Algorithmic game theory

Martin Balko

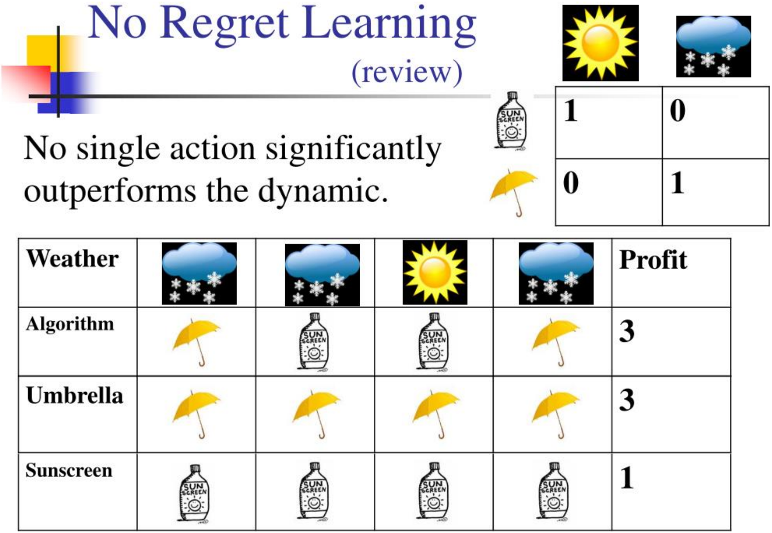
8th lecture

November 28th 2022

Regret minimization

Example

Example



Source: No regret algorithms in games (Georgios Piliouras)

Polynomial weights algorithm

Polynomial weights algorithm

- An algorithm that works with very small external regret.

Polynomial weights algorithm

- An algorithm that works with very small external regret.
- The algorithm gives $L_{\mathrm{PW}}^{T} \leq L_{min}^{T} + 2\sqrt{T \ln N}$.

Polynomial weights algorithm

- An algorithm that works with very small external regret.
- The algorithm gives $L^T_{\mathrm{PW}} \leq L^T_{min} + 2\sqrt{T \ln N}$.

Algorithm 0.4: POLYNOMIAL WEIGHTS ALGORITHM(X, T, η)

Input : A set of actions $X = \{1, \ldots, N\}$, $T \in \mathbb{N}$, and $\eta \in (0, 1/2]$.
Output : A probability distribution p^t for every time step t.
$w_i^1 \leftarrow 1$ for every $i \in X$,
$p^1 \leftarrow (1/N, \ldots, 1/N)$,
for $t = 2, \ldots, T$
do $\begin{cases} w_i^t \leftarrow w_i^{t-1}(1 - \eta \ell_i^{t-1}), \\ W^t \leftarrow \sum_{i \in X} w_i^t, \\ p_i^t \leftarrow w_i^t / W^t \text{ for every } i \in X. \end{cases}$

No-regret dynamics

No-regret dynamics

- The players use PW algorithm against each other.

No-regret dynamics

- The players use PW algorithm against each other.

Algorithm 0.7: NO-REGRET DYNAMICS(G, T, ε)

Input : A game $G = (P, A, C)$ of n players, $T \in \mathbb{N}$ and $\varepsilon > 0$.
Output : A prob. distribution p_i^t on A_i for each $i \in P$ and step t.
for every step $t = 1, \dots, T$
do Each player $i \in P$ independently chooses a mixed strategy p_i^t using an algorithm with average regret at most ε.
Each player $i \in P$ receives a loss vector $\ell_i^t = (\ell_i^t(a_i))_{a_i \in A_i}$, where $\ell_i^t(a_i) \leftarrow \mathbb{E}_{a_{-i}^t \sim \prod_{j \neq i} p_j^t}[C_i(a_i; a_{-i}^t)]$.

No-regret dynamics

- The players use PW algorithm against each other.

Algorithm 0.8: NO-REGRET DYNAMICS(G, T, ε)

Input : A game $G = (P, A, C)$ of n players, $T \in \mathbb{N}$ and $\varepsilon > 0$.
Output : A prob. distribution p_i^t on A_i for each $i \in P$ and step t.
for every step $t = 1, \ldots, T$
do Each player $i \in P$ independently chooses a mixed strategy p_i^t using an algorithm with average regret at most ε.
Each player $i \in P$ receives a loss vector $\ell_i^t = (\ell_i^t(a_i))_{a_i \in A_i}$, where $\ell_i^t(a_i) \leftarrow \mathbb{E}_{a_{-i}^t \sim \prod_{j \neq i} p_j^t}[C_i(a_i; a_{-i}^t)]$.

- Gave a new proof of the Minimax theorem in zero-sum games.

No-regret dynamics

- The players use PW algorithm against each other.

Algorithm 0.9: NO-REGRET DYNAMICS(G, T, ε)

Input : A game $G = (P, A, C)$ of n players, $T \in \mathbb{N}$ and $\varepsilon > 0$.
Output : A prob. distribution p_i^t on A_i for each $i \in P$ and step t.
for every step $t = 1, \ldots, T$
do $\begin{cases} \text{Each player } i \in P \text{ independently chooses a mixed strategy} \\ p_i^t \text{ using an algorithm with average regret at most } \varepsilon. \\ \text{Each player } i \in P \text{ receives a loss vector } \ell_i^t = (\ell_i^t(a_i))_{a_i \in A_i}, \\ \text{where } \ell_i^t(a_i) \leftarrow \mathbb{E}_{a_{-i}^t \sim \prod_{j \neq i} p_j^t}[C_i(a_i; a_{-i}^t)]. \end{cases}$

- Gave a new proof of the Minimax theorem in zero-sum games.
- Today we will see some new applications in general games.

Example: Coarse correlated equilibrium

Example: Coarse correlated equilibrium

- Giving probability $1/6$ to each red outcome gives coarse correlated equilibrium in the Rock-Paper-Scissors game.

	Rock	Paper	Scissors
Rock	(0,0)	(-1,1)	(1,-1)
Paper	(1,-1)	(0,0)	(-1,1)
Scissors	(-1,1)	(1,-1)	(0,0)

Example: Coarse correlated equilibrium

- Giving probability $1/6$ to each red outcome gives coarse correlated equilibrium in the Rock-Paper-Scissors game.

	Rock	Paper	Scissors
Rock	(0,0)	(-1,1)	(1,-1)
Paper	(1,-1)	(0,0)	(-1,1)
Scissors	(-1,1)	(1,-1)	(0,0)

- Then, the expected payoff of each player is 0 and deviating to any pure strategy gives the expected payoff 0.

Example: Coarse correlated equilibrium

- Giving probability $1/6$ to each red outcome gives coarse correlated equilibrium in the Rock-Paper-Scissors game.

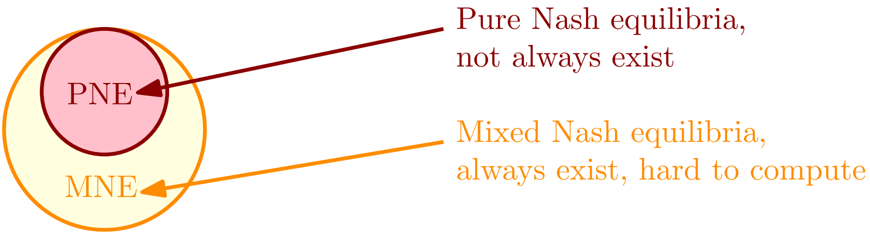
	Rock	Paper	Scissors
Rock	(0,0)	(-1,1)	(1,-1)
Paper	(1,-1)	(0,0)	(-1,1)
Scissors	(-1,1)	(1,-1)	(0,0)

- Then, the expected payoff of each player is 0 and deviating to any pure strategy gives the expected payoff 0.
- It is not a correlated equilibrium though.

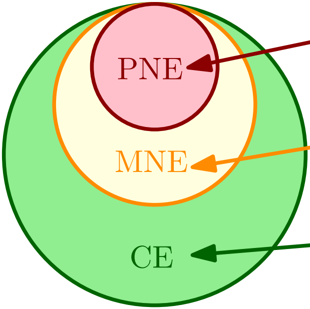
Hierarchy of Nash equilibria

Hierarchy of Nash equilibria

Hierarchy of Nash equilibria



Hierarchy of Nash equilibria

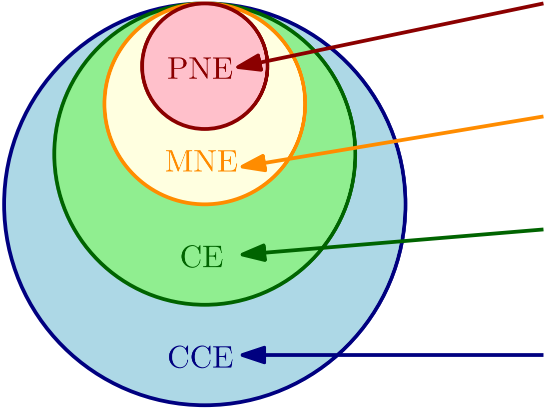


Pure Nash equilibria,
not always exist

Mixed Nash equilibria,
always exist, hard to compute

Correlated equilibria,
easy to compute

Hierarchy of Nash equilibria



Pure Nash equilibria,
not always exist

Mixed Nash equilibria,
always exist, hard to compute

Correlated equilibria,
easy to compute

Coarse correlated equilibria,
even easier to compute

No-swap-regret dynamics

No-swap-regret dynamics

- Exactly the same as No-regret dynamics, but operates with swap regret instead of external regret.

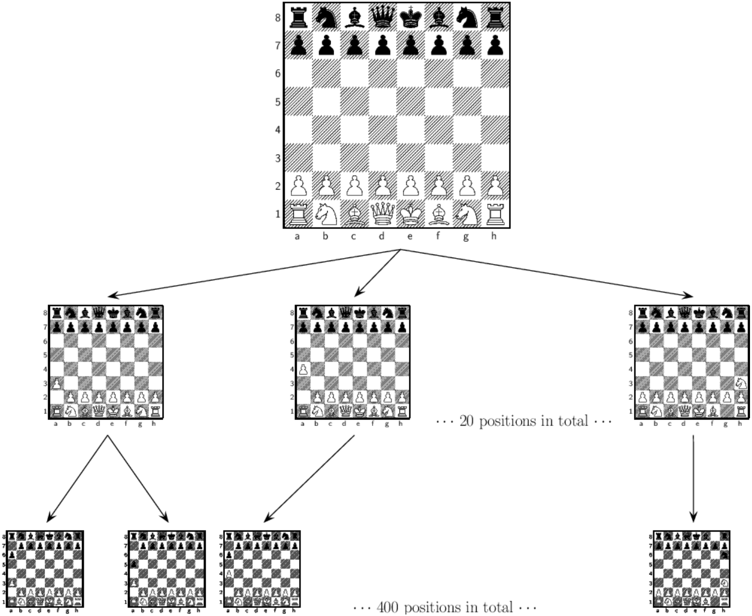
No-swap-regret dynamics

- Exactly the same as No-regret dynamics, but operates with swap regret instead of external regret.

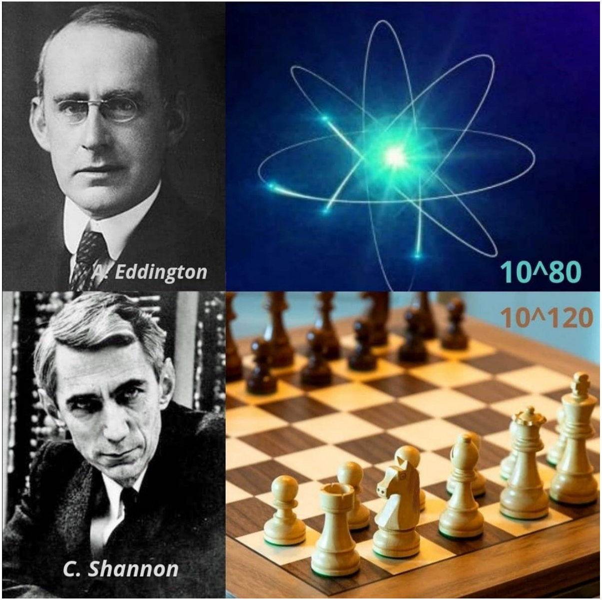
Algorithm 0.12: NO-REGRET DYNAMICS(G, T, ε)

Input : A game $G = (P, A, C)$ of n players, $T \in \mathbb{N}$ and $\varepsilon > 0$.
Output : A prob. distribution p_i^t on A_i for each $i \in P$ and step t.
for every step $t = 1, \ldots, T$
do $\begin{cases} \text{Each player } i \in P \text{ independently chooses a mixed strategy} \\ p_i^t \text{ using an algorithm with average swap regret at most } \varepsilon. \\ \text{Each player } i \in P \text{ receives a loss vector } \ell_i^t = (\ell_i^t(a_i))_{a_i \in A_i}, \\ \text{where } \ell_i^t(a_i) \leftarrow \mathbb{E}_{a_{-i}^t \sim \prod_{j \neq i} p_j^t}[C_i(a_i; a_{-i}^t)]. \end{cases}$

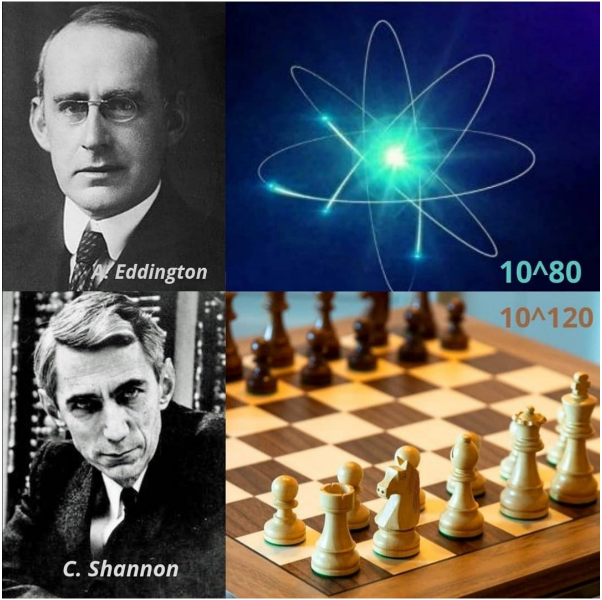
- Next week, we will study extensive form games, that is, games represented by trees.



Source: David Ravn Rasmussen: Parallel Chess Searching and Bitboards



Source: https://twitter.com/curiosite12



Source: https://twitter.com/curiosite12

Thank you for your attention.