

Algorithmic game theory — Homework 4¹

Extensive games and mechanism design

assigned 19.12.2022, deadline 31.1.2023

Homework 1. Construct an extensive form of the Rock-Paper-Scissors game from Table 1 and write its sequence form and the linear program for finding Nash equilibria in this game. [2]

	Rock	Paper	Scissors
Rock	(0,0)	(-1,1)	(1,-1)
Paper	(1,-1)	(0,0)	(-1,1)
Scissors	(-1,1)	(1,-1)	(0,0)

Tabulka 1: A normal form of the game Rock-paper-scissors.

Homework 2. Assume there are k identical items and $n > k$ bidders. Also assume that each bidder can receive at most one item. What is the analog of the second-price auction? Prove that your auction is DSIC. [2]

Homework 3. Let F be the uniform probability distribution on $[0, 1]$. Consider a single-item auction with two bidders 1 and 2 that have probability distributions $F_1 = F$ and $F_2 = F$ on their valuations. Prove that the expected revenue obtained by the Vickrey auction with reserve $1/2$ is $5/12$. [2]

Homework 4. Compute the virtual valuation function of the following probability distributions and show which of these distributions are regular (meaning the virtual valuation function is strictly increasing).

- (a) The distribution given by $F(z) = 1 - \frac{1}{(z+1)^c}$ on $[0, \infty)$, where $c > 0$ is some constant, [1]
- (b) Consider the probability distribution F in part (a), with $c = 1$. Argue that when bidder valuations are drawn from F , it is not necessarily the case that the expected revenue of an auction equals its expected virtual social surplus. To reconcile this observation with the theorem from the lecture about maximizing revenue, identify which assumption of this result is violated in your example. [3]

¹Information about the course can be found at <http://kam.mff.cuni.cz/~balko/>