MA2: Exams Requirements

Metric spaces.

General. Definition, examples, $\mathbb{E}_n$. Subspaces, convergence.
Continuous maps, continuity and convergence.
Neighborhoods, open and closed subsets. Closure.
Continuity and preimages (of open resp. closed sets).
Topological concepts, equivalent and strongly equivalent metrics; strongly equivalent metrics in $\mathbb{E}_n$.
Products and projections.

Compact spaces. Subspaces of compact spaces. Products.
Compact subspaces of $\mathbb{E}_n$.
Maxima and minima of continuous functions on a compact space.
Uniform continuity, on a compact space it coincides with continuity.

Completeness. Cauchy sequences, complete space.
Complete subspaces of complete spaces.
Product of complete spaces.
Compact $\Rightarrow$ complete.

Real functions of several variables.

Why we need to understand continuity in some generality.
Domains.

Partial derivatives.

Definition, how weak it is (not even continuity implied).
Total differential, geometrical interpretation (linear approximation).
Continuous partial derivatives $\Rightarrow$ total differential.

Computation: arithmetical rules.
Composed maps and Chain rule. Lagrange formula.
Higher order partial derivatives. Interchangeability.

Implicit Functions Theorems.

The task, understanding what the problem is.
The simplest case: $F(x, y) = 0$, role of $\frac{\partial F}{\partial y}$.
Jacobian.
General theorem.
Application: Regular maps.
Application: Extremes with constraints, theorem, how it is used.

Riemann integral.

Riemann integral in one variable. Repetition, geometric interpretation, volumes, etc..
Existence for continuous functions.
Fundamental Theorem of Analysis, Riemann integral and primitive functions.

Riemann integral in several variables.
Up to the existence for continuous functions quite analogous with one variable.
Fubini theorem, how it is used.
Note on Lebesgue integral: just the practical fact that we can compute it like Riemann integral plus the rule

$$\int \lim f_n = \lim \int f_n$$

for equally bounded f_n.