Algorithmic game theory

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11th lecture

December 21st 2023

Myerson's lemma

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- Is there awesome mechanism (x, p) for single-parameter environments?
- We started by looking for DSIC mechanisms.

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Figure: Roger Myerson (born 1951) receiving a Nobel prize in economics.

Sources: https://en.wikipedia.org and https://twitter.com

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Myerson's lemma

In a single-parameter environment, the following three claims hold.

(a) An allocation rule is implementable if and only if it is monotone.

(b) If an allocation rule x is monotone, then there exists a unique payment rule p such that (x, p) is DSIC (assuming $b_i = 0$ implies $p_i(b) = 0$).

(c) For every i, the payment rule p is given by the following explicit formula

$$p_i(b_i; b_{-i}) = \int_0^{b_i} z \cdot \frac{\mathrm{d}}{\mathrm{d}z} x_i(z; b_{-i}) \, \mathrm{d}z.$$

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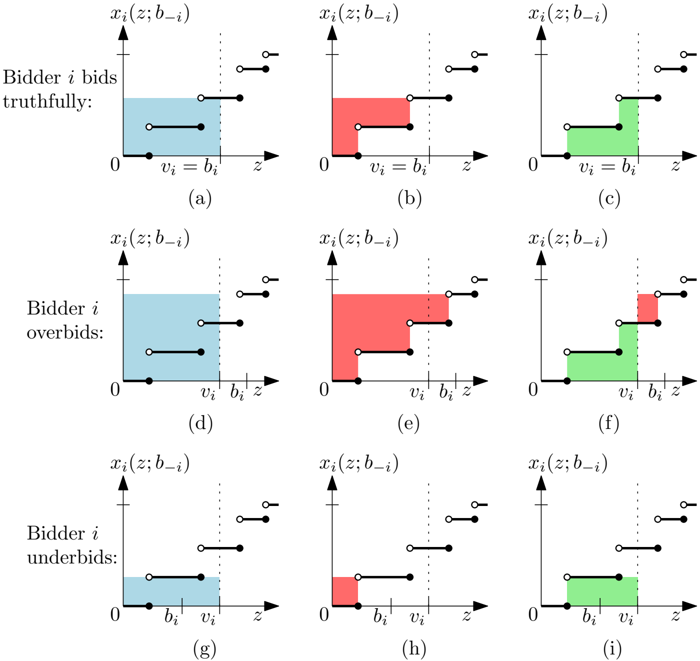

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- Bidders are companies such that each company has its own TV commercial of length w_i and is willing to pay v_i in order to have the commercial presented during a commercial break. The seller is a television station with a commercial break of length W.

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- Can we design an awesome mechanism that assigns the slots?

Knapsack problem

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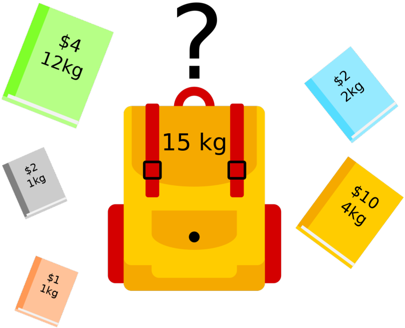
- given a capacity W and n items of values $v_1, \ldots, v_n$ and sizes $w_1, \ldots, w_n$, find a subset of the items having a maximum total value such that the total size is at most W.



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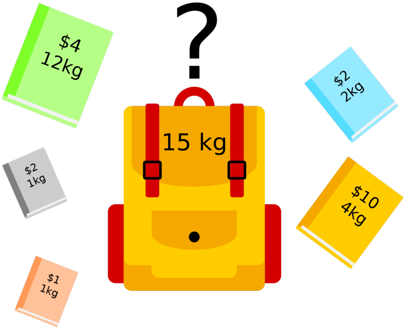

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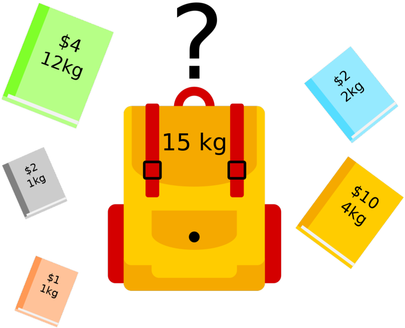

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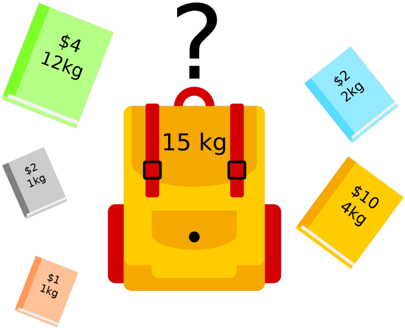

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- There is a pseudo-polynomial time algorithm using dynamic programming and a fully polynomial-time approximation scheme.

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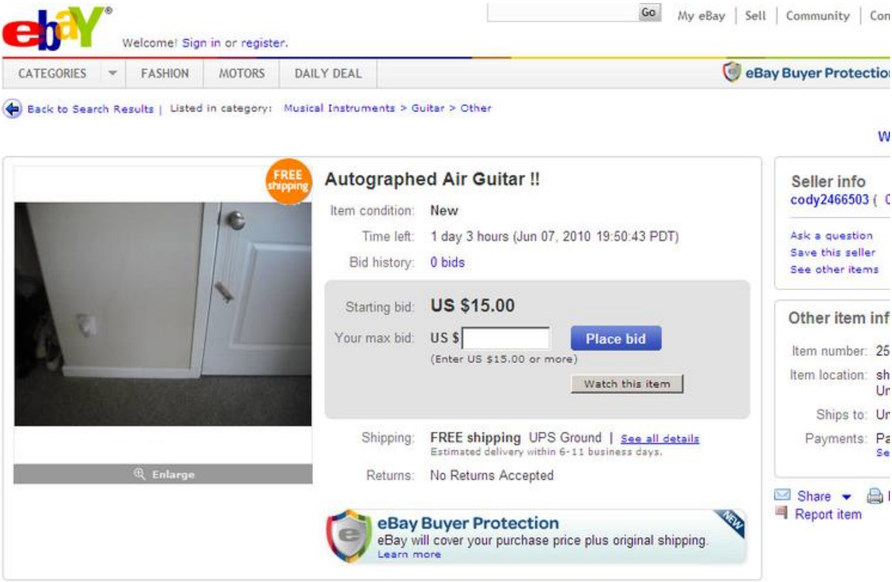

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Thank you for your attention and merry Christmas!