# Algorithmic game theory

Martin Balko

## 3rd lecture

October 24th 2022

# Proof of the Minimax Theorem

# The Minimax Theorem

# The Minimax Theorem

- For every zero-sum game, worst-case optimal strategies for both players exist and can be efficiently computed. There is a number $v$ such that, for any worst-case optimal strategies $x^*$ and $y^*$, the strategy profile $(x^*, y^*)$ is a Nash equilibrium and $\beta(x^*) = (x^*)^\top M y^* = \alpha(y^*) = v$.

# The Minimax Theorem

- For every zero-sum game, worst-case optimal strategies for both players exist and can be efficiently computed. There is a number $v$ such that, for any worst-case optimal strategies $x^*$ and $y^*$, the strategy profile $(x^*, y^*)$ is a Nash equilibrium and $\beta(x^*) = (x^*)^\top M y^* = \alpha(y^*) = v$.

Figure: John von Neumann (1903–1957) and Oskar Morgenstern (1902–1977).

Sources: https://en.wikiquote.org and https://austriainusa.org

# Duality of linear programming

# Duality of linear programming

| | Primal linear program | Dual linear program |
|---|---|---|
| Variables | $x_1, \ldots, x_m$ | $y_1, \ldots, y_n$ |
| Matrix | $A \in \mathbb{R}^{n \times m}$ | $A^\top \in \mathbb{R}^{m \times n}$ |
| Right-hand side | $b \in \mathbb{R}^n$ | $c \in \mathbb{R}^m$ |
| Objective function | $\max c^\top x$ | $\min b^\top y$ |
| Constraints | $i$th constraint has $\leq$ | $y_i \geq 0$ |
| | $\geq$ | $y_i \leq 0$ |
| | $=$ | $y_i \in \mathbb{R}$ |
| | $x_j \geq 0$ | $j$th constraint has $\geq$ |
| | $x_j \leq 0$ | $\leq$ |
| | $x_j \in \mathbb{R}$ | $=$ |

Table: A recipe for making dual programs.

# Nash equilibria in bimatrix games

# Nash equilibria in bimatrix games

# Nash equilibria in bimatrix games

- We try to design an algorithm for finding Nash equilibria in games of two players (bimatrix games).

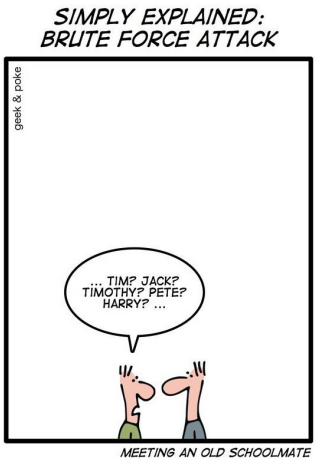
# Nash equilibria in bimatrix games

- We try to design an algorithm for finding Nash equilibria in games of two players (bimatrix games).
- We state some observations that yield a brute-force algorithm.


Source: https://pinterest.com

# Nash equilibria in bimatrix games

- We try to design an algorithm for finding Nash equilibria in games of two players (bimatrix games).
- We state some observations that yield a brute-force algorithm.

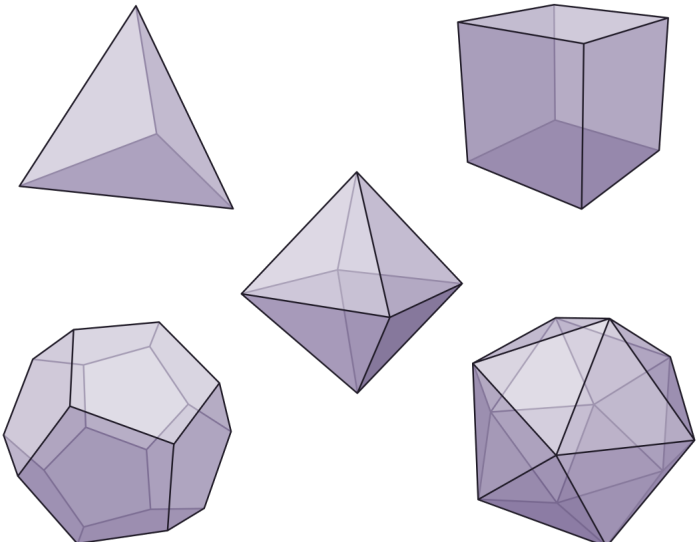



Source: https://pinterest.com

- Later, we show the currently best known algorithm for this problem.

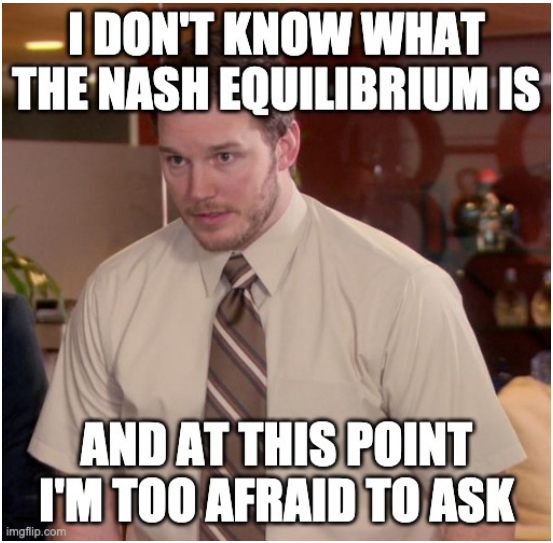
# Examples of polytopes

# Examples of polytopes

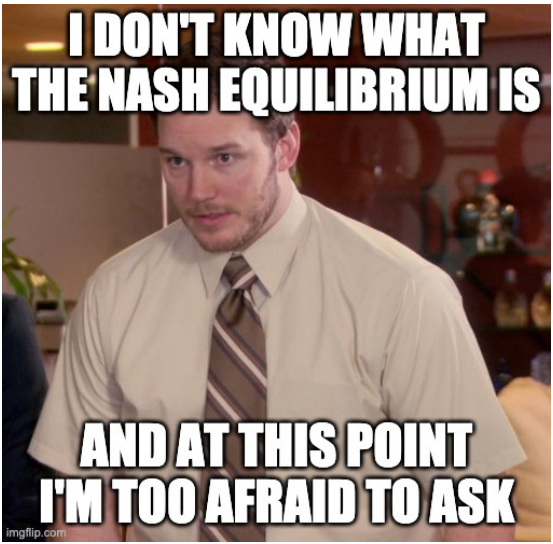


- Next lecture we learn the Lemke–Howson algorithm, the best known algorithm to find Nash equilibria in bimatrix games.

- Next lecture we learn the Lemke–Howson algorithm, the best known algorithm to find Nash equilibria in bimatrix games.



- Next lecture we learn the Lemke–Howson algorithm, the best known algorithm to find Nash equilibria in bimatrix games.




Thank you for your attention.