

Algorithmic game theory

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11th lecture

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Myerson's lemma

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- We started by looking for **DSIC** mechanisms.

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Figure: Roger Myerson (born 1951) receiving a Nobel prize in economics.

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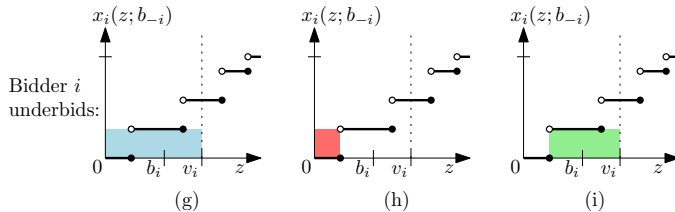
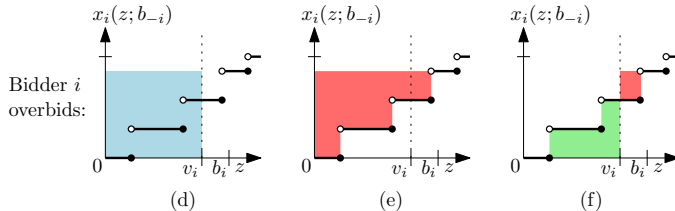
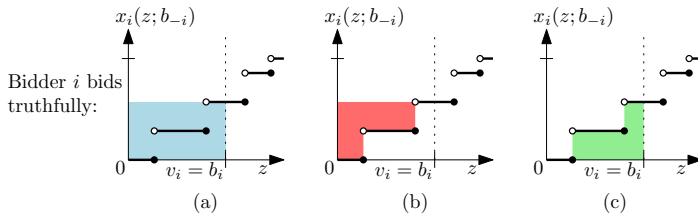
In a single-parameter environment, the following three claims hold.

- (a) An allocation rule is **implementable if and only if it is monotone**.
- (b) If an allocation rule x is monotone, then there exists a **unique payment rule** p such that (x, p) is DSIC (assuming $b_i = 0$ implies $p_i(b) = 0$).
- (c) For every i , the payment rule p is given by the following **explicit formula**

$$p_i(b_i; b_{-i}) = \int_0^{b_i} z \cdot \frac{d}{dz} x_i(z; b_{-i}) dz.$$

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- **Bidders are companies** such that each company has its own TV commercial of length w_i and is willing to pay v_i in order to have the commercial presented during a commercial break. The **seller is a television station** with a commercial break of length W .



Sources: <https://mountain.com/> and <https://www.eq-international.com/>

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- Can we design an awesome mechanism that assigns the slots?

Knapsack problem

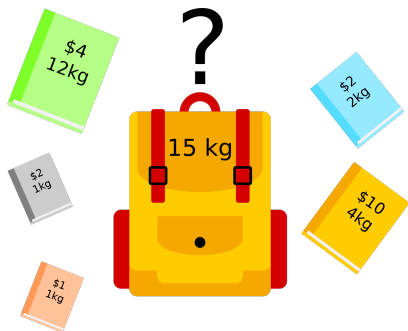
Knapsack problem

- given a capacity W and n items of values v_1, \dots, v_n and sizes w_1, \dots, w_n , find a subset of the items having a maximum total value such that the total size is at most W .



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- This problem is **NP-hard**.
- There is a **pseudo-polynomial time algorithm** using dynamic programming and a **fully polynomial-time approximation scheme**.



- Next lecture, we will focus on **revenue maximization**.

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