

Exercise sheet #1
Set Theory 2024

Exercise 1. Let $a = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$ and $b = \{\{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$.

1. How many elements do a and b contain?
2. Calculate $\bigcup a, \bigcap b$.

Exercise 2. Prove or give a counterexample.

1. $a \subseteq b \leftrightarrow a \cap b = a \leftrightarrow a \cup b = b \leftrightarrow a \setminus b = \emptyset$.
2. $a \setminus b = (a \cup b) \setminus b = a \setminus (a \cup b)$.
3. $a \cap b = a \setminus (a \setminus b)$.
4. $a \Delta (b \cup c) = (a \Delta b) \cup c$.
5. $a \setminus (b \setminus c) = (a \setminus b) \cup (a \cap c)$.

Exercise 3. Prove that $\{(a, b) : a \in b\}$ is not a set.

Exercise 4. Prove that $\bigcap s$ is defined for all $s \neq \emptyset$. What is wrong with $\bigcap \emptyset$?