Algorithmic game theory

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11th lecture

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Myerson's lemma

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- Is there an awesome auction for single-parameter environments?

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Figure: Roger Myerson (born 1951) receiving a Nobel prize in economics.

Sources: https://en.wikipedia.org and https://twitter.com

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Theorem (Myerson's lemma)

In a single-parameter environment, the following three claims hold.

(a) An allocation rule is implementable if and only if it is monotone.

(b) If an allocation rule x is monotone, then there exists a unique payment rule p such that the mechanism (x, p) is DSIC (assuming that $b_i = 0$ implies $p_i(b) = 0$).

(c) For every $i \in \{1, \ldots, n\}$, the payment rule p is given by

$$p_i(b_i; b_{-i}) = \int_0^{b_i} z \cdot \frac{\mathrm{d}}{\mathrm{d}z} x_i(z; b_{-i}) \, \mathrm{d}z.$$

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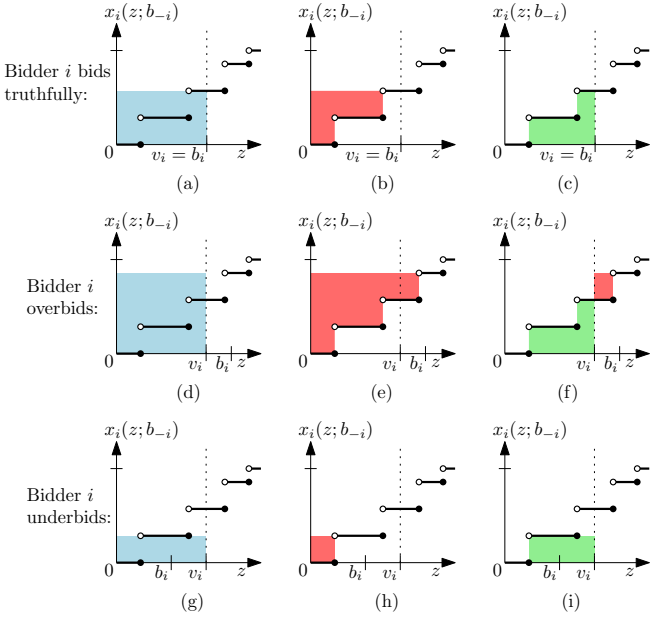

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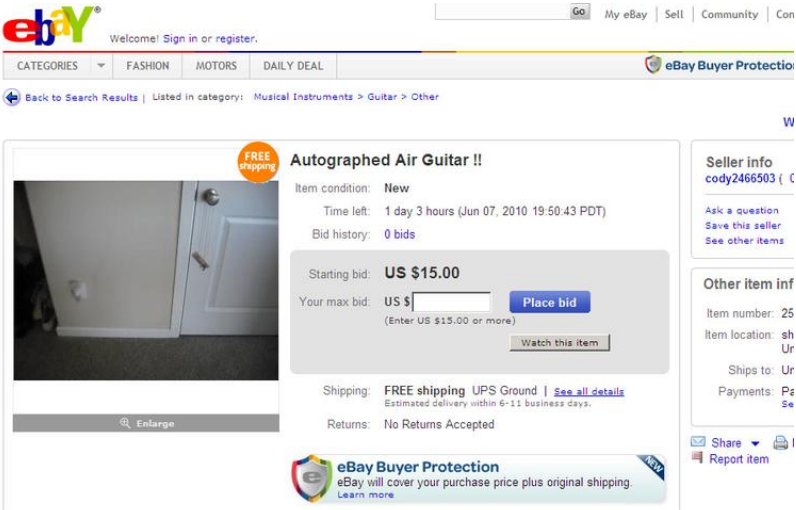

Source: https://goodgearguide.com.au

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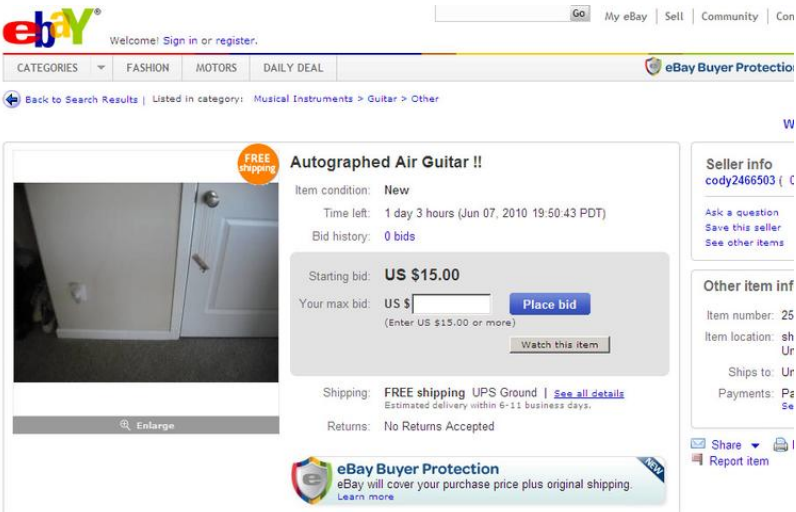

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Thank you for your attention.