

Algorithmic game theory

Martin Balke

9th lecture

December 5th 2022



Games in extensive form

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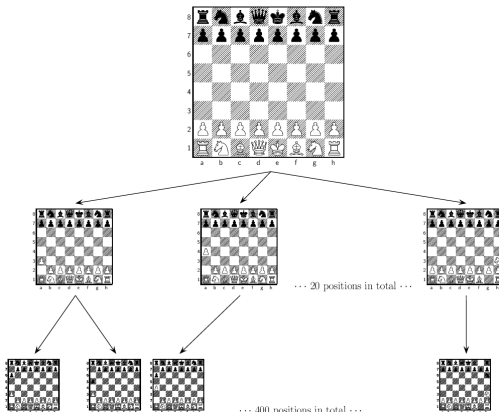
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- Today, we describe a **different representation** of games which provides a **dynamic** description where players act sequentially.

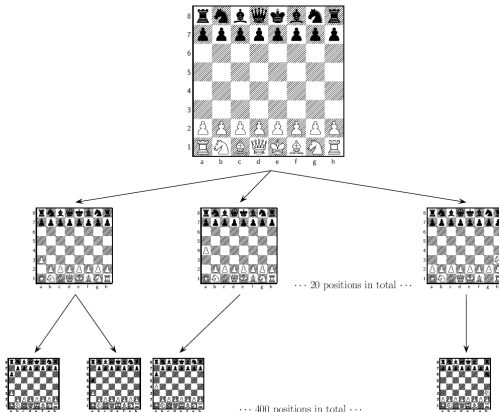
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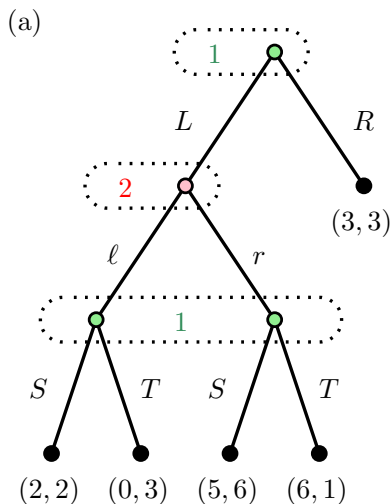


- For some of these games, we **show how to efficiently compute NE**.

Example

Example

- An example of an imperfect-information game in extensive form (**part (a)**) and its normal-form (**part (b)**).



(b)

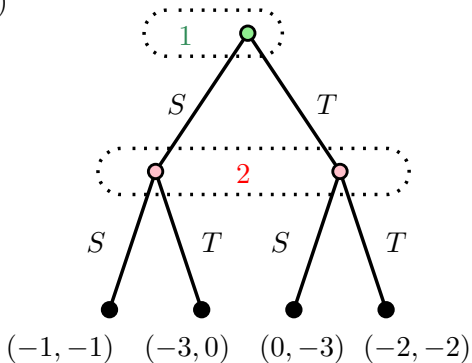
	(ℓ)	(r)
(L, S)	$(2, 2)$	$(5, 6)$
(L, T)	$(0, 3)$	$(6, 1)$
(R, S)	$(3, 3)$	$(3, 3)$
(R, T)	$(3, 3)$	$(3, 3)$

Example: Prisoner's dilemma

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- Prisoner's dilemma in extensive form (**part (a)**) and its normal-form (**part (b)**).

(a)



(b)

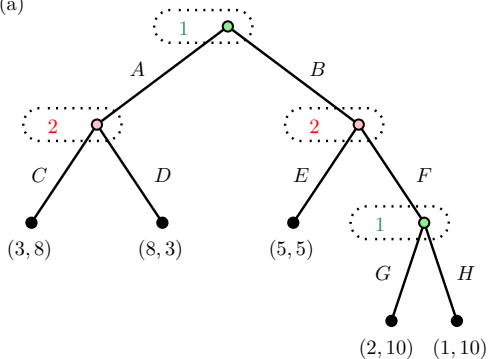
	T	S
T	$(-2, -2)$	$(0, -3)$
S	$(-3, 0)$	$(-1, -1)$

Example: behavioral strategy

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- An example of a perfect-information game in extensive form (**part (a)**) and its normal-form (**part (b)**).

(a)



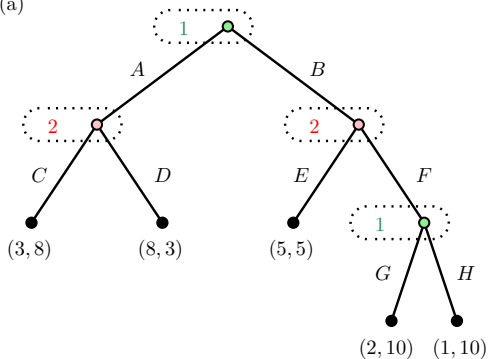
(b)

	(C, E)	(C, F)	(D, E)	(D, F)
(A, G)	(3, 8)	(3, 8)	(8, 3)	(8, 3)
(A, H)	(3, 8)	(3, 8)	(8, 3)	(8, 3)
(B, G)	(5, 5)	(2, 10)	(5, 5)	(2, 10)
(B, H)	(5, 5)	(1, 0)	(5, 5)	(1, 0)

Example: behavioral strategy

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(a)



(b)

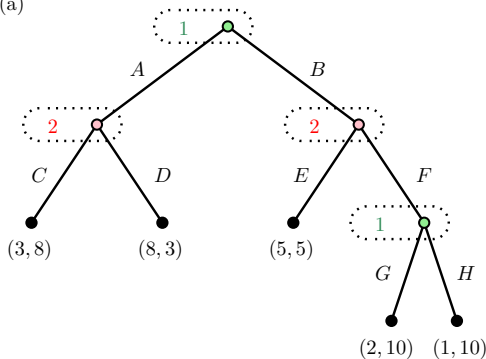
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- A strategy of player 1 that selects A with probability $\frac{1}{2}$ and G with probability $\frac{1}{3}$ is a **behavioral strategy**.

Example: behavioral strategy

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(a)



(b)

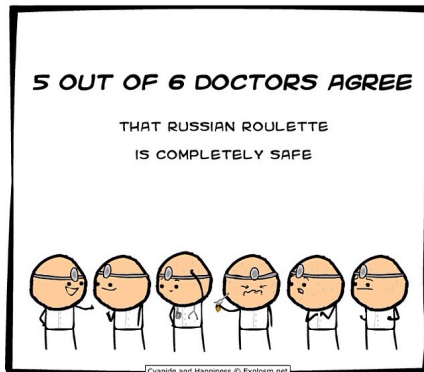
	(C, E)	(C, F)	(D, E)	(D, F)
(A, G)	(3, 8)	(3, 8)	(8, 3)	(8, 3)
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- A strategy of player 1 that selects A with probability $\frac{1}{2}$ and G with probability $\frac{1}{3}$ is a **behavioral strategy**.
- The mixed strategy $(\frac{3}{5}(A, G), \frac{2}{5}(B, H))$ is **not a behavioral strategy** for 1 as the choices made by him at the two nodes are not independent.

Example: Russian roulette

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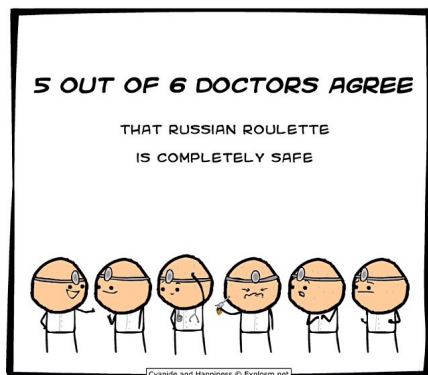
- We have two players with a six-shot revolver containing a single bullet. Each player has two moves: **shoot** or **give up**. If player gives up, he loses the game immediately. If he shoots, then he either dies or survives, in which case the other player is on turn.



Source: <https://www.memedroid.com/>

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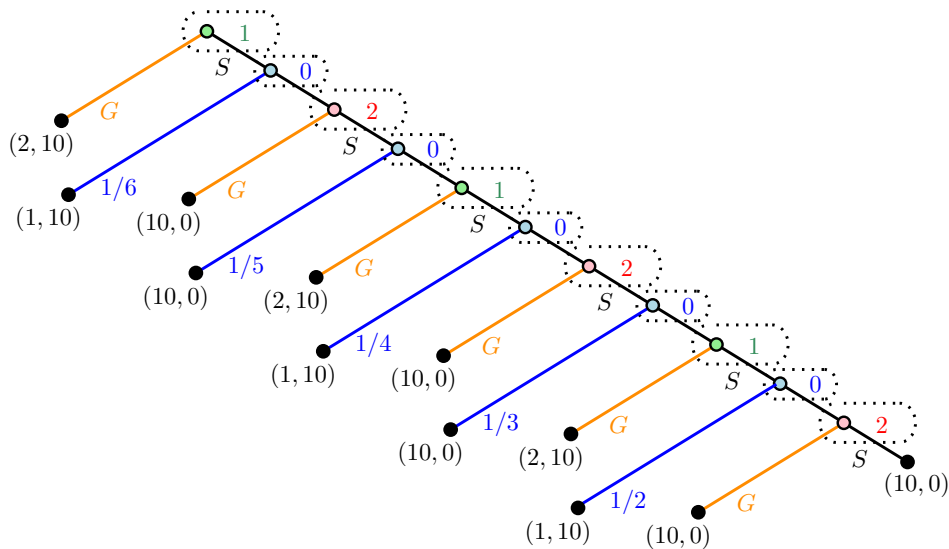
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- Consider that player 1 has payoffs $(10, 2, 1)$ for (Win, Loss, Death) and that player 2 has payoffs $(10, 0, 0)$.

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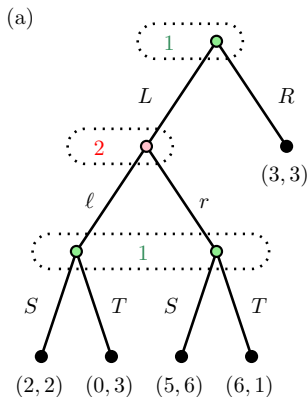
- The **Russian roulette** in the extensive form using the random player.



Example: sequence form constraints

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- An example of an imperfect-information game in extensive form (part (a)) and linear constraints in its sequence form (part (b)).



(b)

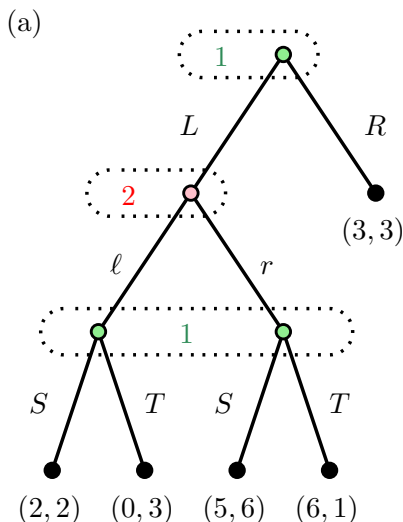
$$E = \begin{pmatrix} 1 & & & \\ -1 & 1 & 1 & \\ & -1 & 1 & 1 \end{pmatrix}, \quad e = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},$$

$$F = \begin{pmatrix} 1 & & \\ -1 & 1 & 1 \end{pmatrix}, \quad f = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Example: sequence form payoff matrices

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- An example of an imperfect-information game in extensive form (part (a)) and its sequence form payoff matrices (part (b)).



(b)

$$A = \begin{pmatrix} \emptyset & l & r \\ 3 & & \end{pmatrix} \begin{matrix} \emptyset \\ L \\ R \\ LS \\ LT \end{matrix}$$

$$B = \begin{pmatrix} \emptyset & l & r \\ 3 & & \end{pmatrix} \begin{matrix} \emptyset \\ L \\ R \\ LS \\ LT \end{matrix}$$

Sequence form payoff matrices A and B are shown, corresponding to the game tree in (a).



- More about games in extensive form + implementation of the algorithms will be taught in a new lecture by [Martin Schmid](#).

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Thank you for your attention.