Exercise sheet #6
Set Theory 2022

This one is a homework. As usual, send your solutions by email to aranda@kam.mff.cuni.cz or bring them to S324. Deadline: 8/4/2022 14:30.

Definition 1.

1. A set X is *countable* if there exist a surjective function $\omega \to X$.
2. A real number a is *algebraic* if there exists a polynomial $p \in \mathbb{Z}[x]$ such that $p(a) = 0$.
3. A *binary sequence* is a function whose codomain is 2.
4. If X and Y are sets, then X^Y is the set of all functions $Y \to X$.

Exercise 1. Decide whether each the following sets is countable or uncountable and prove why. (All worth 11 points. 1 free point for handing in your solutions.)

1. A union of countably many finite sets.
2. The set of all binary sequences whose domain is a natural number.
3. The set of all polynomials in one variable with coefficients in $\mathbb{Q}$.
4. The set of algebraic numbers.
5. The set of all programs in Borland C.
6. A union of countably many countable sets.
7. 2^ω
8. The interval $(0,1) \subset \mathbb{R}$.
9. F, where F is the set of all functions $f\colon \omega \to \omega$ for which there exists some $1 \leq n_f \in \omega$, a strictly increasing sequence of natural numbers $(a_i)_{i \in n_f}$, and $\{b_i : i \in n_f + 1\} \subset \omega$ such that

$$f(j) = \begin{cases} b_0 & \text{if } j \leq a_0 \\ b_1 & \text{if } a_0 < j \leq a_1 \\ \dots & \\ b_{n_f} & \text{if } j > a_{n_f - 1} \end{cases}$$