Abstracts of KAM-DIMATIA Series Year 2007

2007-802 Z. Dvořák, D. Král', and R. Škrekovski

Non-rainbow colorings of 3-, 4- and 5-connected plane graphs

We study vertex-colorings of plane graphs that do not contain a rainbow face, i.e., a face with vertices of mutually distinct colors. If G is 3-connected plane graph with n vertices, then the number of colors in such a coloring does not exceed $\lfloor \frac{7n-8}{9} \rfloor$. If G is 4-connected, then the number of colors is at most $\lfloor \frac{5n-6}{8} \rfloor$, and for $n \equiv 3 \pmod 8$, it is at most $\lfloor \frac{5n-6}{8} \rfloor - 1$. Finally, if G is 5-connected, then the number of colors is at most $\lfloor \frac{43}{100}n - \frac{19}{25} \rfloor$. The bounds for 3-connected and 4-connected plane graphs are the best possible as we exhibit constructions of graphs with colorings matching the bounds.

2007-803 M. Nehéz

Properties of Oblivious End-to-End Communication Protocols in Reliability Networks with Hypercubic and Mesh-like Topology

We design protocols for single-packet oblivious end-to-end communication problem according to induced shortest path requirement for several types of reliability networks modeled by random graphs. The most part of this paper contains an analysis of the protocols. Namely, the protocol for random k-dimensional generalized hypercubes GH uses packets with header size $O(k)$ bits and the protocol for n-node random tori T uses packets with header size $O(\log n)$ bits. Validity of the both protocols is satisfying whenever a necessary condition expressed in the form of the existence of induced shortest paths holds. We also claim the necessary conditions of fault-tolerant end-to-end communication protocols over the random graphs GH and T, respectively. These conditions are described in the terms of threshold functions and phase transition phenomena in random graphs.

2007-804 D. Král', B. Mohar, A. Nakamoto, O. Pangrác, and Y. Suzuki
Coloring Eulerian triangulations of the Klein bottle

We show that an Eulerian triangulation of the Klein bottle has chromatic number equal to six if and only if it contains a complete graph of order six, and it is 5-colorable, otherwise. As a consequence of our proof, we derive that every Eulerian triangulation of the Klein bottle with face-width at least four is 5-colorable.

2007-805 J. Picado, A. Pultr, and A. Tozzi
Ideals in Heyting semilattices and open homomorphisms

Subfitness and its relation to openness and completeness is studied in the context of Heyting semilattices. A formally weaker condition (c-subfitness) is shown to be necessary and sufficient for openness and completeness to coincide. For a large class of spatial frames, c-subfit $\equiv$ subfit.

2007-806 M. Hladík
Description of symmetric and skew–symmetric solution set

We consider a linear system $Ax = b$, where A is varying inside a given interval matrix $\boldsymbol{A}$ and b is varying inside a given interval vector $\boldsymbol{b}$. The solution set of such a system is described by famous Oettli–Prager Theorem. But if we are restricted only on symmetric (or skew–symmetric) matrices $A \in \boldsymbol{A}$, the problem is much more complicated. So far, the symmetric/skew–symmetric solution set description could be obtained only by a lengthy Fourier–Motzkin elimination applied on each orthant. We present a simple explicit description of the symmetric and skew–symmetric solution set by means of nonlinear inequalities the number of which is, however, still exponential.

2007-807 D. Zeps
Classical and quantum self-reference systems in physics and mathematics

Systematic use of the concept of self-reference system is suggested to apply in mathematics and physics. We enter concept of quantum self-references, which describe our most general mathematical distinctions, abstractions. We argue that in nature there are only self-reference systems, that we may distinct properly, and that our natural way of thinking is mathematical.

2007-808 J. Fiala and J. Kratochvíl
Locally injective graph homomorphism: Lists guarantee dichotomy

We prove that in the List version, the problem of deciding the existence of a locally injective homomorphism to a parameter graph H performs a full dichotomy. Namely we show that it is polynomially time solvable if every connected component of H has at most one cycle and NP-complete otherwise.

2007-809 J. Matoušek and P. Škovroň
Removing degeneracy may require unbounded dimension increase

Many geometric algorithms are formulated for input objects in general position; sometimes this is for convenience and simplicity, and sometimes it is essential for the algorithm to work at all. For arbitrary inputs this requires removing degeneracies, which has usually been solved by relatively complicated and computationally demanding perturbation methods.

The result of this paper can be regarded as an indication that the problem of removing degeneracies has no simple “abstract” solution. We consider *LP-type problems*, a successful axiomatic framework for optimization problems capturing, e.g., linear programming and the smallest enclosing ball of a point set. We prove that in order to remove degeneracies of an LP-type problem, we sometimes have to increase its combinatorial dimension by an arbitrarily large amount.

The proof consists of showing that certain posets cannot be covered by pairwise disjoint copies of Boolean algebras under some restrictions on their placement. To this end, we prove that certain systems of linear inequalities are unsolvable, which seems to require surprisingly precise calculations.

2007-810 J. Nešetřil and P. Ossona de Mendez
Induced Matchings and Induced Paths in Graphs

Denote by $\nabla_1(G)$ the maximum of $\frac{|E(H)|}{|V(H)|}$ over all (simple) minors of G obtained by contracting a star forest. We prove that there exists a positive function ϵ such that every graph G of order n has (at least) two clones (that is two vertices with the same neighbours) or an induced matching of size at least $\epsilon(\nabla_1(G))n$ and that this set may be found in linear time.

More generally, we prove that for every integer k there exists a (very slowly growing) positive function ϵ_k such that every graph of order n has an involutive automorphism or includes a set of size at least $k\lfloor\epsilon_k(\nabla_{\lfloor k/2\rfloor}(G))n\rfloor$ inducing $\lfloor\epsilon_k(\nabla_{\lfloor k/2\rfloor}(G))n\rfloor$ disjoint paths on k vertices.

2007-811 E. Ondráčková (ed.)
HOMONOLO 2006

2007-812 J. Nešetřil
A Surprising Permanence of Old Motivations(a not so rigid story)

This is not a survey article. Rather it is a personal statement written for a lifelong friend and collaborator. Still it is an ambition of this article to trace some of the key moments of our development in the past 40 years. In doing so perhaps some evidence has arisen which otherwise seems to be obscured by the hectic day-to-day academic life. Thus the title. This paper is based on the lecture by the author at the meeting in Victoria devoted to 60 birthday of P.Hell.

2007-813 J. Foniok, J. Nešetřil, and C. Tardif
On Finite Maximal Antichains in the Homomorphism Order

The relation of existence of a homomorphism on the class of all relational structures of a fixed type is reflexive and transitive; it is a quasiorder. There are standard ways to transform a quasiorder into a partial order – by identifying equivalent objects, or by choosing a particular representative for each equivalence class. The resulting partial order is identical in both cases.

Properties of this partial order (the *homomorphism order*) have been intensively studied in algebraic, category theory, random and combinatorial context. Particular interest has been paid to density and universality. Here, we are interested in the characterisation of all *finite maximal antichains* in the homomorphism order.

We show that for structures with at most two relations all finite maximal antichains correspond to what is known as finite homomorphism dualities. In addition, we examine the *splitting property* of finite maximal antichains in the homomorphism order. We derive a structural condition which implies that most finite maximal antichains split. This was previously known for digraphs and structures with at most one relation.

2007-814 J. Nešetřil and M. H. Siggers

A New Combinatorial Approach to the Constraint Satisfaction Problem Dichotomy Classification

We introduce a new general polynomial-time construction- the *fibre construction*- which reduces any constraint satisfaction problem CSP(H) to the constraint satisfaction problem CSP(P), where P is any subprojective relational structure. As a consequence we get a new proof (not using universal algebra) that CSP(P) is NP-complete for any subprojective (and so for any projective) relational structure. The fibre construction allows us to prove the NP-completeness part of the conjectured Dichotomy Classification of CSPs, previously obtained by algebraic methods. We show that this conjectured Dichotomy Classification is equivalent to the dichotomy of whether or not the template is subprojective. This approach is flexible enough to yield NP-completeness of coloring problems with large girth and bounded degree restrictions thus reducing the Feder-Hell-Huang and Kostočka-Nešetřil-Smolíková problems to the Dichotomy Classification of coloring problems.

2007-815 M. Hladík

Multiparametric linear programming: support set and optimal partition invariancy

This paper reflects the renascence of sensitivity and parametric analysis in linear programming and extends single-parametric results to the case when there are multiple parameters in the objective function and in the right-hand side of equations. Multiparametric approach enables us to study more complex perturbation occurring in linear programs than the simpler sensitivity analysis does. Interior point methods in linear optimization suppressed the traditional parametric analysis based on preserving optimal basis and gave rise to new ones. In this paper there is presented a description of the set of admissible parameters under so called support set invariancy and optimal partition invariancy and compared with the classical optimal basis concept. Such a description can be used e.g. for tolerance analysis.

2007-816 J. Hladký, D. Král', J. Sereni and M. Stiebitz
List colorings with measurable sets

The measurable list chromatic number of a graph G is the smallest number ξ such that if each vertex v of G is assigned a set $L(v)$ of measure ξ in a fixed atomless measure space, then there exist sets $c(v) \subseteq L(v)$ such that each $c(v)$ has measure one and $c(v) \cap c(v') = \emptyset$ for every pair of adjacent vertices v and v'. We show that the measurable list chromatic number of a finite graph G is equal to its fractional chromatic number. We also apply our method to obtain an alternative proof of a measurable generalization of Hall's theorem due to Hilton and Johnson [J. Graph Theory **54** (2007), 179–193].

2007-817 M. Hladík
Additive and multiplicative tolerance in multiobjective linear programming

We consider a multiobjective linear program and the coefficients of the multiobjective function are subject to some uncertainties. Let x^* be an efficient point. We propose a procedure to compute an additive and multiplicative (percentage) tolerance in which all the objective function coefficients may simultaneously and independently vary while preserving the efficiency of x^*. Although the tolerances are not maximal in general, they are satisfactorily large. If x^* is a nondegenerate basic solution, then the procedure runs in a polynomial time.

2007-818 D. Král', E. Máčajová, O. Pangrác, A. Raspaud, J. Sereni and M. Škoviera
Projective, affine, and abelian colorings of cubic graphs

We develop an idea of a *local* 3*-edge-coloring* of a cubic graph, a generalization of the usual 3-edge-coloring. We allow for an unlimited number of colors but require that the colors of two edges meeting at a vertex always determine the same third color. Local 3-edge-colorings are described in terms of colorings by points of a partial Steiner triple system such that the colors meeting at each vertex form a triple of the system. An important place in our investigation is held by the two smallest non-trivial Steiner triple systems, the Fano plane $PG(2,2)$ and the affine plane $AG(2,3)$. For $i = 4, 5$, and 6 we identify certain configurations F_i and A_i of i lines of the Fano plane and the affine plane, respectively, and prove a theorem saying that a cubic graph admits an F_i-coloring if and only if it admits an A_i-coloring.

Among consequences of this is the result of Holroyd and Škoviera (2004) that the edges of every bridgeless cubic graph can be colored by using points and blocks of any non-trivial Steiner triple system S. Another consequence is that every bridgeless cubic graph has a proper edge-coloring by elements of any abelian group of order at least 12 such that around each vertex the group elements sum to 0.

We also propose several conjectures concerning edge-coloring of cubic graphs and relate them to several well-known conjectures. In particular, we show that both the Cycle Double Cover Conjecture and the Fulkerson Conjecture can be formulated as a coloring problem in terms of known geometric configurations—the Desargues configuration and the Cremona-Richmond configuration, respectively.

2007-819 Z. Dvořák and R. Škrekovski
k-chromatic number of graphs on surfaces

A well-known result (Heawood 1890, Ringel 1974, Ringel and Youngs 1968) states that the maximum chromatic number of a graph embedded in a given surface S coincides with the size of the largest clique that can be embedded in S, and that this number can be expressed as a simple formula in the Eulerian genus of S. We study maximum chromatic number of k edge-disjoint graphs embedded in a surface. We improve the previously known upper bounds, and show that in many cases, the new upper bound coincides with the lower bound obtained from embedding disjoint cliques in the surface. In the proof of this result, we derive a variant of Euler's Formula for union of several graphs that might be interesting independently.

2007-820 D. Král', E. Máčajová, A. Pór and J. Sereni
Characterization results for Steiner triple systems and their application to edge-colorings of cubic graphs

It is known that a Steiner triple system is projective if and only if it does not contain the four-triple configuration C_{14}. We find three configurations such that a Steiner triple system is affine if and only if it does not contain one of these configurations. Similarly, we characterize Hall triple systems using two forbidden configurations.

Our characterizations have several interesting corollaries in the area of edge-colorings of graphs. A cubic graph G is S-edge-colorable for a Steiner triple system S if its edges can be colored with points of S in such a way that the points assigned to three edges sharing a vertex form a triple in S. Among others, we show that all cubic graphs are S-edge-colorable for every non-projective non-affine point-transitive Steiner triple system S.

2007-821 J. Hubička and J. Nešetřil
A Finite Presentation of the rational Urysohn Space

We give a new construction of the rational Urysohn space $\mathbb{U}_{\mathbb{Q}}$, which yields a finite presentation of $\mathbb{U}_{\mathbb{Q}}$. This may be viewed as an extension of the finite presentation of the generic partial order.

2007-822 D. Garijo, J. Nešetřil and M. P. Revuelta
Homomorphisms and Polynomial Invariants of Graphs

This paper initiates a study of the connection between graph homomorphisms and the Tutte polynomial. This connection enables us to extend the study to other important polynomial invariants associated with graphs, and closely related to the Tutte polynomial. We then obtain applications of these relationships in several areas, including Abelian Groups and Statistical Physics. A new type of uniqueness of graphs, strongly related to chromatically-unique graphs and Tutte-unique graphs, is introduced in order to provide a new point of view of the conjectures about uniqueness of graphs stated by Bollobas, Peabody and Riordan.

2007-823 J. Nešetřil and M. Siggers
Combinatorial Proof that Subprojective Constraint Satisfaction Problems are NP-Complete

We introduce a new general polynomial-time construction- the *fibre construction*- which reduces any constraint satisfaction problem CSP($\mathcal{H}$) to the constraint satisfaction problem CSP($\mathcal{P}$), where $\mathcal{P}$ is any subprojective relational system. As a consequence we get a new proof (not using universal algebra) that CSP($\mathcal{P}$) is NP-complete for any subprojective (and thus also projective) relational system. The fibre construction allows us to prove the NP-completeness part of the conjectured Dichotomy Classification of CSPs, previously obtained by algebraic methods. We show that this conjectured Dichotomy Classification is equivalent to the dichotomy of whether or not the template is subprojective. This approach is flexible enough to yield NP-completeness of coloring problems with large girth and bounded degree restrictions thus reducing the Feder-Hell-Huang and Kostočka-Nešetřil-Smolíková problems to the Dichotomy Classification of coloring problems.

2007-824 Milan Hladík
Optimal value range in interval linear programming

We deal with the linear programming problem in which input data can vary in some given real compact intervals. The aim is to compute exact range of the optimal value function. We present a general approach to the situation the feasible set is described by an arbitrary linear interval system. Moreover, there can be certain dependencies between the constraint matrix coefficients. As long as we are able to characterize the primal and dual solution set (the set of all possible primal and dual feasible solutions, respectively), the bounds of the objective function result from two nonlinear programming problems. We demonstrate our approach on various cases of the interval linear programming problem (with and without dependencies).

2007-825 J. Hladký, M. Krčál, B. Lidický (eds.)
Spring School on Combinatorics 2007

2007-826 D. Kráľ, P. Škoda
Bounds for the real number graph labellings and application to labellings of the triangular lattice

We establish new lower and upper bounds for the real number graph labelling problem. As an application, we completely determine the optimum spans of $L(p, q)$-labellings of the infinite triangular plane lattice (solving an open problem of Griggs).

2007-827 D. Piguet, M. J. Stein
An approximate version of the Loebl-Komlós-Sós conjecture

Loebl, Komlós, and Sós conjecture that if at least half of the vertices of a graph G have degree at least some $k \in \mathbb{N}$, then every tree with at most k edges is a subgraph of G. Our main result is an approximate version of this conjecture for large enough $n = |V(G)|$, and k linear in n.

We extend our result to a slightly larger class of subgraphs. Namely, we show that G contains as subgraphs all bipartite connected graphs of order $k+1$ with at most $k+c$ edges, where c is some constant in n.

Also, we derive from our result an asymptotic bound for the Ramsey number of trees. We prove that $r(T_k, T_m) \leq k + m + o(k+m)$, provided that $\liminf(k/m), \liminf(m/k) > 0$.

2007-828 J. Nešetřil, T. Valla
On Ramsey-type Positional Games

Beck introduced the concept of Ramsey games by studying the game versions of Ramsey and van der Waerden theorems. We contribute to this topic by investigating games corresponding to structural extensions of Ramsey and van der Waerden theorems—the theorem of Brauer, structural and restricted Ramsey theorems.

2007-829 S. Gago
Eigenvalues of scale free graphs

Scale free graphs can be found very often as models of real networks and are characterized by a power law degree distribution, that is, for a constant $\gamma \geq 1$ the number of vertices of degree d is proportional to $d^{-\gamma}$. Experimental studies show that the eigenvalue distribution also follows a power law for the highest eigenvalues. Hence it has been conjectured that the power law of the degrees determines the power law of the eigenvalues. In this paper we show that we can construct a scale free graph with non highest eigenvalue power law distribution. For $\gamma = 1$ we can construct a scale free graph with small spectrum and a regular graph with eigenvalue power law distribution.

2007-830 Milan Hladík
Solution set of complex linear interval systems of equations

We present a solution set description for the system of complex interval equations, where complex intervals have a rectangular form. The solution set is described by a system of nonlinear inequalities, which can be used to obtain very accurate approximation of interval hull of the solution set. In our numerical experiments we exploit this approximation to study overestimation for common complex interval equations solvers (Gauss elimination, Hansen-Bliek-Rohn-Ning-Kearfott method).

2007-831 Jan Kára, ed.
Third Workshop on Graph Classes, Optimization, and Width Parameters, GROW '07, Eugene, October 18–20, 2007

2007-832 R. N. Ball, A. Pultr, J. Sichler
A Priestley Sum of Finite Trees is Acyclic

We show that the Priestley sum of finite trees contains no cyclic finite poset.

2007-833 J. Hladký, D. Král', and S. Norine
Rank of divisors on tropical curves

We investigate, using purely combinatorial methods, structural and algorithmic properties of linear equivalence classes of divisors on tropical curves. In particular, an elementary proof of the Riemann-Roch theorem for tropical curves, similar to the recent proof of the Riemann-Roch theorem for graphs by Baker and Norine, is presented. In addition, a conjecture of Baker asserting that the rank of a divisor D on a (non-metric) graph is equal to the rank of D on the corresponding metric graph is confirmed, and an algorithm for computing the rank of a divisor on a tropical curve is constructed.

2007-834 Jan Foniok
Homomorphisms and Structural Properties of Relational Systems

We investigate, using purely combinatorial methods, structural and algorithmic properties of linear equivalence classes of divisors on tropical curves. In particular, an elementary proof of the Riemann-Roch theorem for tropical curves, similar to the recent proof of the Riemann-Roch theorem for graphs by Baker and Norine, is presented. In addition, a conjecture of Baker asserting that the rank of a divisor D on a (non-metric) graph is equal to the rank of D on the corresponding metric graph is confirmed, and an algorithm for computing the rank of a divisor on a tropical curve is constructed.

2007-835 R. N. Ball, J. Nešetřil, and A. Pultr
Finite dualities, in particular in full homomorphisms

This paper is a survey of several results concerning finite dualities, a special case of the famous Constraint Satisfaction Problem (CSP). In CSP, the point is to characterize a class $\mathcal{C}$ of objects X determined by constraints represented by the requirement of the *existence* of structure preserving mappings *from* X into special ones. In a finite duality, such a class $\mathcal{C}$ is characterized by the *non-existence* of special maps *into* X from a finite system of objects.

In the first third of the article we recall some well-known facts concerning constraints represented by classical homomorphisms of relational systems. In the second part we present several results, not yet published but mostly already submitted, concerning the variant of full homomorphisms. The third part contains a few results on hypergraphs and complexes in this context. These form part of an investigation recently undertaken, and appear here first.

2007-836 K. Kawarabayashi, D. Král, J. Kynčl, and B. Lidický
6-critical graphs on the Klein bottle

We provide a complete list of 6-critical graphs that can be embedded on the Klein bottle settling a problem of Thomassen [Problem 3, J. Comb. Theory Ser. B 70 (1997), 67–100]. The list consists of nine non-isomorphic graphs which have altogether 18 non-isomorphic 2-cell embeddings and one embedding that is not 2-cell.

2007-837 J. Fiala, D. Kráľ, and J. Kratochvíl (eds.)
Workshop on Frequency Assignment Problems, Sádek (Třebíč), September 23–27, 2007

2007-838 S. Gago and D. Schlatter
Bounded expansion in web graphs

In this paper we study various models for web graphs with respect to bounded expansion. All the deterministic models even have constant expansion, whereas the copying model has unbounded expansion. The most interesting case turns out to be the preferential attachment model—which we conjecture to have unbounded expansion, too.

2007-839 F. Kardoš, D. Kráľ, J. Miškuf, and J. Sereni
Fullerene graphs have exponentially many perfect matchings

A fullerene graph is a planar cubic 3-connected graph with only pentagonal and hexagonal faces. We show that fullerene graphs have exponentially many perfect matchings.

2007-840 J. Fiala, P. Golovach, and J. Kratochvíl
Distance constrained labelings of trees

An $H(p, q)$-labeling of a graph G is a vertex mapping $f : V_G \to V_H$ such that the distance (in the graph H) of $f(u)$ and $f(v)$ is at least p (at least q) if the vertices u and v are adjacent in G (are at distance two in G, respectively). This notion generalizes the notions of $L(p, q)$- and $C(p, q)$-labelings of graphs studied as a graph model of the Frequency Assignment Problem. We study the computational complexity of the problem of deciding the existence of such a labeling when the graphs G and H come from restricted graph classes. In this way we are extending known results for linear and cyclic labelings of trees, with a hope that our results would help to open a new angle of view on the still open open problem of $L(p, q)$-labeling of trees for fixed $p > q > 1$ (i.e., when G is a tree and H a path). Our main results are a polynomial time algorithm for $H(p, 1)$-labeling of trees for arbitrary H, and NP-completeness results for $H(p, q)$-labeling of trees when H is a q-caterpillar, and $L(p, q)$-labeling of trees for fixed $q > 1$ and p part of input.

2007-841 J. Fiala and D. Paulusma
Comparing universal covers in polynomial time

The universal cover T_G of a connected graph G is the unique (possible infinite) tree covering G, i.e., that allows a locally bijective homomorphism from T_G to G. Universal covers have major applications in the area of distributed computing. It is well-known that if a graph G covers a graph H then their universal covers are isomorphic, and that the latter can be tested in polynomial time by checking if G and H share the same degree refinement matrix. We extend this result to locally injective and locally surjective homomorphisms by following a very different approach. Using linear programming techniques we design two polynomial time algorithms that check if there exists a locally injective or a locally surjective homomorphism, respectively, from a universal cover T_G to a universal cover T_H. This way we obtain two heuristics for testing the corresponding locally constrained graph homomorphisms. As a consequence, we have obtained a new polynomial time algorithm for testing (subgraph) isomorphism between universal covers, and for checking if there exists a role assignment (locally surjective homomorphism) from a given tree to an arbitrary fixed graph H.

2007-842 D. Piguet and M. Stein

The Loebl–Komlós–Sós conjecture for trees of diameter 5 and for certain caterpillars

Loebl, Komlós, and Sós conjectured that if at least half the vertices of a graph G have degree at least some $k \in \mathbb{N}$, then every tree with at most k edges is a subgraph of G.

We prove the conjecture for all trees of diameter at most 5 and for a class of caterpillars. Our result implies a bound on the Ramsey number $r(T, T')$ of trees T, T' from the above classes.

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