

Algorithmic game theory

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2nd lecture

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Proof of Nash's Theorem

Nash's Theorem

- Every normal-form game has a Nash equilibrium.

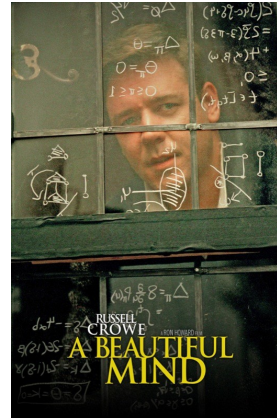


Figure: John Forbes Nash Jr. (1928–2015) and his depiction in the movie *A Beautiful mind*.

Brouwer's Fixed Point Theorem

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Figure: L. E. J. Brouwer (1881–1966).

Pareto optimality

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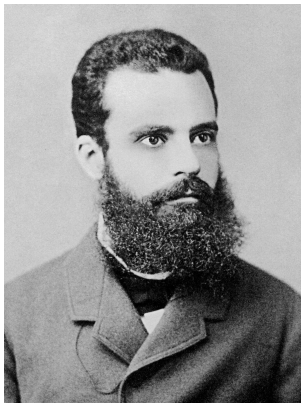


Figure: Vilfredo Pareto (1848–1923).

Sources: <https://en.wikipedia.org>

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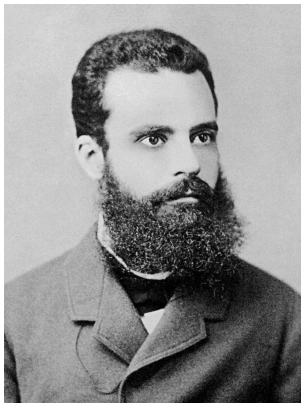


Figure: Vilfredo Pareto (1848–1923).

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- **Pareto principle**: for many outcomes roughly 80% of consequences come from 20% of the causes.

Zero-sum games

The Minimax Theorem

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Figure: John von Neumann (1903–1957) and Oskar Morgenstern (1902–1977).

