

Assignment mechanisms

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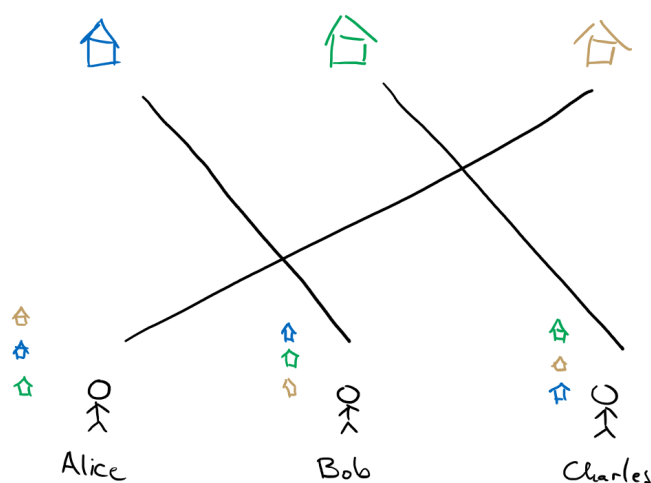
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Assignment mechanism

Definition (Assignment mechanism)

Let N be a number of players and indivisible objects (houses). Each player has a list of preferences i.e. strict ordering of houses. An assignment mechanism is a mechanism which assigns (possibly stochastically) each house to a different player.



Mechanism constraints

We want the mechanism to satisfy:

- ① Ex-post efficiency,
- ② Equal treatment of equals,
- ③ Strategy proofness.

Ex-post efficiency

No subgroup of players can exchange houses amongst themselves to make everyone happier.
Randomize over efficient assignments.

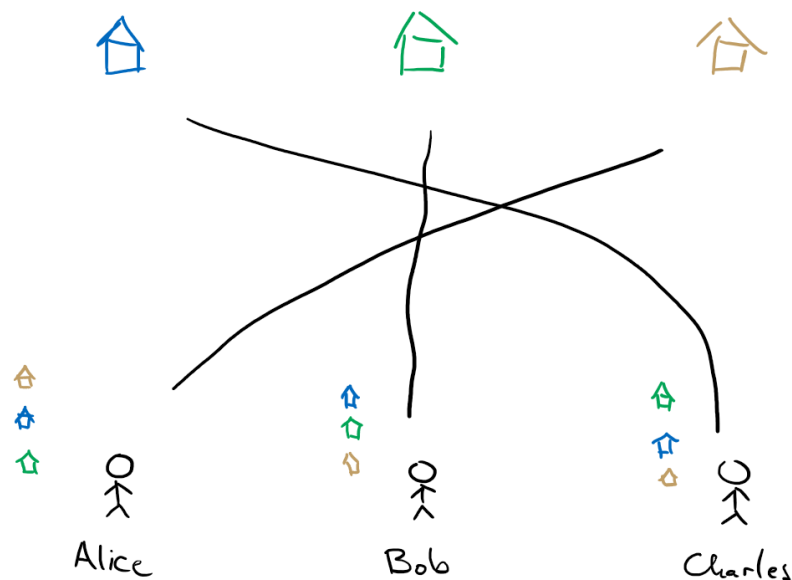


Figure: Example of a non efficient assignment.

Equal treatment of equals

If any two players have the same preferences they have the same chances to receive a given house.

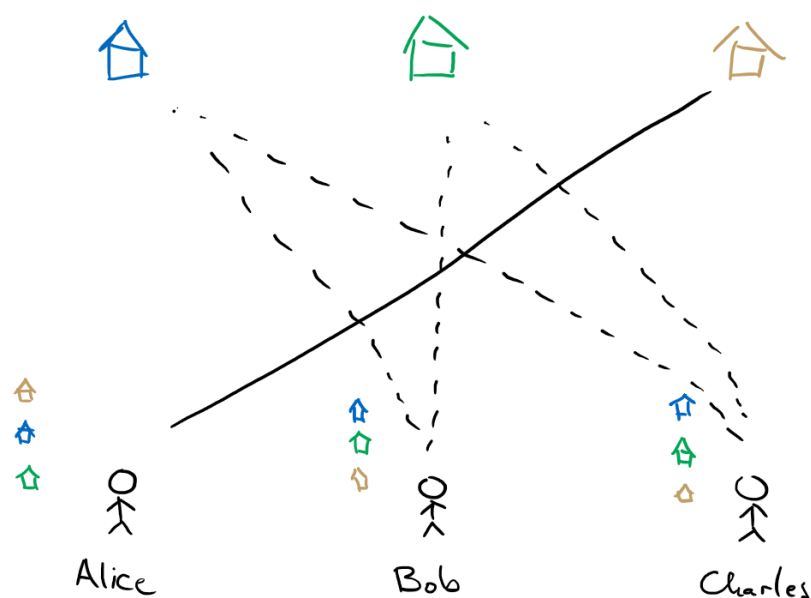


Figure: Example of preferences that cannot be resolved deterministically.

Strategy proofness

No player may gain by lying about his/her preferences.

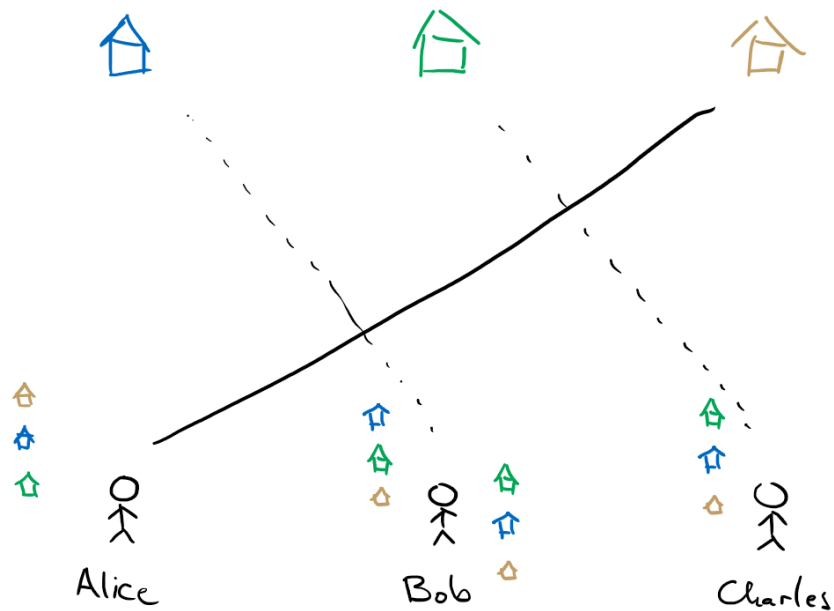


Figure: Example of Bob lying about his preferences.

Random serial dictatorship

Definition (RSD)

RSD is an assignment mechanism with steps:

- 1 Choose uniformly randomly one of the $\frac{1}{N!}$ possible order of players.
- 2 First player in the ordering gets the most preferable house.
- 3 For $1 < r \leq N$ the r -th player gets the most preferable house from the houses left after $r - 1$ players get their houses.

Theorem

RSD satisfies Ex-post efficiency, equal treatments of equals and strategy proofness.

Open question

Conjecture

RSD is the only assignment mechanism for N players satisfying the following:

- *Equal treatment of equals,*
- *Strategy proofness,*
- *Ex-post efficiency.*

The above conjecture was proven to be true for $N < 4$.

References

- [1] Abdulkadiroglu, A. and Sonmez, T., *Random serial dictatorship and the core from random endowments in house allocation problems*, *Econometrica* 66 (1998) 689-701.
- [2] Bogomolnaia, A. and Moulin, H., *A new solution to the random assignment problem*, *Journal of Economic Theory* 100 (2001) 295-328.
- [3] Nesterov, A., *Fairness and efficiency in strategy-proof object allocation mechanisms*, *Journal of Economic Theory* 170 (2017) 145-168.

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