

Exercise sheet #6

Set Theory 2023

Exercise 1. Let S be a set. Prove that there exists a set T such that for all $X \in S$, $|X| < |Y|$.

Exercise 2. Prove that if X and Y are sets which satisfy $|X| \leq |Y|$ and $X \neq \emptyset$, then there exists a surjective function $Y \rightarrow X$.

Exercise 3. Suppose that A is at most countable and B is uncountable. Prove that $B \setminus A$ is uncountable.

Exercise 4. If S is uncountable and $S \subseteq T$, then T is uncountable.

Exercise 5. Show that $|\mathbb{R}| = |\mathcal{P}(\mathbb{N})|$.

Definition 1.

1. A real number a is *algebraic* if there exists a polynomial $p \in \mathbb{Z}[x]$ such that $p(a) = 0$.
2. A *binary sequence* is a function whose codomain is 2.
3. If X and Y are sets, then X^Y is the set of all functions $Y \rightarrow X$.

Exercise 6. Decide whether each of the following sets is finite, countable or uncountable and argue why.

1. The set of all binary sequences whose domain is a natural number.
2. The set of all polynomials in one variable with coefficients in \mathbb{Q} .
3. The set of all algebraic numbers.
4. 2^ω .