Algorithmic game theory

Martin Balko

5th lecture

November 7th 2022

Nash equilibria in bimatrix games

What have we learned so far

What have we learned so far

- We have seen two algorithms to find NE in bimatrix games:

What have we learned so far

- We have seen two algorithms to find NE in bimatrix games:
  - the greedy algorithm,

What have we learned so far

- We have seen two algorithms to find NE in bimatrix games:
  - the greedy algorithm,
  - the Lemke–Howson algorithm.

What have we learned so far

- We have seen two algorithms to find NE in bimatrix games:
  - the greedy algorithm,
  - the Lemke–Howson algorithm.
- Both these algorithms have exponential running time in the worst case.

What have we learned so far

- We have seen two algorithms to find NE in bimatrix games:
  - the greedy algorithm,
  - the Lemke–Howson algorithm.
- Both these algorithms have exponential running time in the worst case.

- Is there a chance to get an efficient algorithm?

What have we learned so far

- We have seen two algorithms to find NE in bimatrix games:
  - the greedy algorithm,
  - the Lemke–Howson algorithm.
- Both these algorithms have exponential running time in the worst case.

- Is there a chance to get an efficient algorithm?
- NASH = the problem of finding NE in bimatrix games.

The class PPAD

The class PPAD

- The class PPAD was introduced in 1994 by Papadimitrou.

The class PPAD

- The class PPAD was introduced in 1994 by Papadimitrou.

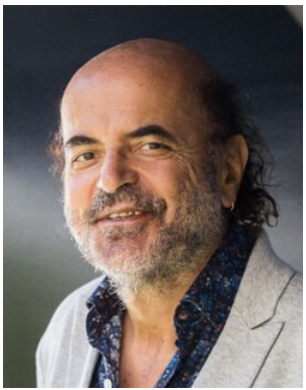


Figure: Christos Papadimitriou (born 1949).

Source: https://cs.columbia.edu

The class PPAD

- The class PPAD was introduced in 1994 by Papadimitrou.

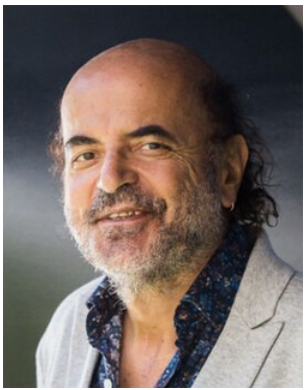


Figure: Christos Papadimitriou (born 1949).

Source: https://cs.columbia.edu

- Abbreviation for "Polynomial Parity Arguments on Directed graphs".

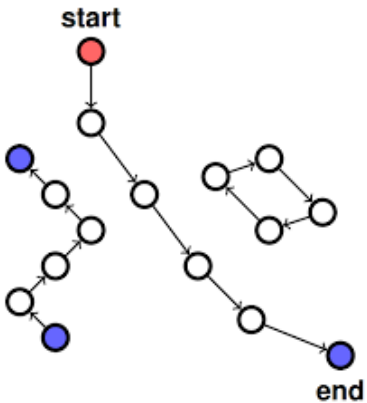
Problems from PPAD: End-of-the-line

Problems from PPAD: End-of-the-line

- For an oriented graph G with max. indegree and outdegree 1 and a source in G, find a target in G. The graph is given by a polynomial-time computable function $f(v)$ that returns predecessor and successor of v.

Problems from PPAD: End-of-the-line

- For an oriented graph G with max. indegree and outdegree 1 and a source in G, find a target in G. The graph is given by a polynomial-time computable function $f(v)$ that returns predecessor and successor of v.



Source: R. Savani "Polymatrix Games" Tutorial at WINE 2015

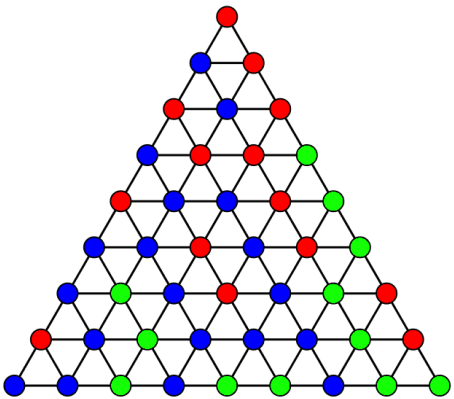
Problems from PPAD: Sperner's lemma

Problems from PPAD: Sperner's lemma

- Given a legal 3-coloring of a triangulated triangle, find a triangle with vertices colored by all 3 colors.

Problems from PPAD: Sperner's lemma

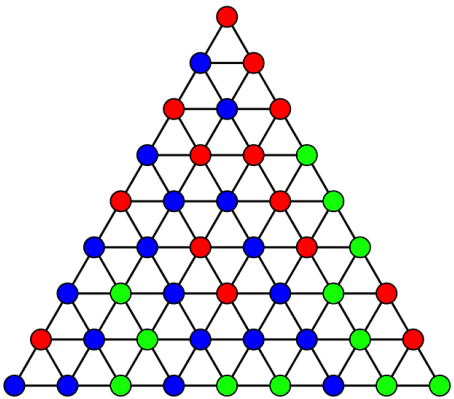
- Given a legal 3-coloring of a triangulated triangle, find a triangle with vertices colored by all 3 colors.



Source: https://lesswrong.com

Problems from PPAD: Sperner's lemma

- Given a legal 3-coloring of a triangulated triangle, find a triangle with vertices colored by all 3 colors.



Source: https://lesswrong.com

- Discrete version of the Brouwer's fixed point theorem.

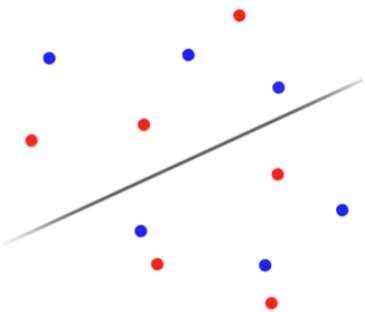
Problems from PPAD: Ham sandwich theorem

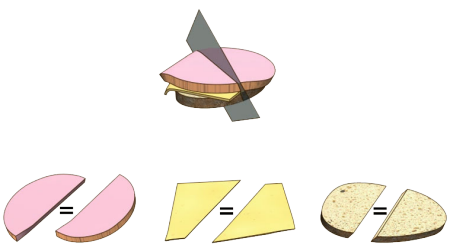
Problems from PPAD: Ham sandwich theorem

- Given n sets of $2n$ points in $\mathbb{R}^n$, find a hyperplane H that contains exactly n points from each of the sets in each open halfspace determined by H.

Problems from PPAD: Ham sandwich theorem

- Given n sets of $2n$ points in $\mathbb{R}^n$, find a hyperplane H that contains exactly n points from each of the sets in each open halfspace determined by H.





Sources: https://ejarzo.github.io and https://curiosamathematica.tumblr.com

Other notions of equilibria

- The concept of correlated equilibria was introduced by Robert Aumann, who received a Nobel prize in economics for his work in game theory.

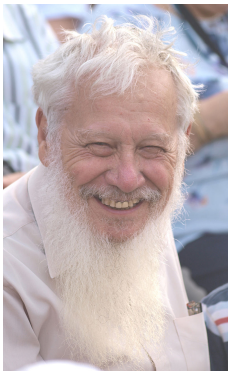


Figure: Robert Aumann (born 1930).

Sources: https://en.wikipedia.org and https://slideslive.com/38910863/strategic-information-theory

- The concept of correlated equilibria was introduced by Robert Aumann, who received a Nobel prize in economics for his work in game theory.

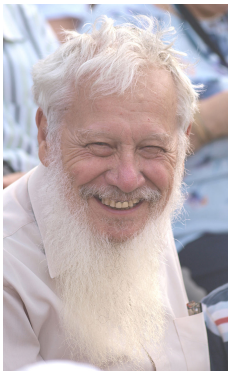


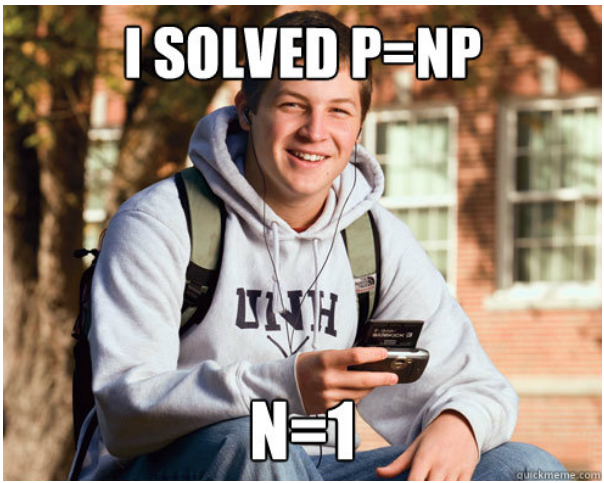
Figure: Robert Aumann (born 1930).

Sources: https://en.wikipedia.org and https://slideslive.com/38910863/strategic-information-theory

- In 2018, Robert Aumann visited Prague and gave a lecture at Prague mathematical colloquium. You can see the lecture here: https://slideslive.com/38910863/strategic-information-theory.

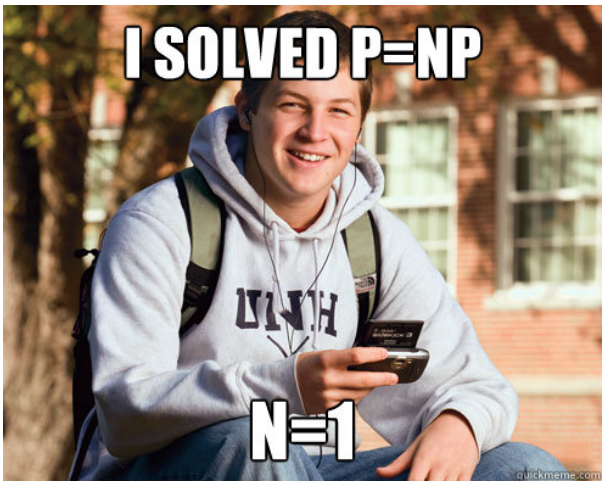
- "P=NP" is one of the most important problems in computer science. The website https://www.win.tue.nl/~gwoegi/P-versus-NP.htm contains a collection of over 100 attempts to solve it.

- "P=NP" is one of the most important problems in computer science. The website https://www.win.tue.nl/~gwoegi/P-versus-NP.htm contains a collection of over 100 attempts to solve it.



Source: https://quickmeme.com

- "P=NP" is one of the most important problems in computer science. The website https://www.win.tue.nl/~gwoegi/P-versus-NP.htm contains a collection of over 100 attempts to solve it.



Source: https://quickmeme.com

Thank you for your attention.