Algorithmic game theory

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1st lecture

October 5th 2023

Basic info

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- Webpage: *https://kam.mff.cuni.cz/~balko/ath2324/ATH.html*
  - lecture info, topics covered, presentations, lecture notes ...

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  - lecture info, topics covered, presentations, lecture notes ...
- Recommended literature:
  - M. Balko: Algorithmic game theory: lecture notes.
  - The notes are still under construction. Comments are welcome.

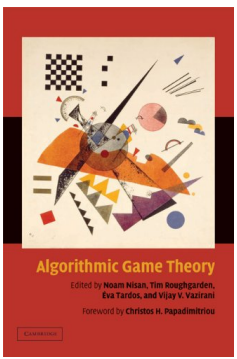

Figure: Algorithmic game theory by Nisan et al.

Source: https://amazon.com

Game theory

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- study of mathematical models of strategic interaction among rational decision-makers.

Zdroj: https://quantamagazine.org

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- We focus on the algorithmic side of the game theory.
- Several real-word applications.
- More than ten game theorists have won the Nobel Prize in economics.

Sylabus

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- Preliminary plan:

Sylabus

- Preliminary plan:
  - Finding Nash equilibria
    - Nash equilibria and Nash's Theorem,
    - zero-sum games,
    - bimatrix games and the Lemke–Howson algorithm,
    - other notions of equilibria,
    - regret minimization.
  - Mechanism design,
    - auctions (Vickrey),
    - Myerson's lemma and its applications,
    - revenue maximization.

Finding Nash equilibria

Normal-form games: Rock-Paper-Scissors

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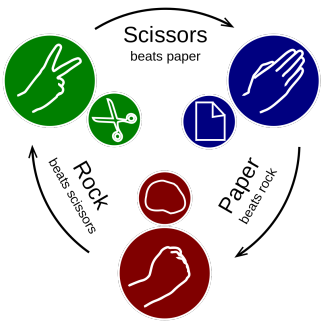
	Rock	Paper	Scissors
Rock	(0,0)	(-1,1)	(1,-1)
Paper	(1,-1)	(0,0)	(-1,1)
Scissors	(-1,1)	(1,-1)	(0,0)



Sources: https://en.wikipedia.org/

Normal-form games: Rock-Paper-Scissors-Lizard-Spock

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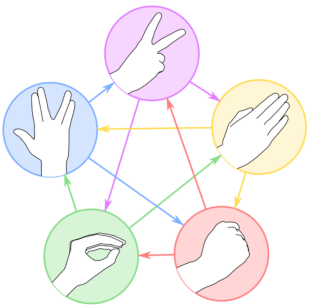
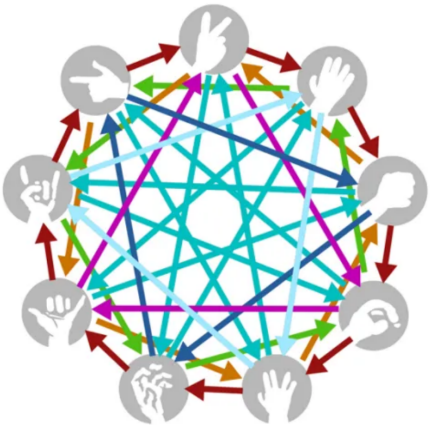
	Rock	Paper	Scissors	Lizard	Spock
Rock	(0,0)	(-1,1)	(1,-1)	(1,-1)	(-1,1)
Paper	(1,-1)	(0,0)	(-1,1)	(-1,1)	(1,-1)
Scissors	(-1,1)	(1,-1)	(0,0)	(1,-1)	(-1,1)
Lizard	(-1,1)	(1,-1)	(-1,1)	(0,0)	(1,-1)
Spock	(1,-1)	(-1,1)	(1,-1)	(-1,1)	(0,0)



Sources: https://bigbangtheory.fandom.com/

- "Scissors cuts Paper, Paper covers Rock, Rock crushes Lizard, Lizard poisons Spock, Spock smashes Scissors, Scissors decapitates Lizard, Lizard eats Paper, Paper disproves Spock, Spock vaporizes Rock (and as it always has) Rock crushes Scissors."

Normal-form games: Rock-Paper-Scissors-Lizard-Spock



ROCK PAPER SCISSORS
LIZARD SPOCK
SPIDER-MAN BATMAN
WIZARD GLOCK

Scissors cuts paper.
Paper covers rock.
Rock crushes lizard.
Lizard poisons Spock.
Spock zaps wizard.
Wizard stuns Batman.
Batman scares Spider-Man.
Spider-Man disarms glock.
Glock breaks rock.
Rock interrupts wizard.
Wizard burns paper.
Paper disproves Spock.
Spock befuddles Spider-Man.
Spider-Man defeats lizard.
Lizard confuses Batman
(because he looks like Killer Croc).
Batman dismantles scissors.
Scissors cut wizard.
Wizard transforms lizard.
Lizard eats paper.
Paper jams glock.
Glock kills Batman's mom.
Batman explodes rock.
Rock crushes scissors.
Scissors decapitates lizard.
Lizard is too small for glock.
Glock shoots Spock.
Spock vaporizes rock.
Rock knocks out Spider-Man.
Spider-Man rips paper.
Paper delays Batman.
Batman hangs Spock.
Spock smashes scissors.
Scissors cut Spider-Man.
Spider-Man annoys wizard.
Wizard melts glock.
Glock dents scissors.

ROCK PAPER SCISSORS SPOCK LIZARD by Sam Kass and Karen Bryla, and then, Brian Yan messed it up into this.

Source: https://www.naturphilosophie.co.uk/

Normal-form games: Chess

Normal-form games: Chess

Source: https://edition.cnn.com/

- Chess as a normal-form game: Each action of player $i \in \{\text{black}, \text{white}\}$ is a list of all possible situations that can happen on the board together with the move player i would make in that situation. Then we can simulate the whole game of chess in one round.

Nash's Theorem

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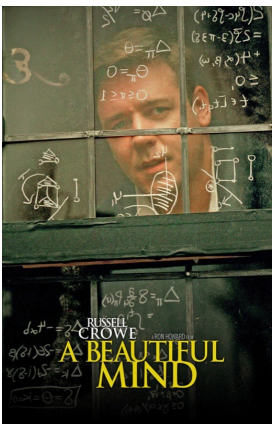

Figure: John Forbes Nash Jr. (1928–2015) and his depiction in the movie A Beautiful mind.

Sources: https://britannica.com and https://medium.com

Brouwer's Fixed Point Theorem

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- For each $d \in \mathbb{N}$, let K be a non-empty compact convex set in $\mathbb{R}^d$ and $f : K \to K$ be a continuous mapping. Then, there exists a fixed point $x_0 \in K$ for f, that is, $f(x_0) = x_0$.

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Figure: L. E. J. Brouwer (1881–1966).

Source: https://arxiv.org/pdf/1612.06820.pdf

Figure: John Forbes Nash Jr. receiving a Nobel prize for economics.

Source: https://pbs.org

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Thank you for your attention.