

Algorithmic game theory

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6th lecture

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Nash equilibria in bimatrix games

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- **NASH** = the problem of finding NE in bimatrix games.

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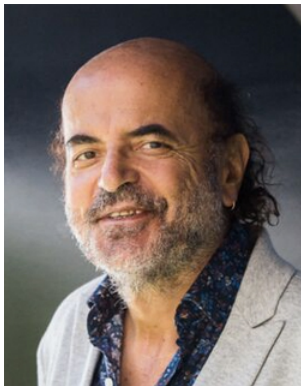


Figure: Christos Papadimitriou (born 1949).

Source: <https://cs.columbia.edu>

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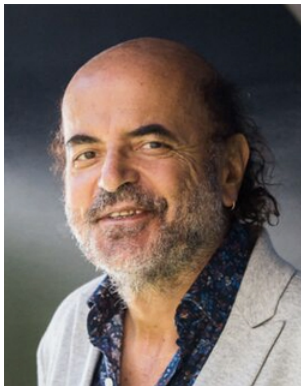


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- Abbreviation for “Polynomial Parity Arguments on Directed graphs”.

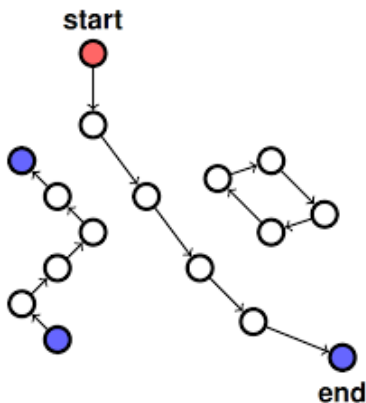
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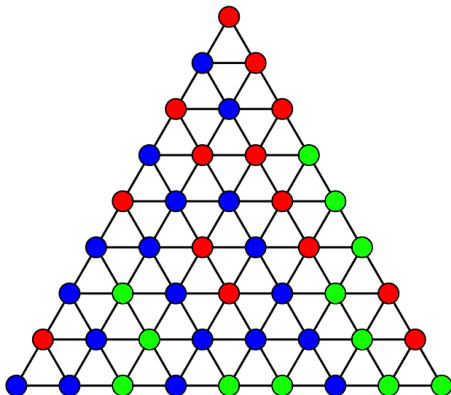
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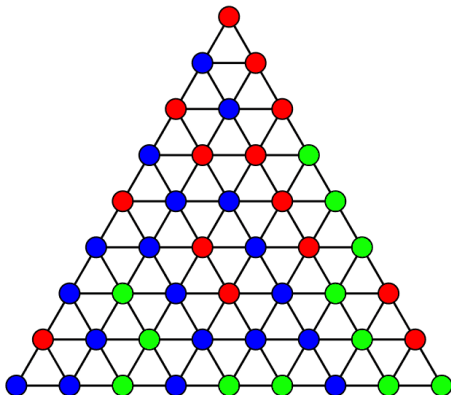
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Source: <https://lesswrong.com>

Problems from PPA: Sperner's lemma

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- Discrete version of the **Brouwer's fixed point theorem**.

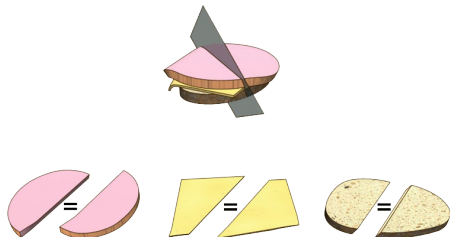
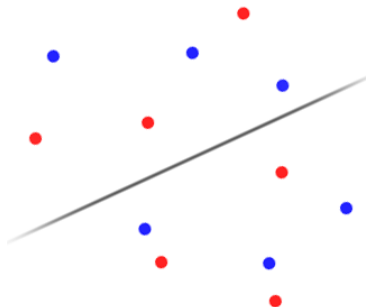
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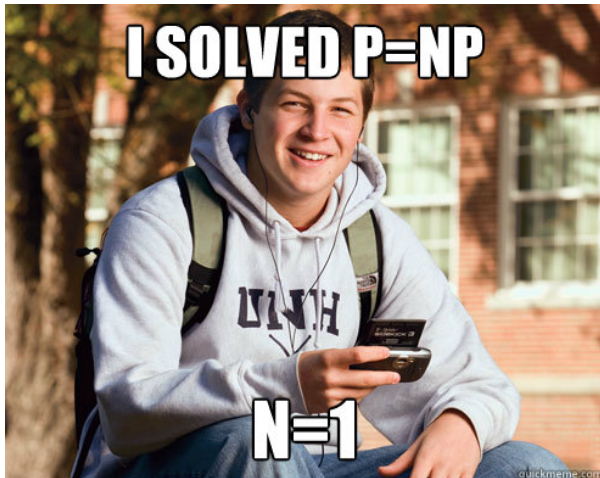


Sources: <https://ejarzo.github.io> and <https://curiosamathematica.tumblr.com>

Other notions of equilibria

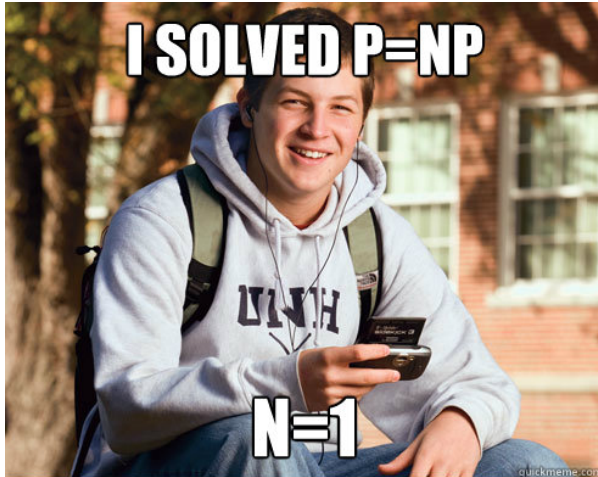


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Thank you for your attention.