

Cayley sum graphs and eigenvalues of (3, 6)-fullerenes

Matt DeVos Luis Goddyn* Bojan Mohar^{†‡}
Robert Šámal^{§¶}

Department of Mathematics
Simon Fraser University
Burnaby, B.C. V5A 1S6

email: {mdevos,goddyn,mohar,rsamal}@sfu.ca

Abstract

We determine the spectra of cubic plane graphs whose faces have sizes 3 and 6. Such graphs, “(3,6)-fullerenes”, have been studied by chemists who are interested in their energy spectra. In particular we prove a conjecture of Fowler, which asserts that all their eigenvalues come in pairs of the form $\{\lambda, -\lambda\}$ except for the four eigenvalues $\{3, -1, -1, -1\}$. We exhibit other families of graphs which are “spectrally nearly bipartite” in the sense that nearly all of their eigenvalues come in pairs $\{\lambda, -\lambda\}$. Our proof utilizes a geometric representation to recognize the algebraic structure of these graphs, which turn out to be examples of Cayley sum graphs.

*Supported by a Canada NSERC Discovery Grant

[†]Supported in part by ARRS Research Grant P1-0297, by an NSERC Discovery Grant, and by the CRC program.

[‡]On leave from: IMFM & FMF, Department of Mathematics, University of Ljubljana, Ljubljana, Slovenia.

[§]Supported by PIMS postdoctoral fellowship.

[¶]On leave from Institute for Theoretical Computer Science (ITI), Charles University, Prague, Czech Republic.