

## Exercise sheet #5

### Set Theory 2022

A set  $X$  is *Dedekind finite* if there is no bijection from  $X$  to any proper  $Y \subset X$ . If  $X$  is not Dedekind finite, then it is Dedekind infinite.

**Exercise 1.** Prove that all finite sets are Dedekind finite. Is the converse true?

**Exercise 2.** If  $X$  is a Dedekind finite set and  $Y \subset X$ , then  $Y$  is Dedekind finite.

**Exercise 3.** Prove that  $X$  is Dedekind infinite if and only if there exists an injective  $f: \omega \rightarrow X$ .

**Exercise 4.** Prove that if  $X$  and  $Y$  are finite, then so are  $X \cup Y$  and  $X \times Y$ .

**Exercise 5.** If  $X \neq \emptyset$  and  $|X| \leq |Y|$ , then there exists a function  $Y \rightarrow X$ . Think about the converse.