Simple Treewidth

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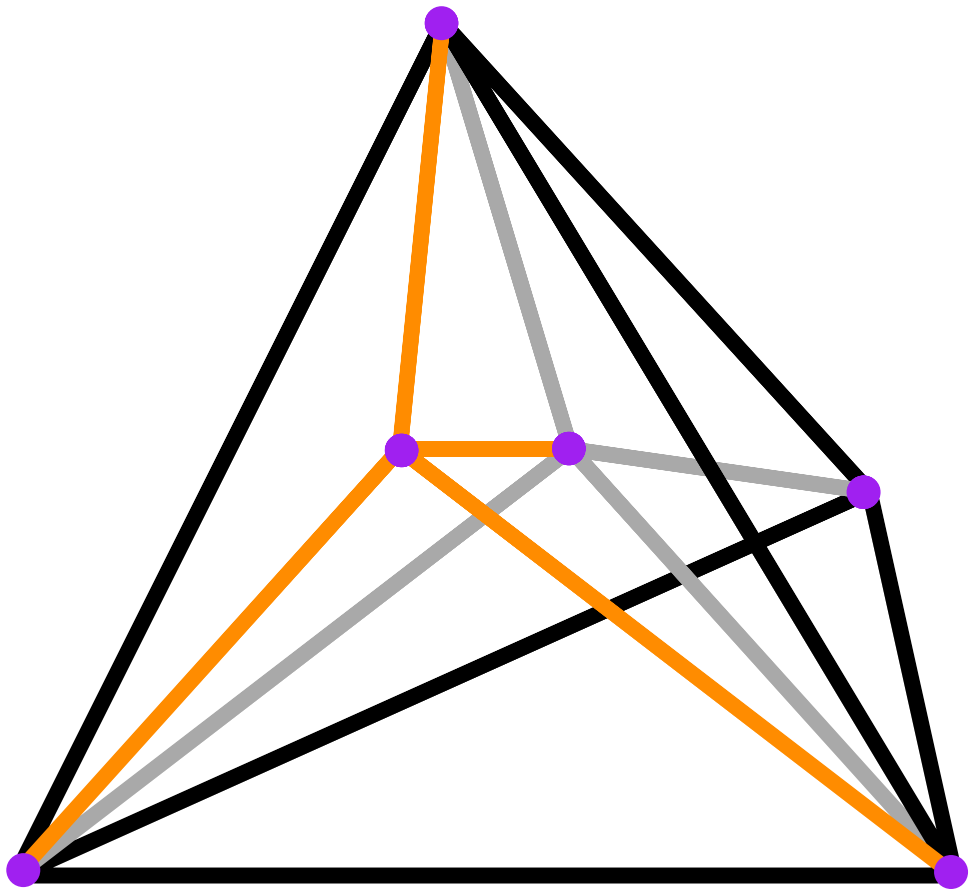
Technische Universität Berlin

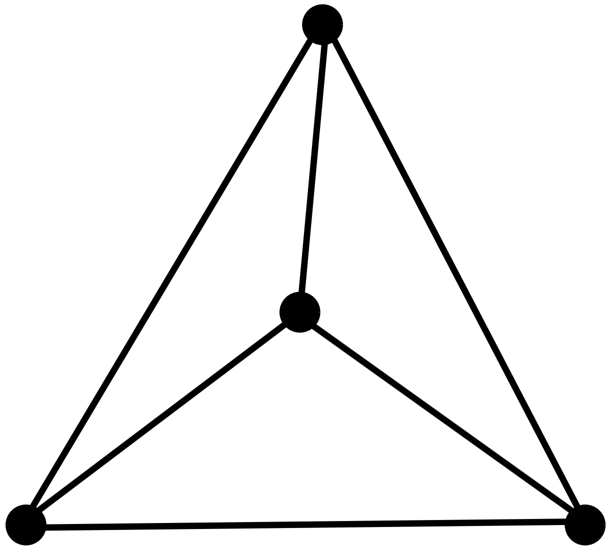


Midsummer Combinatorial Workshop XVIII, Prague, August 1st, 2012

Treewidth

k-tree

- start with K_{k+1}
- connect new vertex to sub-K_k



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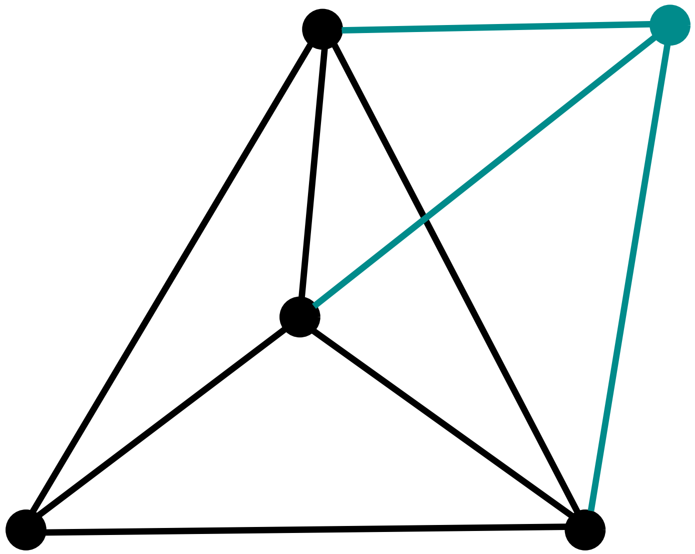

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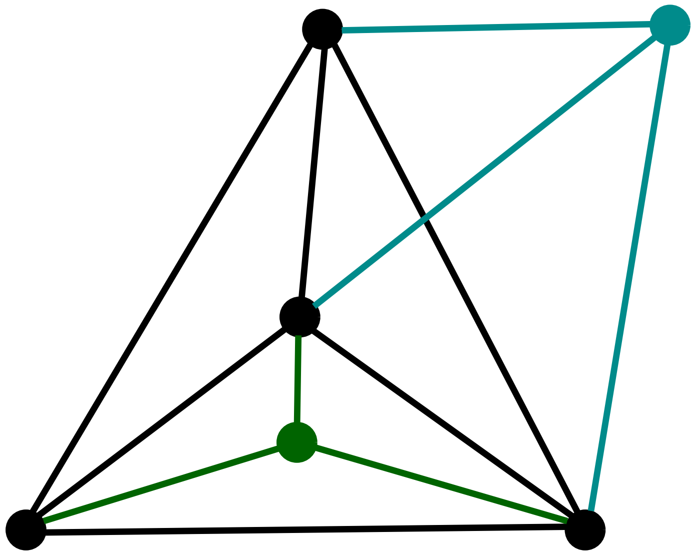

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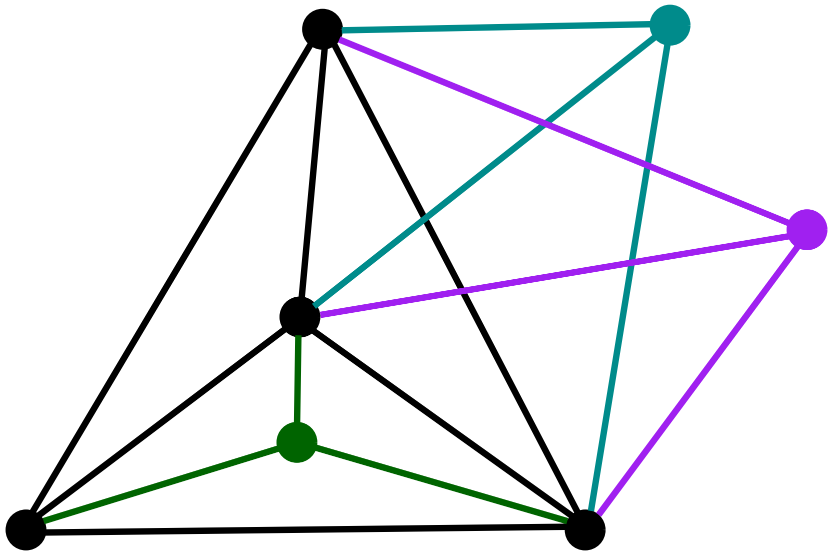

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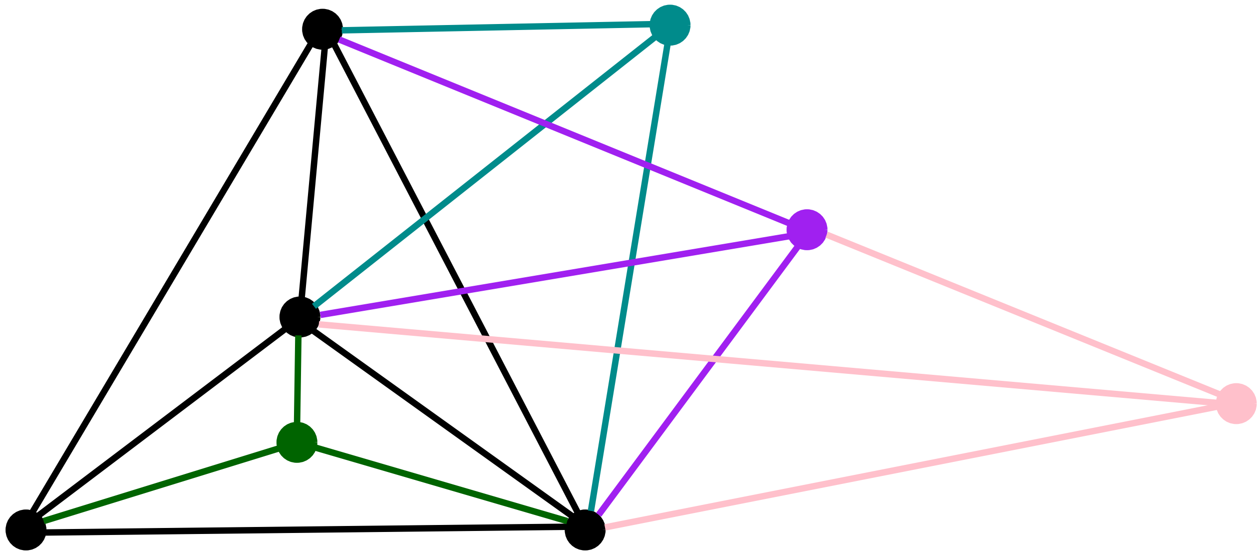

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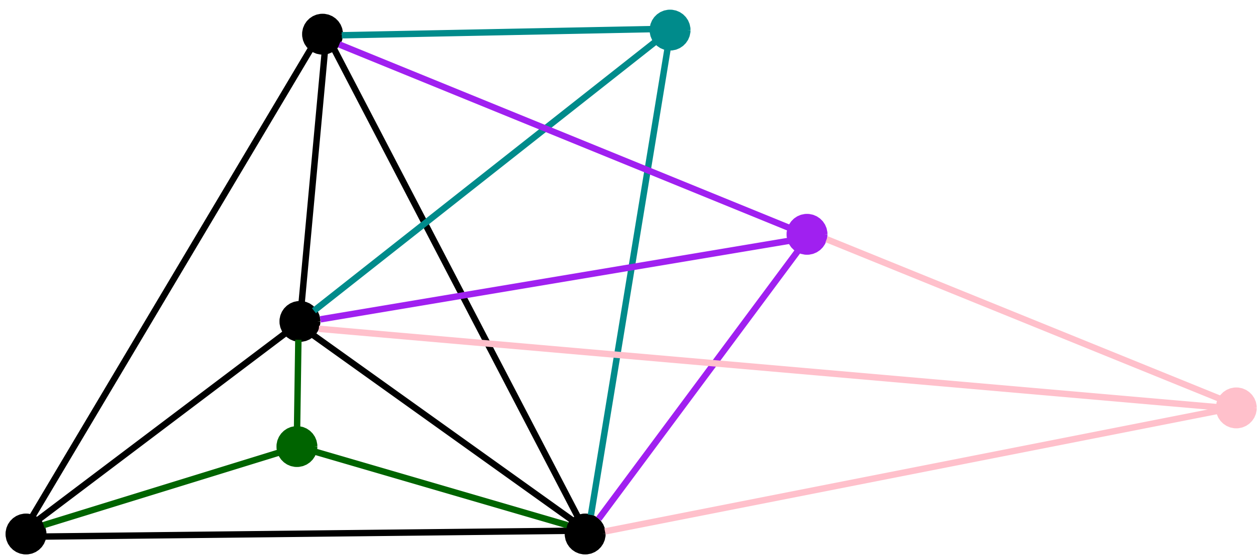
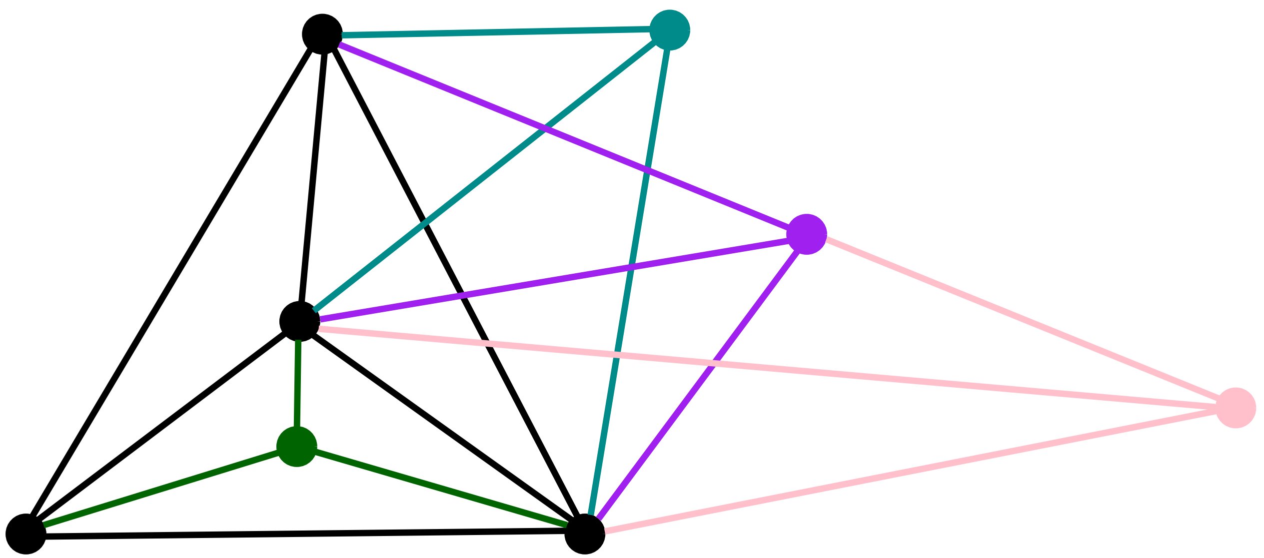


$\mathrm{tw}(G) \leq k \Leftrightarrow G$ subgraph of k-tree

simple Treewidth

simple k-tree

- start with K_{k+1}
- connect new vertex to sub-K_k
  use no K_k twice



stw$(G) \leq k \Leftrightarrow G$ subgraph of k-tree (*simple*)

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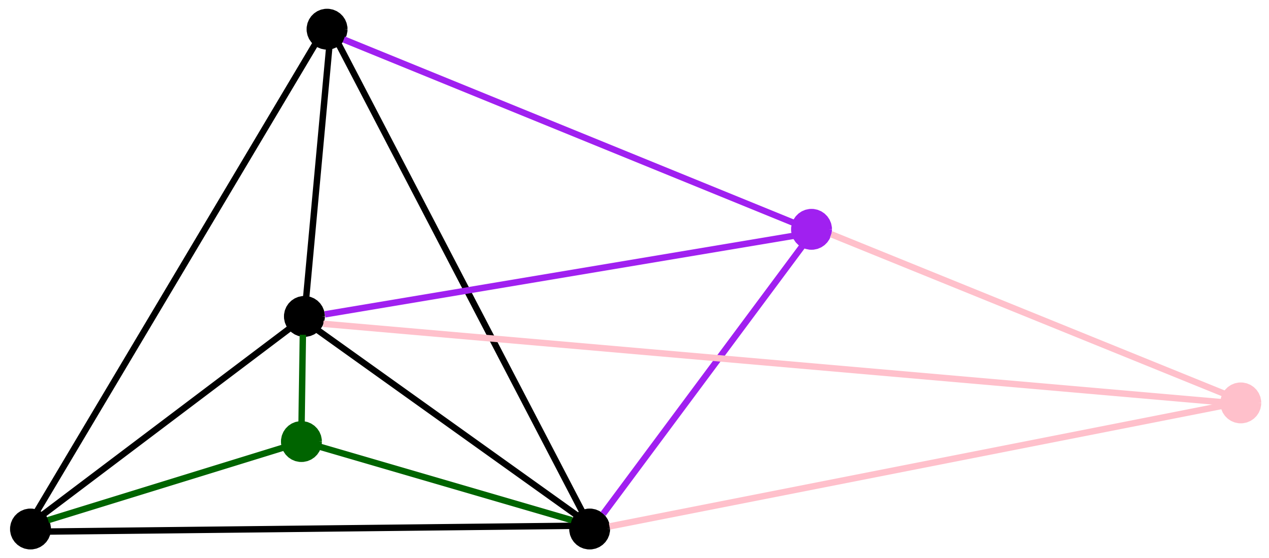

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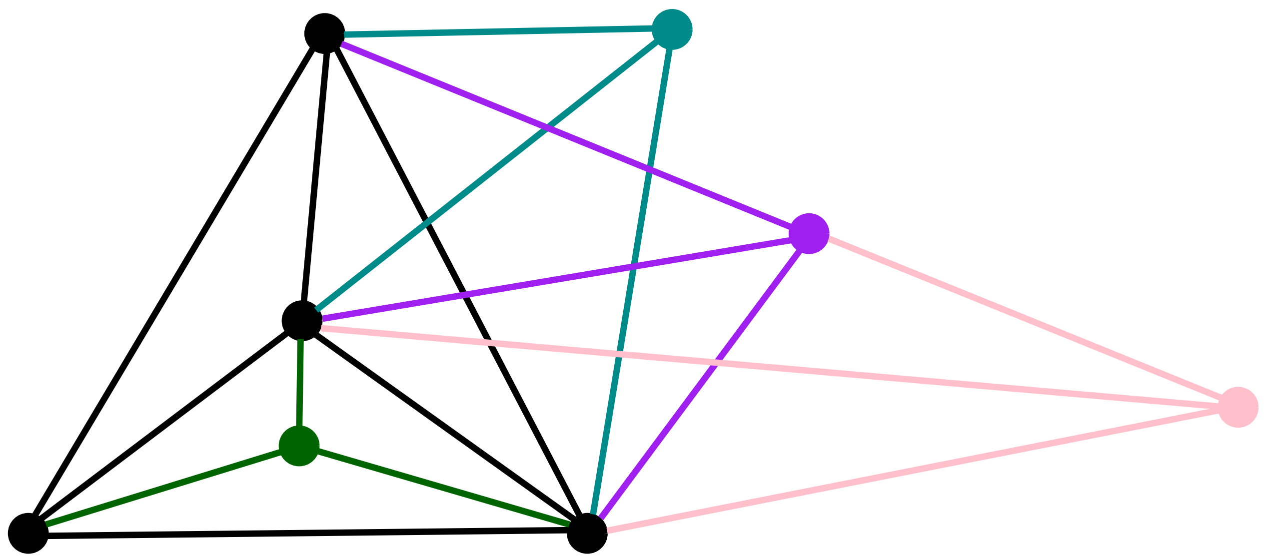
Overview

- Definitions *(done)*
- Why *stw* is not interesting
- Why *stw* is interesting
  - How we came across it
  - Relations to Geometry and Topology
- Problems...

Why *stw* is not interesting

$\mathrm{tw}(G) \leq \mathrm{stw}(G) \leq \mathrm{tw}(G) + 1$



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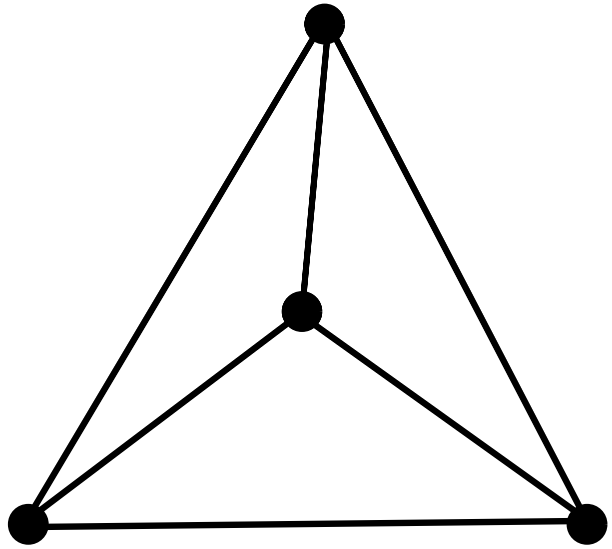

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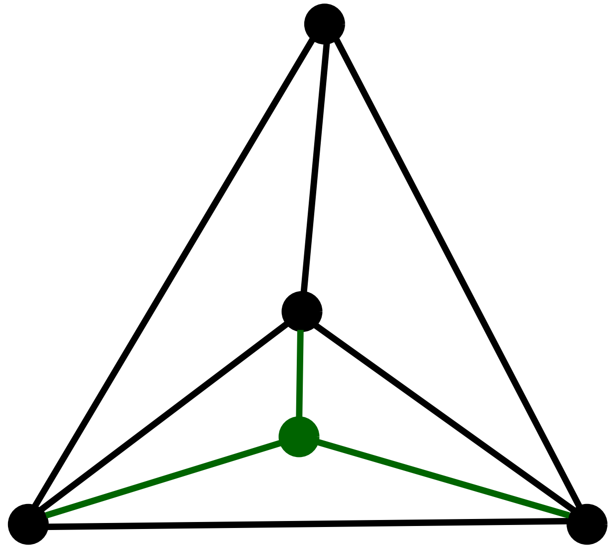

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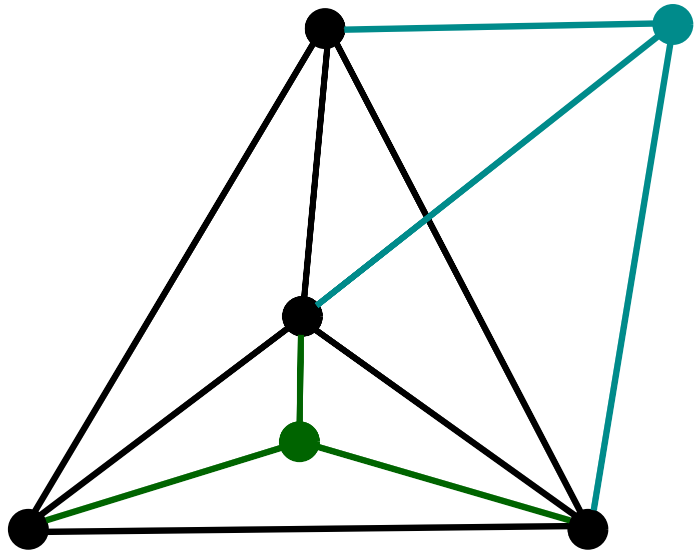

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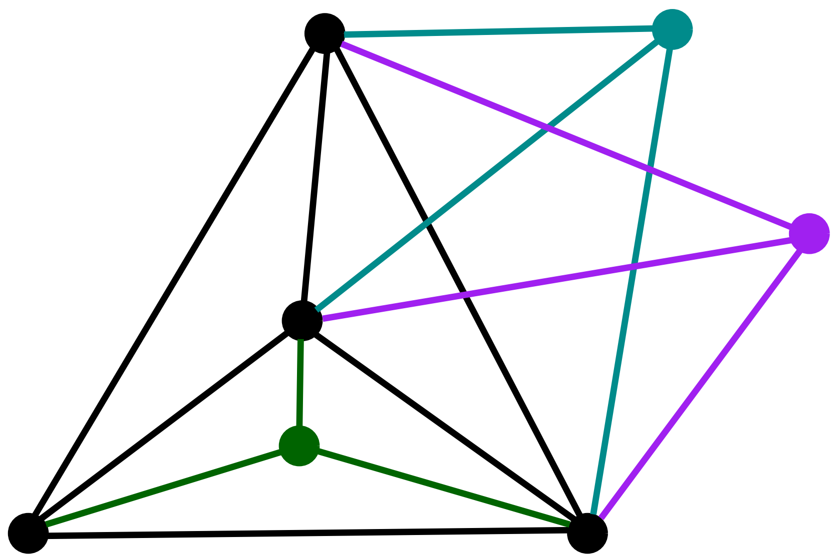

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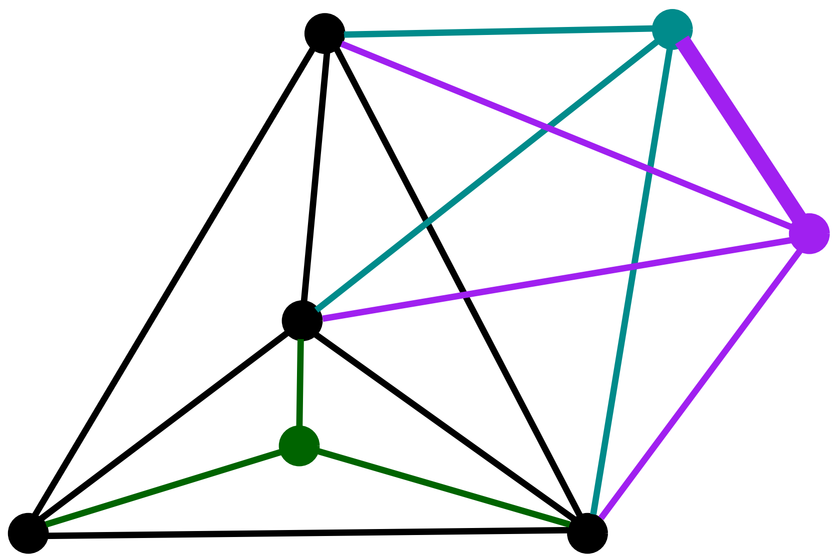

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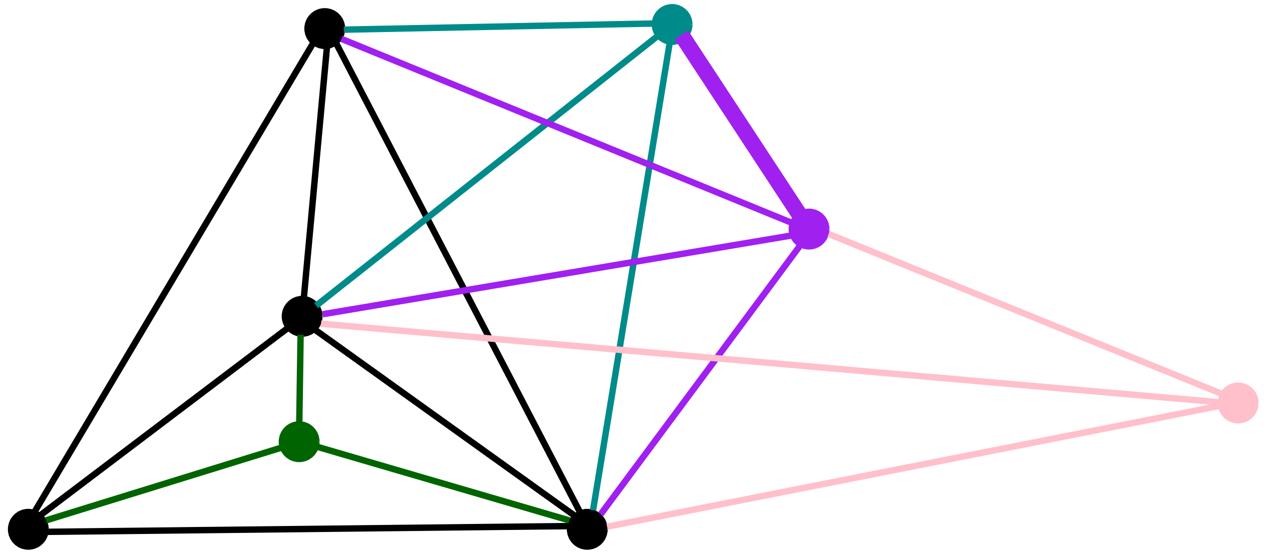

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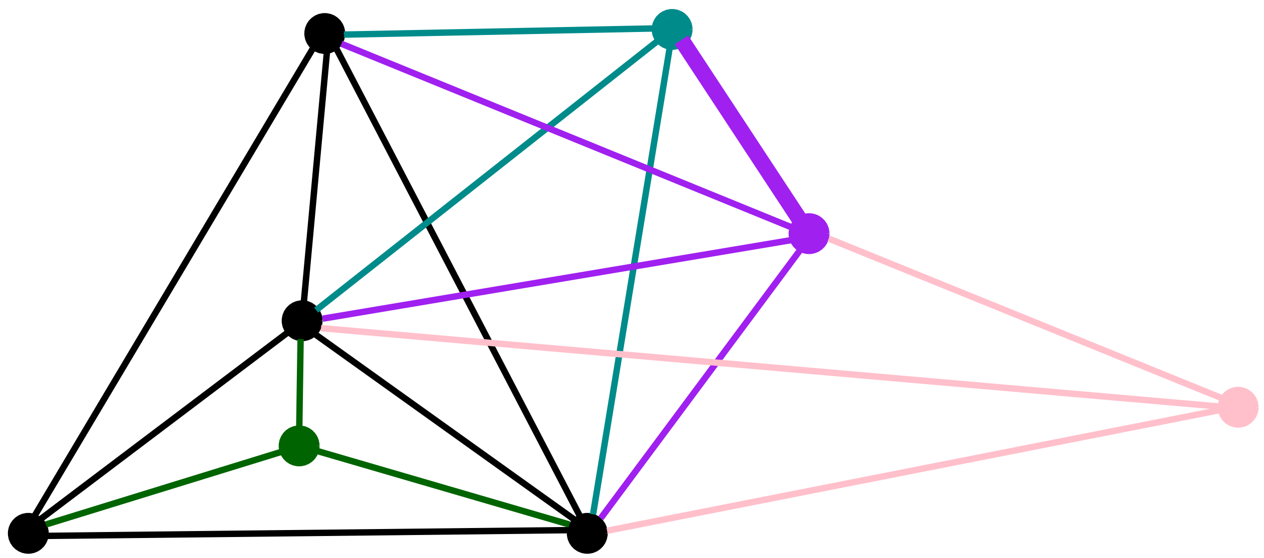
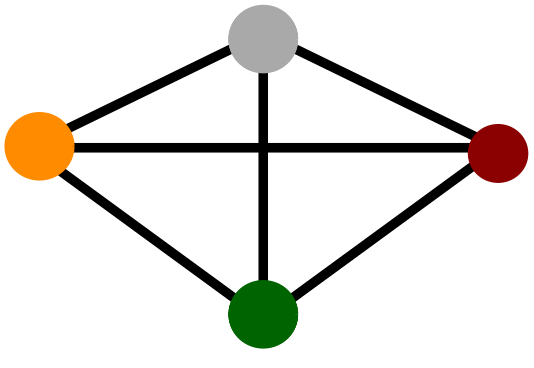


stw not interesting for asymptotical questions

How we came across *stw*

Intersection graphs of systems of intervals

Interval number
Harary, Trotter '79



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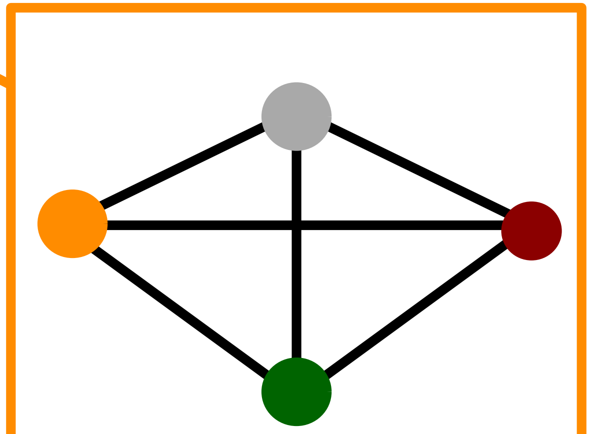
how many intervals per vertex such that G is their intersection graph



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THM
tw$\leq k \Rightarrow$ max. interval nr. $= k+1$
stw$\leq k \Rightarrow$ max. interval nr. $= k$

upper bounds: build representation along construction sequence

Relations to Geometry and Topology

simple k-trees form nice simplicial complexes

Def [Below, De Loera, Richter-Gebert '00]:
Polytope is *stacked* if it has a triangulation whose dual graph is a tree.

Obs: A d-dimensional polytope is stacked iff its graph has stw$\leq d$.

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Obs: A d-dimensional polytope is stacked iff its graph has stw$\leq d$.

Quest: Does stw$\leq d \leq$ connectivity imply polytope graph?

Relations to Geometry and Topology

	≤ 1	≤ 2	≤ 3
stw	paths	outerplanar	planar $\&$ tw≤ 3
tw	trees	*series parallel*	$\ldots$

Colbourn et. al.

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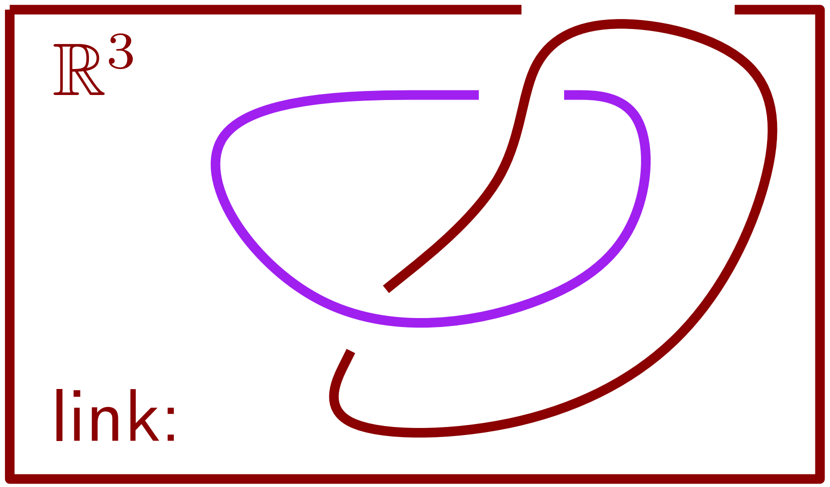
Quest: For $k \geq 3$, does planar & tw$\leq k \Rightarrow$ stw$\leq k$?

Quest: For $k \geq 3$, no $K_{3,k}$ minor & tw$\leq k \Rightarrow$ stw$\leq k$

Conjecture: stw≤ 4 iff linkless embeddable & tw≤ 4



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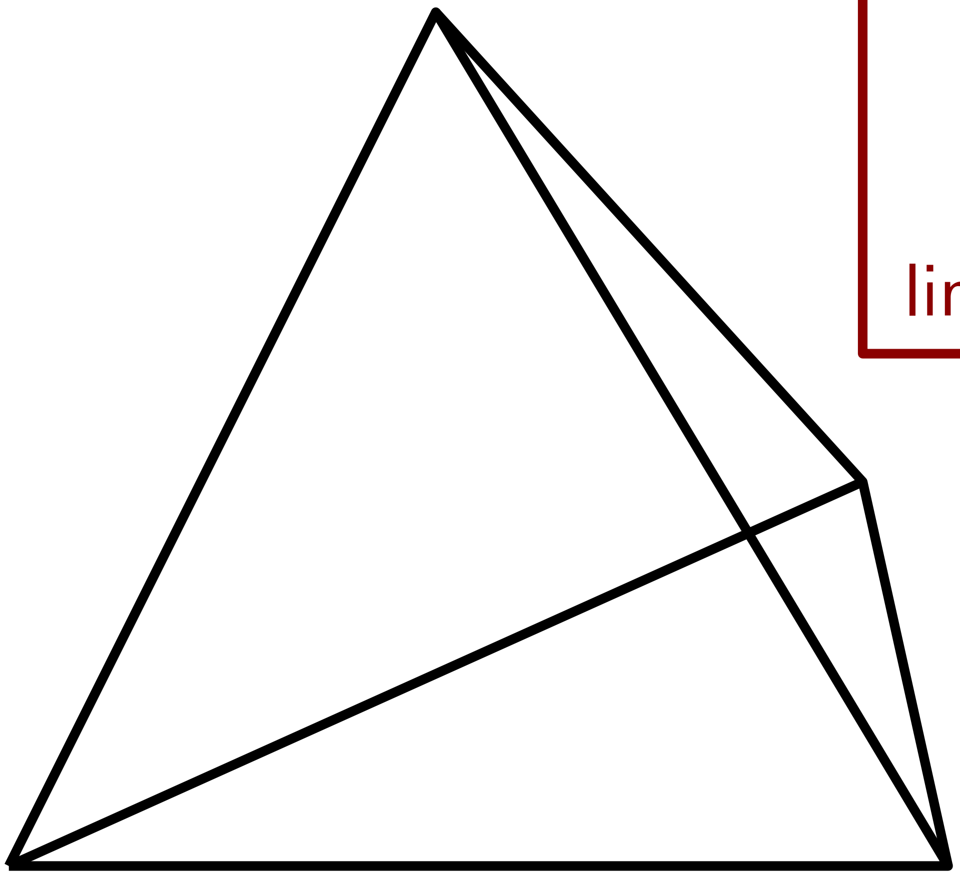

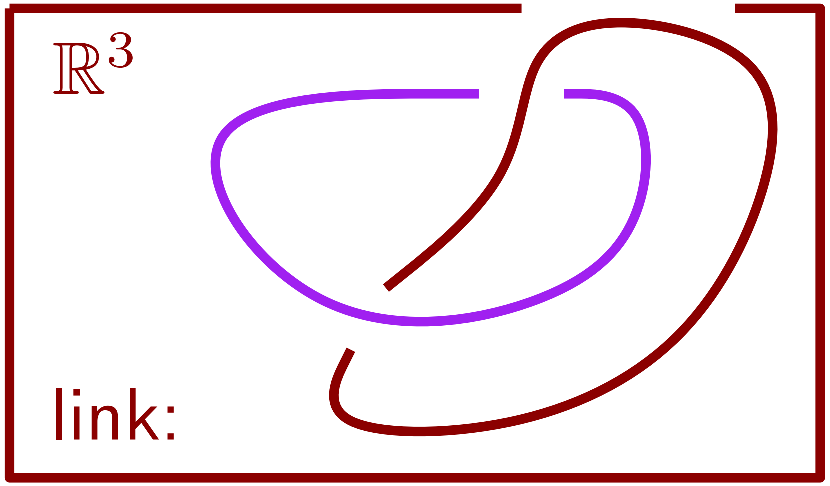

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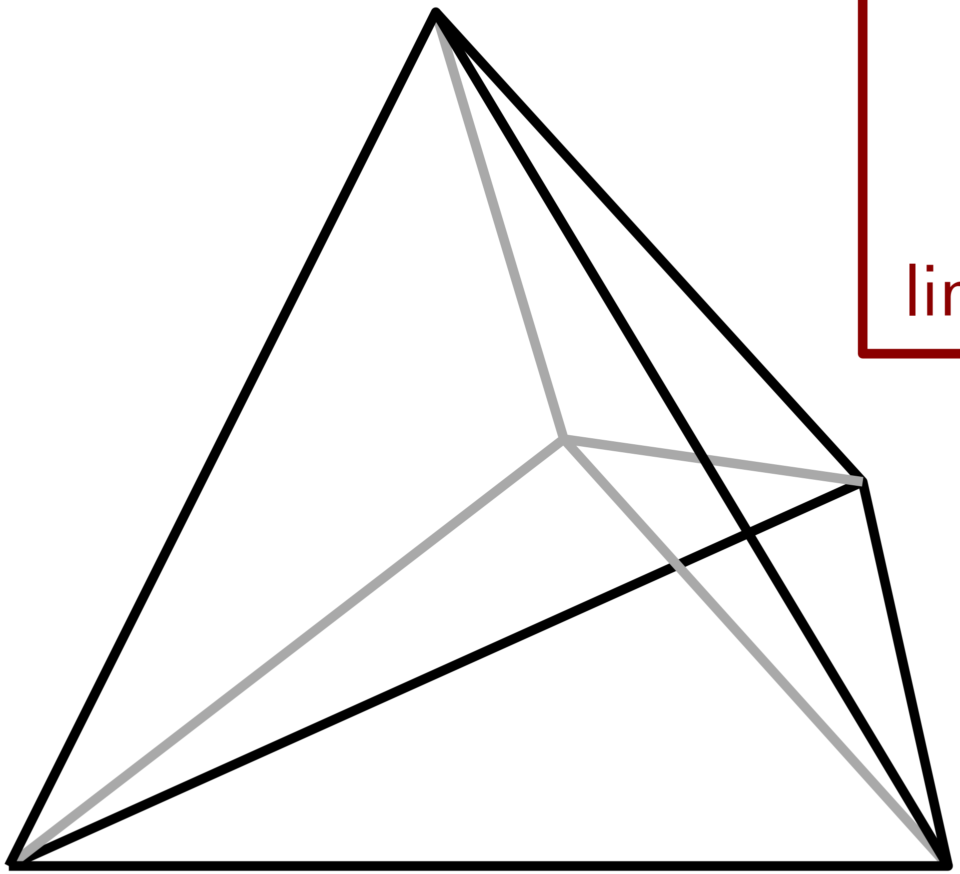

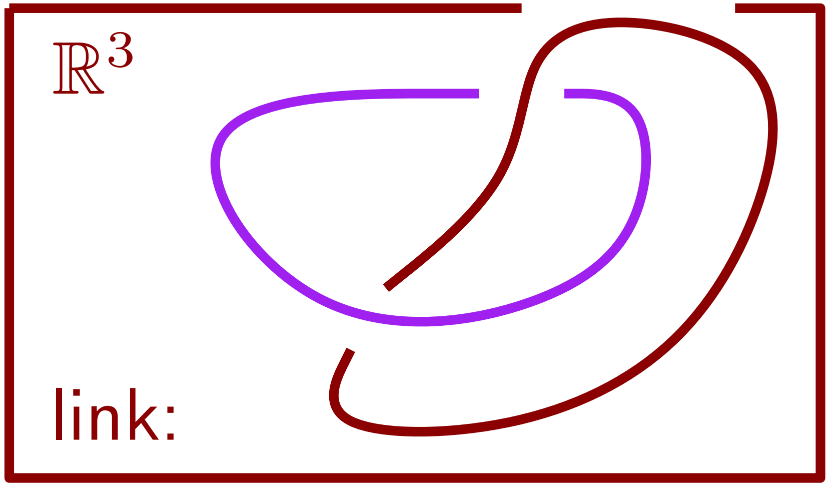

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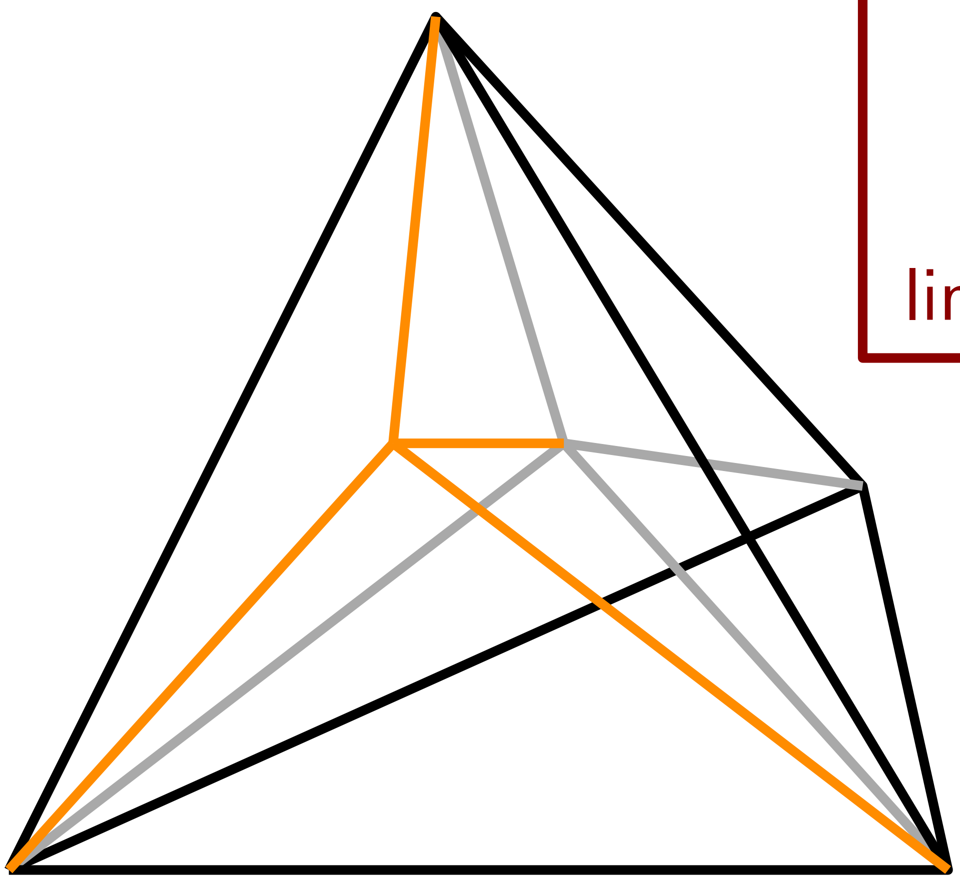

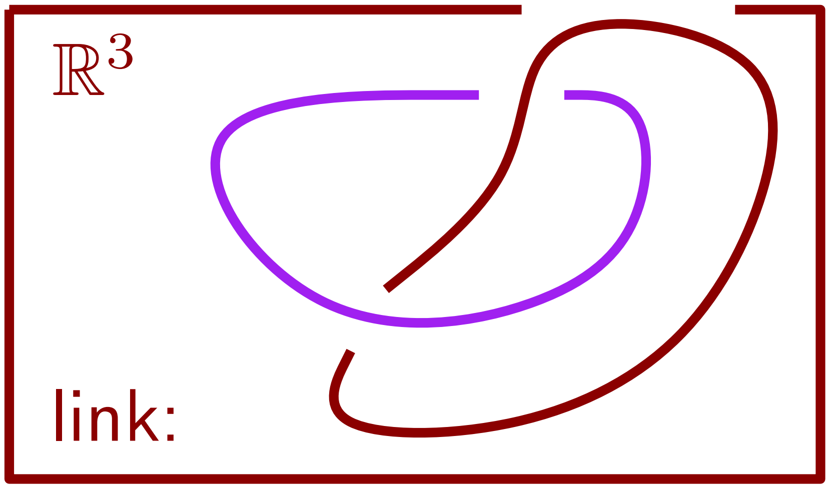

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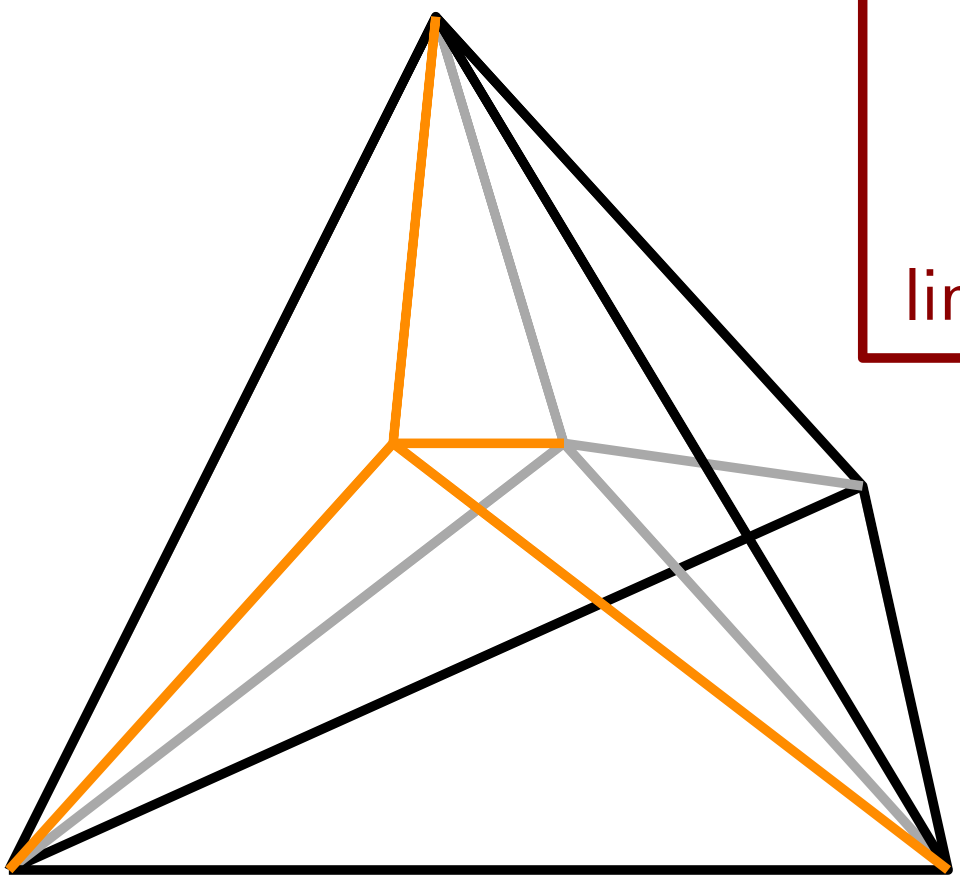

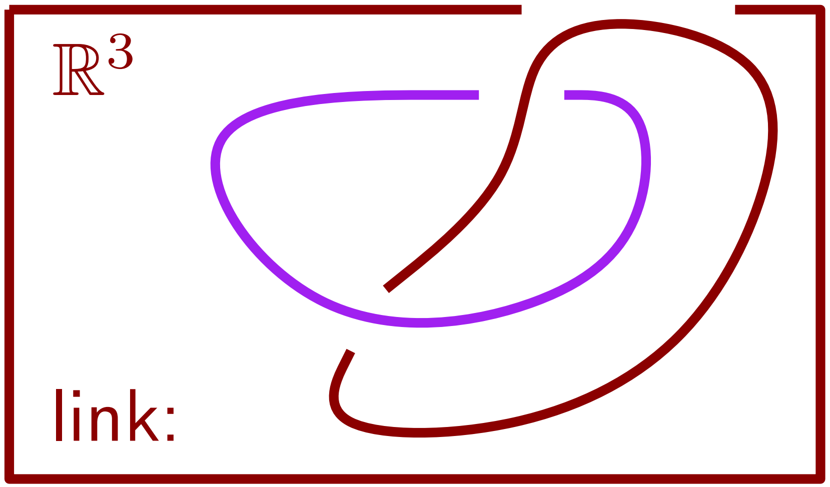

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- easy to see that they have no links

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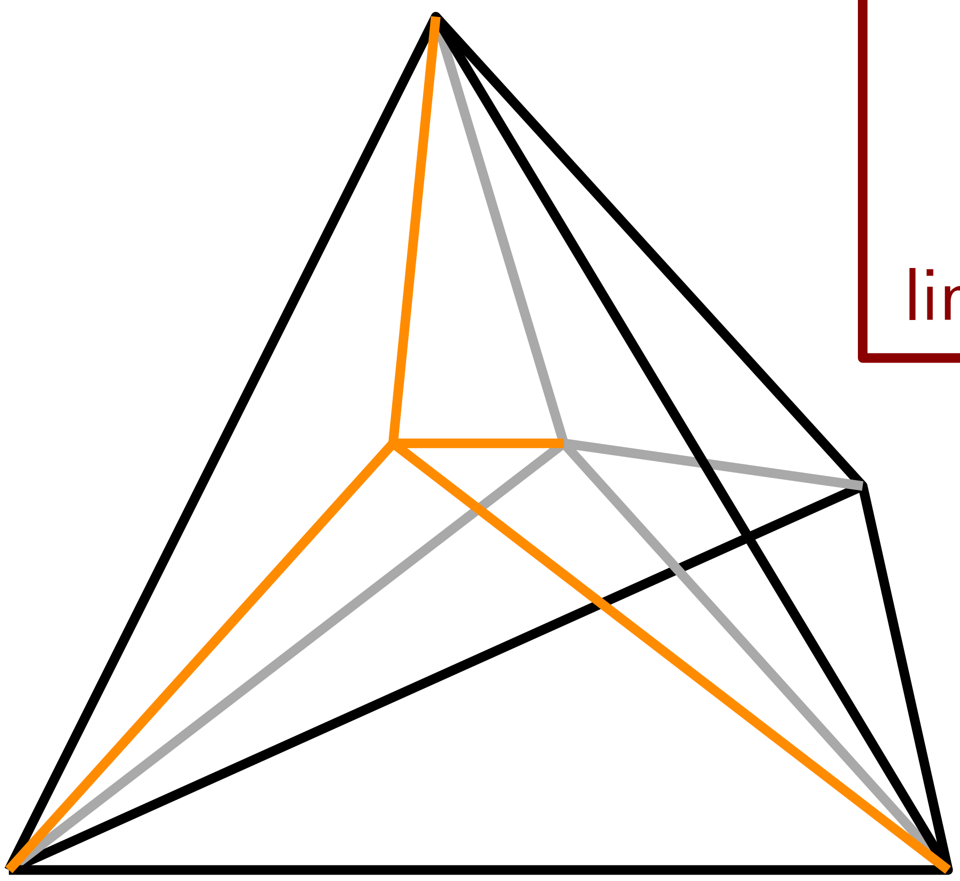

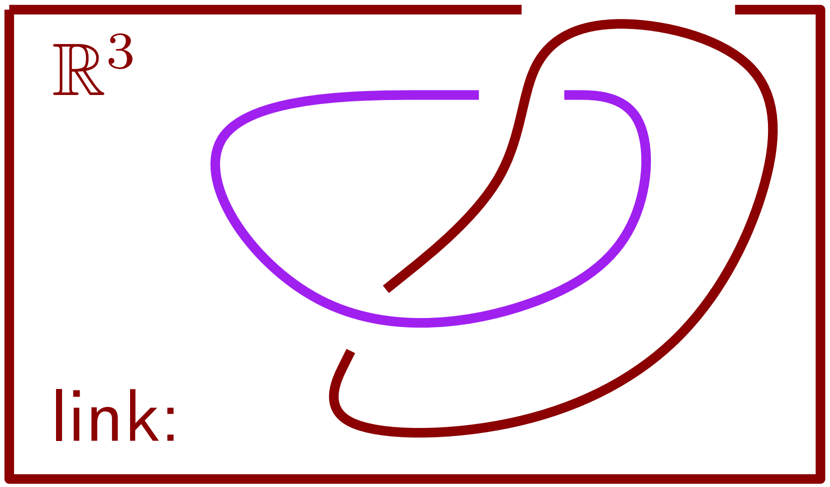

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Problems

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STW$_{\leq k}$ minor-closed:

forbidden minors ...

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stw$\leq k$ for fixed and variable k

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