### Algorithmic game theory

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# Mechanism design basics

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Source: Innovations in Defense Acquisition: Asymmetric Information and Incentive Contract Design

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- We start with single item auctions.
- We then extend these desired properties to a more general setting of single-parameter environments using so-called Myerson's lemma.

### Single item auctions



Source: https://www.widewalls.ch

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### Figure: William Vickrey (1914–1996).

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- Vickrey posthumously received a Nobel prize in Economic Sciences.

### Sponsored search

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Figure: Roger Myerson (born 1951) receiving a Nobel prize in economics.

Sources: https://en.wikipedia.org and https://twitter.com

#### Myerson's lemma

In a single-parameter environment, the following three claims hold.

- (a) An allocation rule is implementable if and only if it is monotone.
- (b) If an allocation rule x is monotone, then there exists a unique payment rule p such that the mechanism (x, p) is DSIC (assuming that  $b_i = 0$  implies  $p_i(b) = 0$ ).
- (c) The payment rule p is given by the following explicit formula

$$p_i(b_i; b_{-i}) = \int_0^{b_i} z \cdot \frac{\mathrm{d}}{\mathrm{d}z} x_i(z; b_{-i}) \,\mathrm{d}z$$

for every  $i \in \{1, ..., n\}$ .



Source: https://businessinsider.com



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# Thank you for your attention.