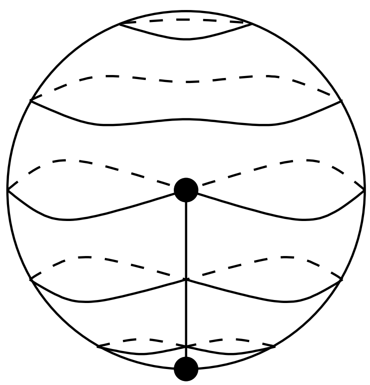
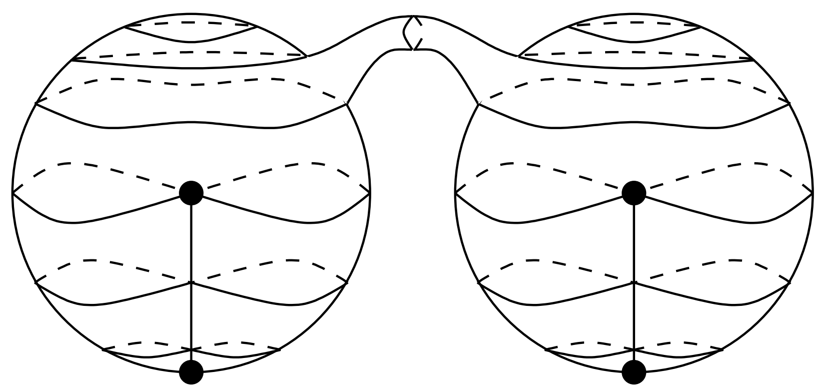
ARRANGEMENTS OF DP-RIBBONS

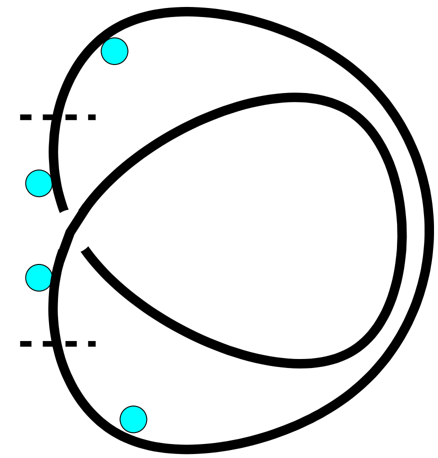
Michel Pocchiola, IMJ, U. Pierre & Marie Curie
(pocchiola@math.jussieu.fr, http://people.math.jussieu.fr/ pocchiola/)





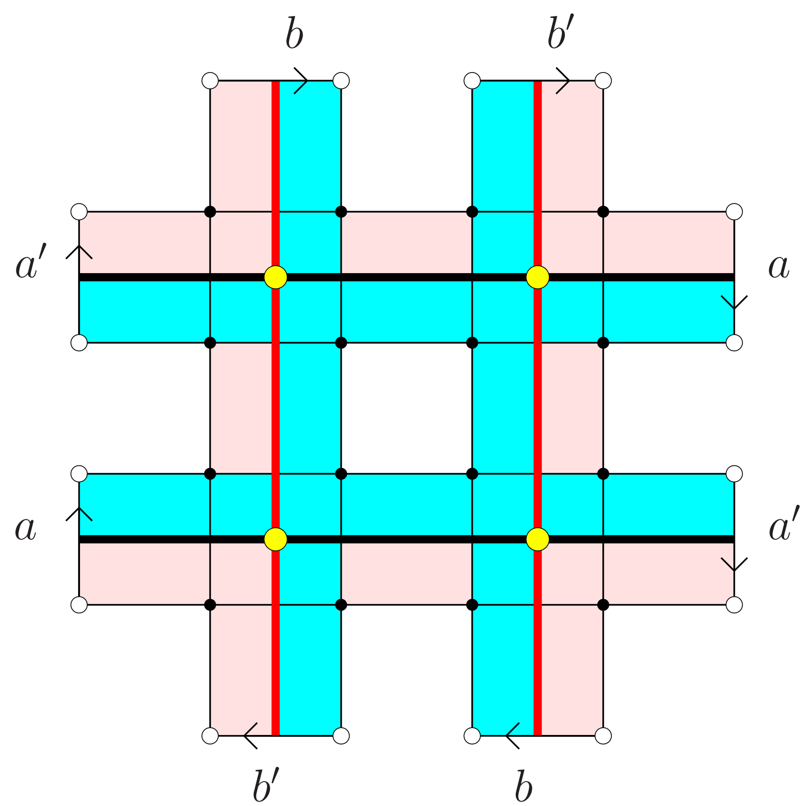
NOTES

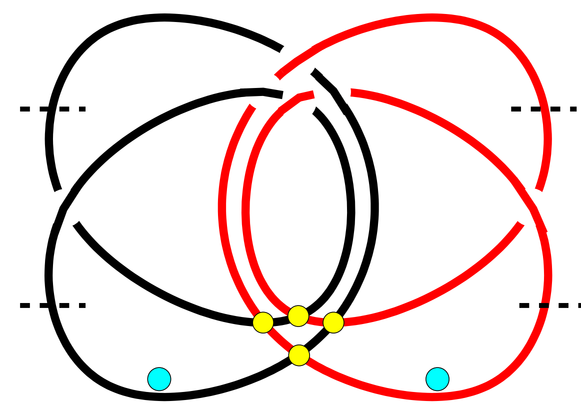
DP-ribbons



DF 1. *A DP-ribbon is a cylinder with a distinguished core circle with a distinguished side.*

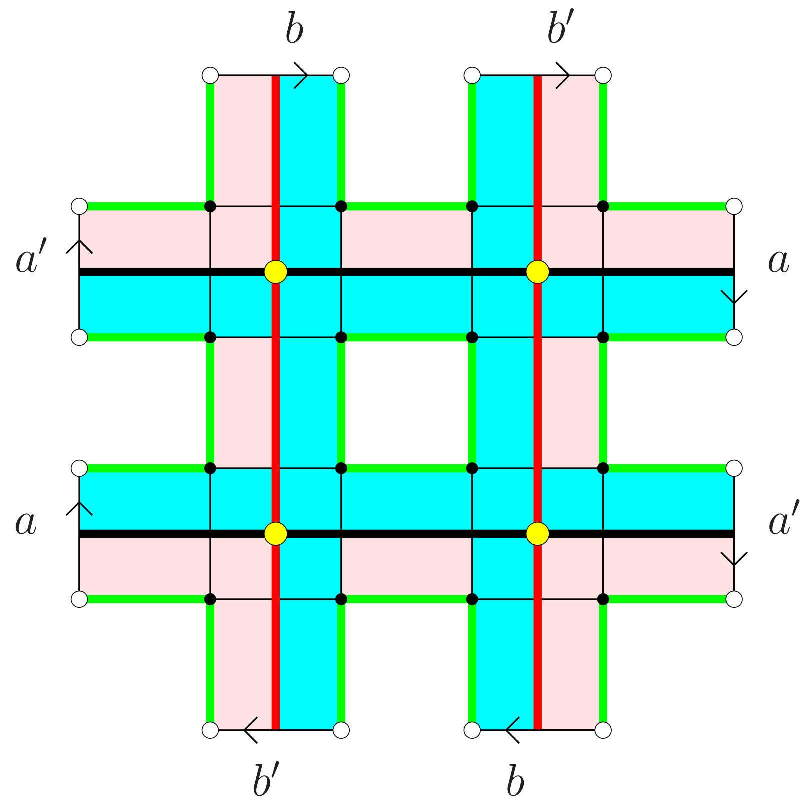
ARRANGEMENTS OF DP-RIBBONS

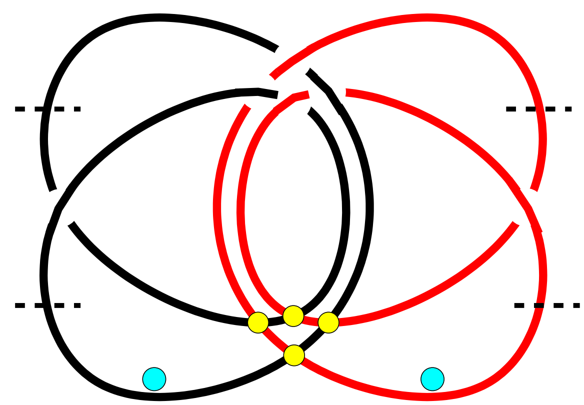




DF 2. *An arrangement of DP-ribbons is a finite family of DP-ribbons pairwise attached as shown in the above figure.*

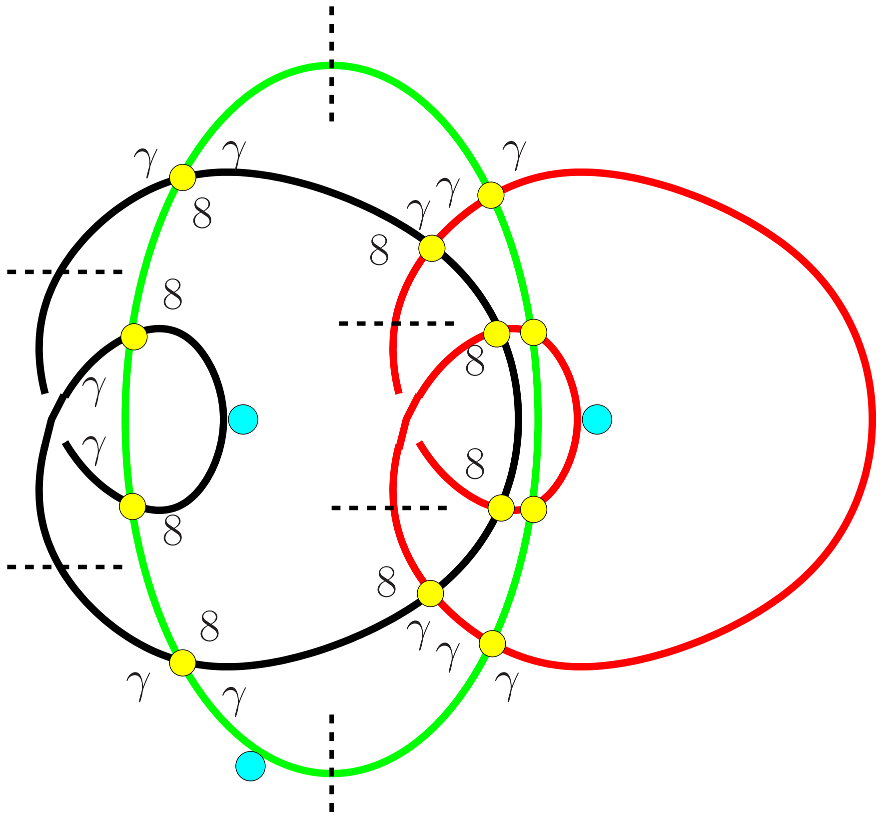
ARRANGEMENTS OF DP-RIBBONS





PP 1. *An arrangement of two DP-ribbons lives in a sphere with 1 crosscap and 5 boundaries (3 tetragons and 2 digons).*

AN ARRANGEMENT OF THREE DP-RIBBONS



10 boundaries

2 digons

6 tetragons

1 octogon

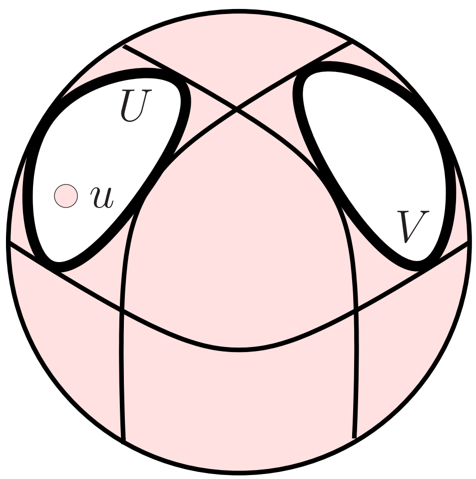
1 dodecagon

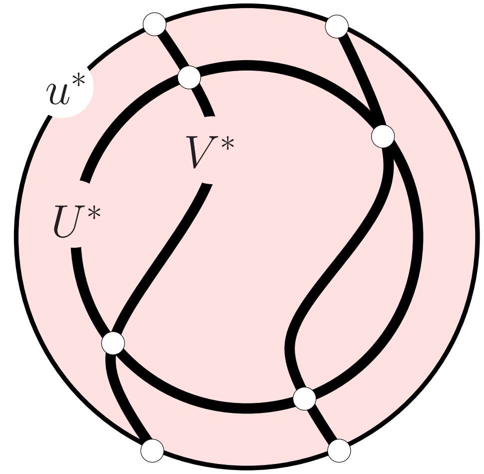
12 vertices

double Klein bottle

genus = 2 − #boundaries + #vertices

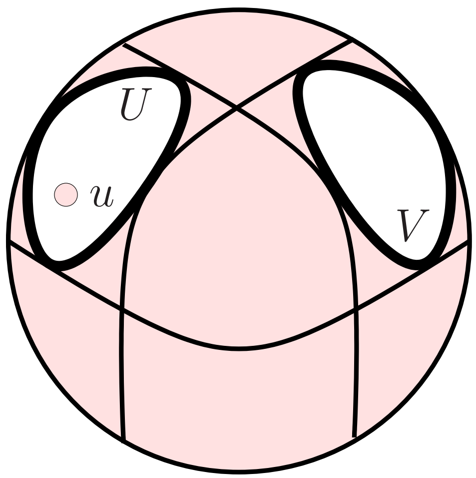
ARRANGEMENTS OF DOUBLE PSEUDOLINES

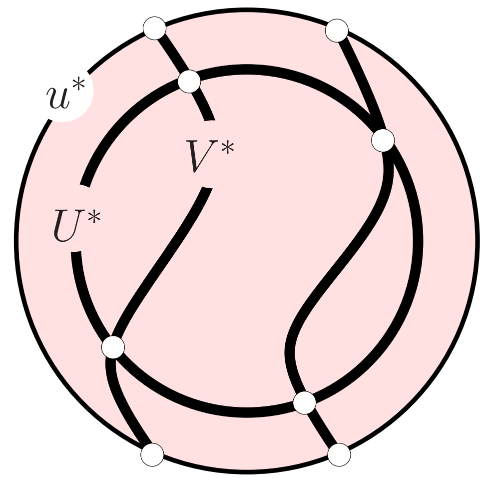




TH 1 (Habert and P. 2006). *Arrangements of DP-ribbons of genus* 1 *are exactly, modulo the adjunction of topological disks along their boundaries, the arrangements of double pseudolines, i.e., the dual arrangements of finite families of pairwise disjoint convex bodies of (real two-dimensional) projective geometries.*

ARRANGEMENTS OF DOUBLE PSEUDOLINES



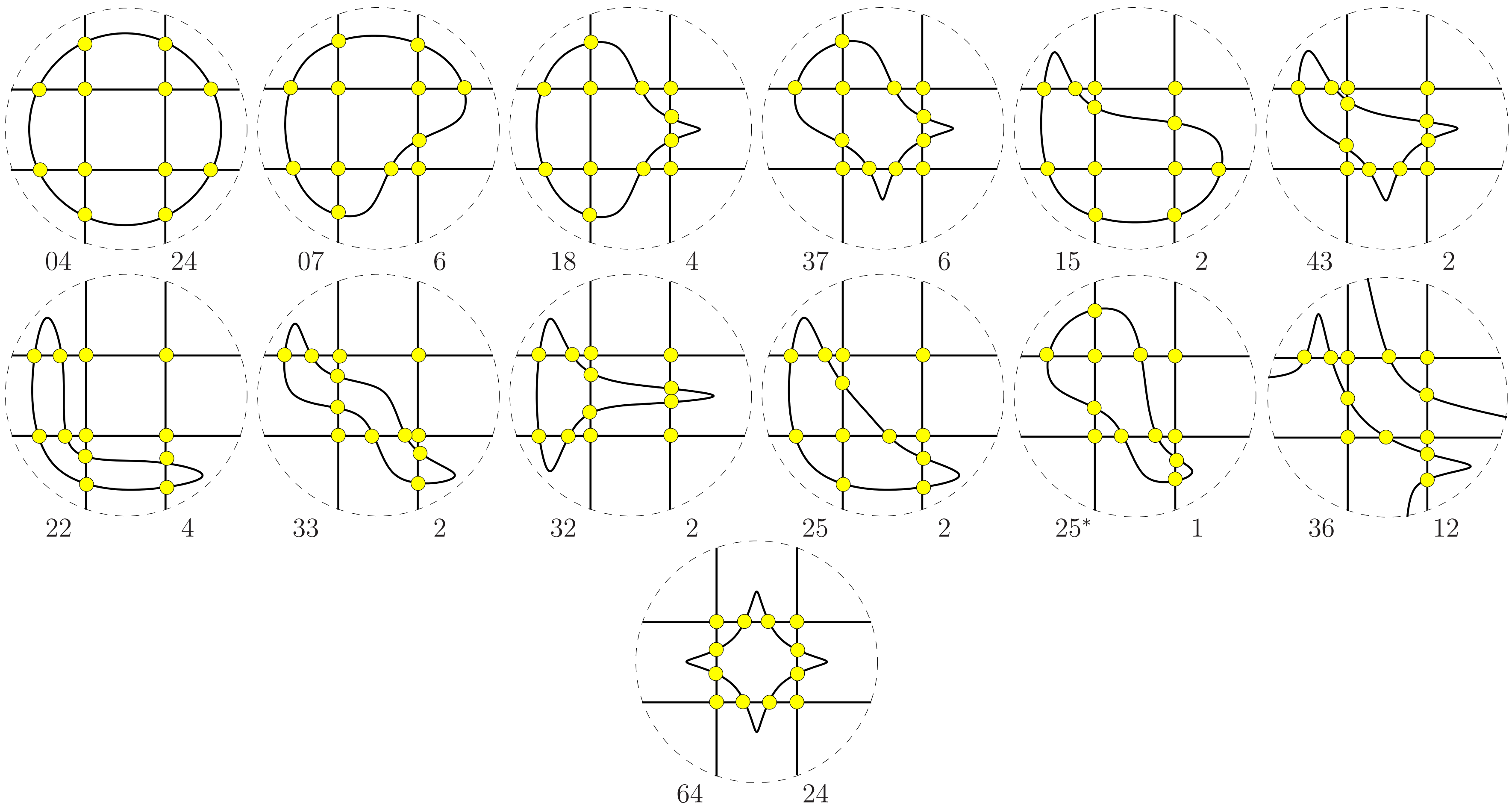


TH 1 (Habert and P. 2006). *Arrangements of DP-ribbons of genus* 1 *are exactly, modulo the adjunction of topological disks along their boundaries, the arrangements of double pseudolines, i.e., the dual arrangements of finite families of pairwise disjoint convex bodies of (real two-dimensional) projective geometries.*

n	2	3	4	5
$a_n(1)$	1	13	6570	181 403 533
$b_n(1)$	1	216	2 415 112	nc

J. Ferté, V. Pilaud and M. P. 2011

SIMPLE ARRANGEMENTS OF THREE DOUBLE PSEUDOLINES



n	2	3	4	5
$a_n(1)$	1	13	6570	181 403 533
$b_n(1)$	1	216	2 415 112	nc

J. Ferté, V. Pilaud and M. P. 2008

LR CHARACTERIZATION

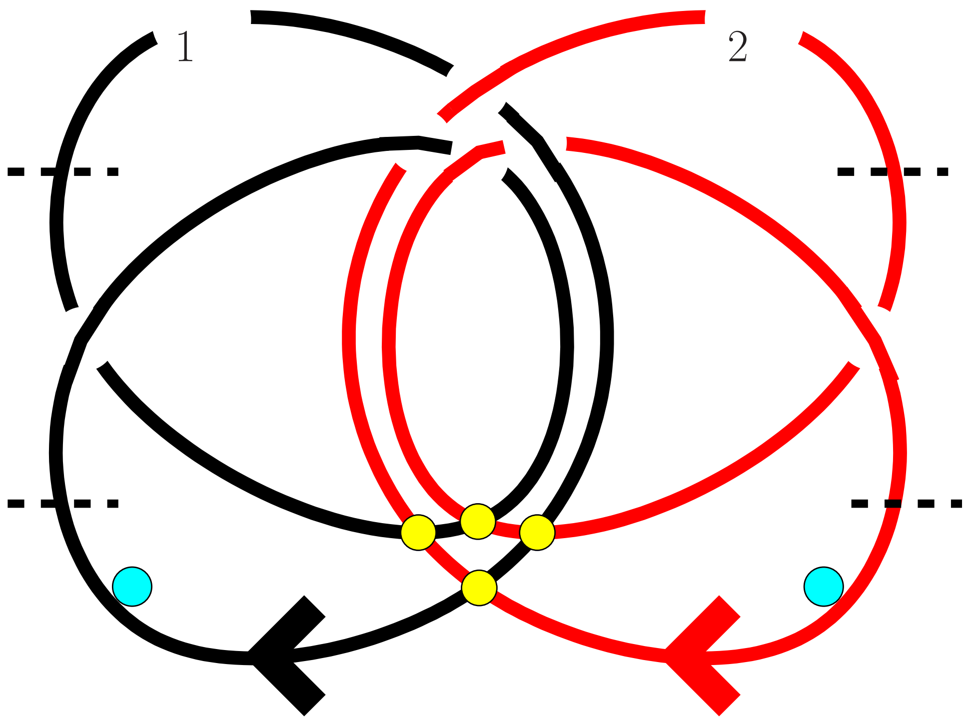
TH 2 (Habert and P. 2006). *An arrangement of DP-ribbons is of genus 1 if and only if its subarrangements of size 3, 4 and 5 are of genus 1.*

LR CHARACTERIZATION

TH 2 (Habert and P. 2006). *An arrangement of DP-ribbons is of genus 1 if and only if its subarrangements of size 3, 4 and 5 are of genus 1.*

TH 3 (P. 2013). *An arrangement of 5 DP-ribbons whose subarrangements of size 4 are of genus 1 is of genus* 1 *or its subarrangements of size* 4 *belong to a well-defined family of few tens of arrangements.*

ENUMERATION



$1: \overline{22}22$
$2: \overline{11}11$

PP 2. *There is a natural correspondence between indexed arrangements of n oriented DP-ribbons and the n-tuples of suffles of the $n-1$ circular sequences $\overline{jj}jj$, $j = 2, 3, \ldots, n$. Furthermore ...*

ENUMERATION (CONTINUED)

PP 2. *There is a natural correspondence between indexed arrangements of n oriented DP-ribbons and the n-tuples of suffles of the $n-1$ circular sequences $\overline{jj}jj$, $j = 2, 3, \ldots, n$. Furthermore the number b_n of indexed arrangements of n oriented DP-ribbons is*

$$\left\{4^{n-2}\binom{4n-5}{3,4,4,\ldots,4}\right\}^n$$

and the number a_n arrangements of n DP-ribbons is bounded from below by

$$b_n/(2^n n!).$$

$$b_3 = \left\{4^1\tbinom{7}{3,4}\right\}^3 = 140^3 = 2\,744\,000 \quad \left\| \, \lceil b_3/(2^3 3!)\rceil = 57167 \, \right\| \quad a_3 = 58042$$

$$b_4 = \left\{4^2\tbinom{11}{3,4,4}\right\}^4 = 184800^4$$

$$b_5 = \left\{4^3\tbinom{15}{3,4,4,4}\right\}^5 = 1009008000^5$$

ENUMERATION (CONTINUED)

$$a_n(g) = \#\text{ arrangements of size } n \text{ and genus } g$$
$$b_n(g) = \#\text{ indexed and oriented arrangements of size } n \text{ and genus } g$$

g	1	2	3	4	5	6	7	8	9	10	11	12	13
$a_3(g)$	13	20	77	197	674	1127	2707	5173	10073	11943	13633	9115	3290
$b_3(g)$	216	636	2756	8292	29032	50848	123240	240196	475920	565016	653528	436496	157824

C. Lange and M.P. 2013

OPEN PROBLEMS

Problem 1. *Give formulae for the numbers $b_n(g)$.*

$$\sum_g b_n(g) = b_n = \left\{ 4^{n-2} \binom{4n-5}{3, 4, 4, \ldots, 4} \right\}^n$$

$$\ln b_n = \Theta(n^2 \log n)$$

$$\ln b_n(1) = \Theta(n^2)$$

Problem 2. *Devise a quadratic time algorithm to compute an arrangement of n double pseudolines presented by its subarrangements of size* 3.

BIBLIOGRAPHY

[1] Helmut Salzmann, Dieter Betten, Theo Grundhöfer, Hermann Hähl, Rainer Löwen, and Markus Stroppel. Compact projective planes. Number 21 in De Gruyter expositions in mathematics. Walter de Gruyter, 1995.

[2] J. E. Goodman, R. Pollack, R. Wenger, and T. Zamfirescu. Arrangements and topological planes. Amer. Math. Monthly, 101(9):866–878, November 1994.

[3] J. Ferté, V. Pilaud, and M. Pocchiola. On the number of simple arrangements of five double pseudolines. Disc. Comput. Geom. 45 (2): 279-302, 2011.

[4] L. Habert and M. Pocchiola. Computing pseudotriangulations via branched covering. Disc. Comput. Geom., 48(3):518–579, 2012.

[5] L. Habert et M. Pocchiola. LR characterization of chirotopes of finite planar families of pairwise disjoint convex bodies. Disc. Comput. Geom., 2013. http://dx.doi.org/10.1007/s00454-013-9532-y.

[6] J. Bokowski. Computational Oriented Matroids. Cambridge, 2006.

[7] A. Björner, M. Las Vergnas, B. Sturmfels, N. White, and G. M. Ziegler. Oriented Matroids. Cambridge University Press, 2nd edition, 1999.