Exercises for Combinatorial and Computational Geometry

Series 5 — Polytopes, arrangements, and Voronoi diagrams

deadline 10. 1. 2020

If you submit solutions on paper, please separate problems 1,2,5 from problems 3,4,6; each subset will be handled by a different corrector.

1. Count the number of k-dimensional faces, for $k = 1, 2, 3$, of a 4-dimensional cyclic polytope on n vertices. **[2]**

2. Count the number of 1- and 2-dimensional faces in an arrangement of n planes in general position in $\mathbb{R}^3$. **[2]**

3. Let $P = \{p_1, p_2, \dots, p_n\}$ be a set of n points in the plane. We say that points x, y have the *same view* of P if the points of P are visible in the same cyclic order from x and y. That is, if we rotate light rays that emanate from x and y, respectively, the points of P are lit in the same order by these rays. We assume that neither x nor y is in P and that neither of them can see two points of P in occlusion.

   Show that the maximum number of points with mutually distinct views of P is $O(n^4)$. **[2]**

4. (a) How many cells are there in the arrangement of $\binom{d}{2}$ hyperplanes in $\mathbb{R}^d$ with equations $x_i = x_j$, where $1 \leq i < j \leq d$? **[2]**

   (b) How many cells are there in the arrangement of hyperplanes in $\mathbb{R}^d$ with equations $x_i + x_j = 0$ and $x_i = x_j$, where $1 \leq i < j \leq d$? **[2]**

5. Show that for $n \geq 2$ the Voronoi diagram of a $2n$-point set $A_{2n} := \{(i, 0, 0) : i = 1, 2, \dots, n\} \cup \{(0, n, j) : j = 1, 2, \dots, n\}$ in $\mathbb{R}^3$ has at least cn^2 vertices for some positive constant c. **[2]**

6. Let P be a finite point set in the plane with no three points on a line and no four points on a circle. Define a graph DT (called the *Delaunay triangulation*) on P as follows: two points a, b are connected by an edge if and only if there exists a circular disk with both a and b on the boundary and no point of P in its interior.

   Prove that DT is a connected plane graph where every inner face is a triangle. **[3]**

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