Exercise sheet #3
Set Theory 2023

Definition 1.

1. If a is a set, we define $s(a)$ as $a \cup \{a\}$.
2. A set a is said to be *inductive* if $\varnothing \in a$ and $\forall b \in a(s(b) \in a)$.
3. A set a is said to be *transitive* if $b \in c \in a$ implies $b \in a$.

Exercise 1. Show that if a and b are inductive sets, then $a \cap b$ is also inductive.

Exercise 2. A set n is said to be a *natural number* if it belongs to all inductive sets. Let ω denote the set of all natural numbers; prove

1. ω is transitive,
2. ω is inductive,
3. ω is linearly ordered by $\in$.

Exercise 3. Prove that a set a is transitive if and only if $\bigcup a \subseteq a$.

Exercise 4. Prove that if t is a transitive set, then $\bigcup s(t) = t$.

Exercise 5. Prove that if a is an inductive set, then $b \coloneqq \{c \in a : c \subseteq a\}$ is inductive.