Algorithmic game theory — Homework 4[1]
Revenue maximizing auctions

assigned 17.12.2020, deadline 7.1.2021

Homework 1. *Let F be the uniform probability distribution on $[0,1]$. Consider a single-item auction with two bidders 1 and 2 that have probability distributions $F_1 = F$ and $F_2 = F$ on their valuations. Prove that the expected revenue obtained by the Vickrey auction with reserve $1/2$ is $5/12$.* **[3]**

Homework 2. *Compute the virtual valuation function of the following probability distributions and show which of these distributions are regular (meaning the virtual valuation function is strictly increasing).*

(a) *The distribution given by $F(z) = 1 - \frac{1}{(z+1)^c}$ on $[0,\infty)$, where $c > 0$ is some constant,* **[2]**

(b) *Consider the probability distribution F in part (a), with $c = 1$. Argue that when bidder valuations are drawn from F, it is not necessarily the case that the expected revenue of an auction equals its expected virtual social surplus. To reconcile this observation with the theorem from the lecture about maximizing revenue, identify which assumption of this result is violated in your example.* **[3]**

Homework 3. *Consider an arbitrary single-parameter environment with feasible set X and n bidders. For every bidder i, the valuation of i is drawn from a regular probability distribution F_i, so the virtual valuation function φ_i of i is strictly increasing. Consider the allocation rule x that maximizes the virtual social surplus for any given input v. That is,*

$$x(v) = \mathrm{argmax}_{(x_1,\ldots,x_n)\in X} \sum_{i=1}^{n} \varphi_i(v_i) x_i.$$

Prove that this allocation rule is monotone. **[2]**

Remark: You should assume that ties are broken in a deterministic and consistent way, such as lexicographically.

[1] Information about the course can be found at http://kam.mff.cuni.cz/~balko/