### Algorithmic game theory

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1st lecture

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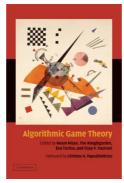
### Basic info

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- Webpage: https://kam.mff.cuni.cz/~balko/ath2223/ATH.html
  - $\,\circ\,$  lecture info, topics covered, presentations, lecture notes  $\ldots\,$

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- Recommended literature:
  - M. Balko: Algorithmic game theory: lecture notes.
  - $\circ\,$  The notes are still under construction. Comments are welcome.



#### Figure: Algorithmic game theory by Nisan et al.

Source: https://amazon.com

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Zdroj: https://quantamagazine.org

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- Several real-word applications.
- More than ten game theorists have won the Nobel Prize in economics.



## Sylabus

• Preliminary plan:

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- Finding Nash equilibria
  - Nash equilibria and Nash's Theorem,
  - zero-sum games,
  - o bimatrix games and the Lemke-Howson algorithm,
  - $\circ\;$  other notions of equilibria,
  - regret minimization.
- Mechanism design,
  - $\circ\,$  auctions (Vickrey),
  - $\circ~$  Myerson's lemma and its applications,
  - revenue maximization.

# Finding Nash equilibria

### Nash's Theorem

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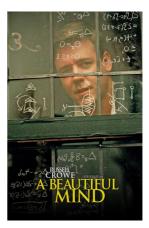


Figure: John Forbes Nash Jr. (1928–2015) and his depiction in the movie A Beautiful mind.

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#### Figure: L. E. J. Brouwer (1881–1966).

Source: https://arxiv.org/pdf/1612.06820.pdf



#### Figure: John Forbes Nash Jr. receiving a Nobel prize for economics.

Source: https://pbs.org



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Thank you for your attention.