIC social-surplus-maximizing mechanism.

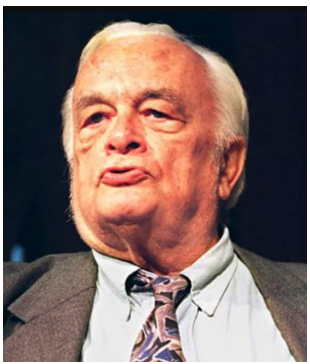

Figure: William Vickrey, Edward H. Clarke, and Theodore Groves.

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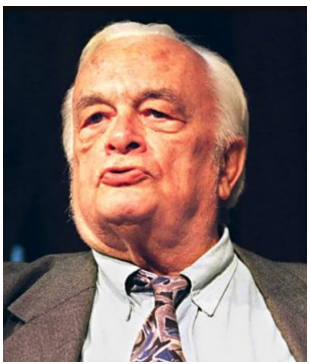

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- We can still do DSIC social surplus maximization.

VCG auction example

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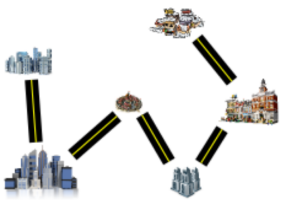
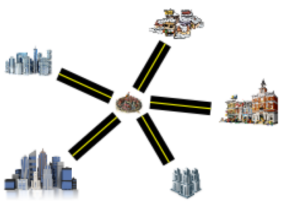


Road Network 1

Road Network 2



Road Network 3

Road Network 4

Sources: https://www.science4all.org/article/auction-design/

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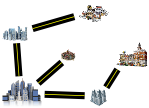
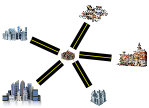
	 Road Network 1	 Road Network 2	 Road Network 3	 Road Network 4
	6 M$	14 M$	2 M$	**16 M$**
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	4 M$	**6 M$**	3 M$	5 M$
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Total (social welfare)	19 M$	32 M$	37 M$	**42 M$**

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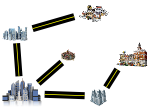
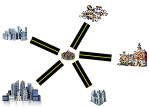
	 Road Network 1	 Road Network 2	 Road Network 3	 Road Network 4
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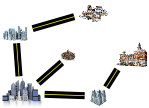
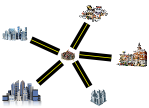
	 Road Network 1	 Road Network 2	 Road Network 3	 Road Network 4
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- Cities pay their negative externalities on the collectivity. Other cities would be happier without the biggest city (NYC, say). How much happier they would be is exactly what NYC must pay.

VCG auction example

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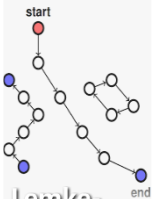
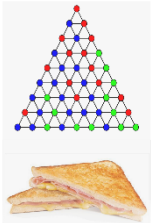
- If NYC was not there, then road network number 3 (RN3) would have been chosen, as opposed to RN4. The value of RN3 for the other cities would be 35 M$, as opposed to the 26 M$ of RN4. Therefore, the negative externality of NYC is $35 - 26 = 9$ M$.

Nash equilibria

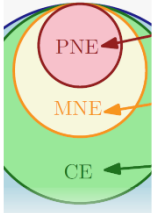
Minimax theorem



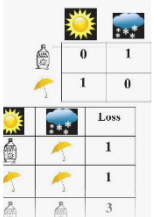
Lemke-Howson algorithm



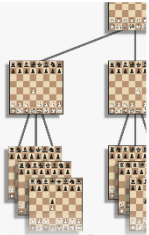
Complexity of NASH



Variants of NE



Regret minimization



Extensive games

Mechanism design

Revenue maximization

VCG mechanism

Nash equilibria

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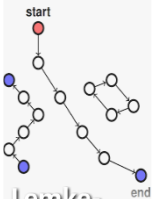

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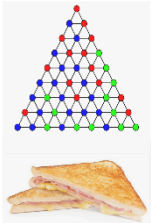

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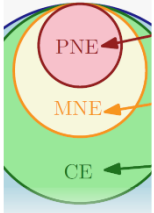

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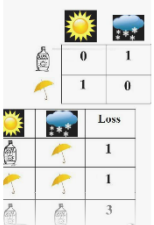

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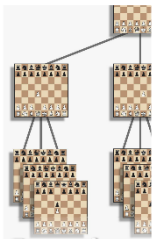

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