Exercise sheet #1
Set Theory 2023

Exercise 1. Let $|$ be a logical operator such that $P|Q$ is false only when both P and Q are true. Show that $\neg P$, $P \wedge Q$, $P \vee Q$, and $P \Rightarrow Q$ are equivalent to statements using only $P, Q, |$.

Exercise 2. Compute the following truth tables:

1. $P \vee \neg P$
2. $\neg(P \wedge \neg P)$
3. $(P \Rightarrow \neg\neg P) \wedge (\neg\neg P \Rightarrow P)$
4. $(\neg P \vee Q) \wedge (\neg R \vee P)$
5. $P \Rightarrow (P \vee Q)$
6. $(P \vee Q) \Rightarrow P$
7. $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$

Exercise 3. If w is a term and w' is a proper initial segment of w then w' is not a term.

Exercise 4. Prove that $P(x) \subset x$ is false for all sets. Conclude that there is no "set of all sets".

Exercise 5. Show that if a is a set, then $\{x | x \notin a\}$ is not a set.

Exercise 6. If a is a set, then $\bigcap a$ denotes the collection of all sets x which belong to every element of a. Prove that $\bigcap a$ is a set for all nonempty a. What is wrong with $\bigcap \varnothing$?

Exercise 7. Prove or give a counterexample.

1. $a \subseteq b \cap c \Leftrightarrow a \subset b \wedge a \subset c$
2. $a = b \Leftrightarrow a \Delta b = \varnothing$
3. $a \subseteq b \cup c \Rightarrow a \subset b \vee a \subset c$
4. $a \cap b = a \setminus (a \setminus b)$
5. $a \setminus (b \setminus c) = (a \setminus b) \cup (a \setminus c)$
6. $a \setminus b = (a \cup b) \setminus b = a \setminus (a \cup b)$