

Algorithmic game theory

Martin Balko

11th lecture

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Myerson's lemma

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- We then generalized single-item auctions to **single-parameter environments**.
- Is there an awesome auction for single-parameter environments?

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Figure: Roger Myerson (born 1951) receiving a Nobel prize in economics.

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Theorem (Myerson's lemma)

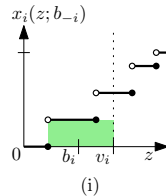
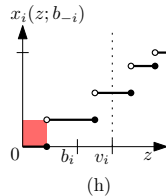
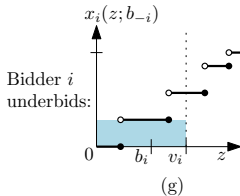
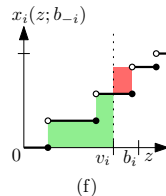
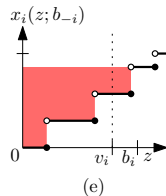
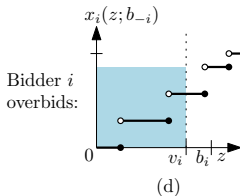
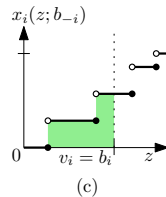
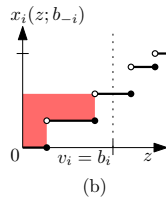
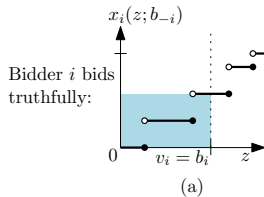
In a single-parameter environment, the following three claims hold.

- (a) An allocation rule is implementable if and only if it is monotone.
- (b) If an allocation rule x is monotone, then there exists a unique payment rule p such that the mechanism (x, p) is DSIC (assuming that $b_i = 0$ implies $p_i(b) = 0$).
- (c) For every $i \in \{1, \dots, n\}$, the payment rule p is given by

$$p_i(b_i; b_{-i}) = \int_0^{b_i} z \cdot \frac{d}{dz} x_i(z; b_{-i}) dz.$$

Proof of Myerson's lemma

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The screenshot shows an eBay listing for an "Autographed Air Guitar !!". The listing includes a photograph of the guitar, a starting bid of US \$15.00, and a "Place bid" button. The seller is "cody2466503". The listing also features "eBay Buyer Protection" and "FREE shipping" badges. The navigation bar at the top includes the eBay logo, a search bar, and links for "My eBay", "Sell", "Community", and "Con". The breadcrumb trail indicates the item is in the "Musical Instruments > Guitar > Other" category.

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CATEGORIES FASHION MOTORS DAILY DEAL eBay Buyer Protection

Back to Search Results | Listed in category: Musical Instruments > Guitar > Other

Autographed Air Guitar !!

Item condition: **New**

Time left: 1 day 3 hours (Jun 07, 2010 19:50:43 PDT)

Bid history: 0 bids

Starting bid: **US \$15.00**

Your max bid: US \$ **Place bid**

(Enter US \$15.00 or more)

Watch this item

Shipping: **FREE shipping** UPS Ground | [See all details](#)
Estimated delivery within 6-11 business days.

Returns: No Returns Accepted

eBay Buyer Protection
eBay will cover your purchase price plus original shipping.
[Learn more](#)

Seller info
cody2466503 ()

[Ask a question](#)
[Save this seller](#)
[See other items](#)

Other item info

Item number: 25
Item location: sh
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Ships to: Ur
Payments: P
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Source: <https://goodgearguide.com.au>

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Thank you for your attention.