Continuous DCPOs as complete information systems

In honour of Dieter Pumplün on the occassion of his 70th birthday

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Abstract

Continuous DCPOs are shown to be precisely the complete objects (in the sense of the completions of Brümmer, Giuli and Herrlich, [2], [1]) in a certain category of generalized information systems (related to those of Vickers, [9]), with naturally defined morphisms and embeddings.

Generalizing the Scott information systems ([8]), Vickers has introduced in [9] the continuous information system (infosys) as a set with a transitive interpolative relation (see also Hoofman, [4]). This simple notion allowed to represent adequately the Scott's approximable maps, and to show (among many other results) that the obtained category was equivalent to the category of continuous DCPOs. In [6] we have restricted the Vickers' information systems to more special ld-sets (not much more special: the interpolativity condition has been just replaced by a simultaneous interpolation for couples – see 1.2 below). Thus obtained formally smaller category is still equivalent to that of continuous DCPOs (and hence in fact equivalent to the original one). Moreover, there are naturally defined morphisms between the ld-sets (the ld-maps, see 1.2 below, other than the "approximable maps" that are relations, not mappings) such that in the obtained category the translation to the DCPOs yields a monad the Kleisli category of which

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*The first author would like to express his thanks for the support by the Italian C.N.R. and by the project LN 00A056 of the Ministry of Education of the Czech Republic.

coincides with the original category of approximable maps. It turned out, a.o., that the category of continuous DCPOs was a reflective subcategory of that of ld-sets and ld-maps. The aim of this paper is to show that this reflection has the properties required by Brümmer, Giuli and Herrlich ([2], [1]) of a *completion*. That is, there is a natural notion of a dense embedding (dense in the Lawson topology modified for ld-sets) such that for any ld-set $(X, \prec)$ there is, up to isomorphism, exactly one dense embedding into a continuous DCPO, namely the mentioned reflection. We also prove some further facts typical for completions, like e.g. that an ld-set is a continuous DCPO iff each of its embeddings has a closed image.

1 Preliminaries

1.1. A binary relation R is *interpolative* if $R \circ R \supseteq R$.

The notion of a *directed (sub)set* will be used in any set with a transitive (not necessarily reflexive) relation. Thus, D is directed in $(X, \prec)$ if it is non-void and if for any $x_1, x_2 \in D$ there is an $x \in D$ such that $x_1, x_2 \prec x$.

In a partially ordered set $(X, \leq)$ we denote by $\bigvee^{\uparrow} D$ the supremum of a directed $D \subseteq X$, if it exists (thus, the use of the symbol $\bigvee^{\uparrow}$ instead of $\bigvee$ or sup indicates the nature of the set that follows, not a change in the definition of supremum).

We will usually denote by $\ll$ the standard *way-below* relation ($x \ll y$ if $(y \leq \bigvee^{\uparrow} D \ \Rightarrow\ \exists d \in D, x \leq d)$).

1.2. A *locally directed set* (briefly, an *ld-set*) $(X, \prec)$ is a set with a transitive relation $\prec$ such that each

$$\prec x = \{y \mid y \prec x\}$$

is directed. More explicitly, we assume that

(ld1) for each $x \in X$ there is a y such that $y \prec x$, and

(ld2) if $y_1, y_2 \prec x$ then there is a y such that $y_1, y_2 \prec y \prec x$.

Note that, in particular, $\prec$ is interpolative. We have used this notion in [6] as a slightly stronger form of Vickers' *continuous information system*, or *infosys* (see [9] – a set with a transitive interpolative relation). It comes closer to the Scott information system (which is being generalized) in that it includes a modification of the axiom "if $u \vdash v_1$ and $u \vdash v_2$ then $u \vdash (v_1 \cup v_2)$", and the category of ld-sets with approximable maps as in [9], similarly like the formally larger **Infosys**, is still equivalent with that of continuous domains.

In an ld-set $(X, \prec)$ we introduce a relation $\precsim$ by setting

$$x \precsim y \quad \text{iff} \quad {\prec} x \subseteq {\prec} y.$$

We also write $x \sim y$ for ${\prec} x = {\prec} y$. We have $(x \sim y \;\; \Rightarrow \;\; x = y)$ iff the preorder $\precsim$ is a partial order. Obviously we have

$$\begin{array}{ll} x \prec y \quad \Rightarrow \quad x \precsim y, & \text{and} \\ x \prec y \precsim z \quad \Rightarrow \quad x \prec z, & \end{array}$$

while in general $x \precsim y \prec z$ does not necessarily imply $x \prec z$.

An *ld-map* $f : (X, \prec) \to (Y, \prec)$ is a mapping $f : X \to Y$ such that

(m1) $x \precsim y \quad \Rightarrow \quad f(x) \precsim f(y)$, and

(m2) if $y \prec f(x)$ then $y \prec f(x')$ for some $x' \prec x$.

The resulting category will be denoted by

LDir

(this is not the category of approximable maps mentioned above; see 1.4.1 below). Note that

> *the isomorphisms in* **LDir** *do not generally preserve the relation* $\prec$*; they preserve the weaker* ${\prec'} = {\precsim} \circ {\prec}$.

1.3. A *continuous domain* is a partially ordered set $(X, \prec)$ such that

- every directed $D \subseteq X$ has a supremum in X, and
- for any $x \in X$, $x = \bigvee^{\uparrow}\{y \mid y \ll x\}$.

(In [9], Vickers used the term *continuous poset.* We have chosen this modified term to prevent a first impression that it might mean just a poset in which every x is the supremum of the set $\{y \mid y \ll x\}$; a more standard term is *continuous DCPO.*)

A *domain map* $f : (X, \leq) \to (Y, \leq)$ preserves all directed suprema. The resulting category will be denoted by **CDom**. It is easy to see that

(1.3.1) *if* $(X,\leq)$ *is a continuous domain then* $(X \ll)$ *is an ld-set, and the original* $\leq$ *coincides with the associated* $\precsim$ *as in 1.2.1.*

Thus, without loss of information, a continuous domain can be regarded as the ld-set $(X \ll)$ (when stating that $(X,\prec)$ is a continuous domain we will mean that $(X,\precsim)$ is a continuous domain *and that* $\prec$ *is the associated well-below relation*). Furthermore, it is easy to check that

> *the ld-maps* $f : (X,\ll) \to (Y,\ll)$ *between continuous domains are precisely the domain maps* $(X,\leq) \to (Y,\leq)$.

Thus,

CDom *is a full subcategory of* **LDir**.

1.3.2. Note that in a continuous domain one has also the implication $x \precsim y \prec z \;\Rightarrow\; x \prec z$.

1.4. An *ideal* J in an ld-set $(X,\prec)$ is a non-void $J \subseteq X$ such that

- if $x \prec y \in J$ then $x \in J$, and
- if $x_1, x_2 \in J$ then there is an x such that $x_1, x_2 \prec x \in J$

(cf. [9]). Note that, in particular,

for each $x \in X$, ${\prec}x$ *is an ideal in* $(X,\prec)$.

The set of all ideals in $(X,\prec)$ ordered by inclusion will be denoted by

$$\mathfrak{I}(X,\prec).$$

The following facts can be found in [6] (they are easy to prove as an exercise, too).

- $\mathfrak{I}(X,\prec)$ is a continuous domain, and the way-below relation $J \ll K$ is given by $(\exists x \in K,\ J \subseteq {\prec}x)$.
- The mappings $\eta_{(X,\prec)} = (x \mapsto {\prec}x); (X,\prec) \to \mathfrak{I}(X,\prec)$ are ld-maps.
- The construction $\mathfrak{I}$ can be extended to a functor by setting $\mathcal{J}(f)(J) = \bigcup\!\!\!\uparrow \{{\prec}f(x) \mid x \in J\}$. The system $\eta = (\eta_{(X,\prec)})_{(X,\prec)}$ is then a transformation $\mathrm{Id} \to \mathfrak{I}$ and constitutes a reflection of **LDir** onto **CDom**.

1.4.1. Note. The approximable maps $(X,\prec)$ to $(Y,\prec)$ from [9] (generalizing those from [8]) can be viewed as ld-maps $f : (X,\prec) \to \mathfrak{I}(Y,\prec)$ (representing $f \subseteq Y \times X$ by $y \in f(x) \;\equiv\; yfx$). In the monad $\mathbb{I}$ obtained from $\mathfrak{I}$ and η, the category of ld-sets and approximable maps is in fact the Kleisli category $\mathbf{LDir}_{\mathbb{I}}$ (see [6]).

2 Lawson topology on ld-sets

2.1. Let $\prec$ be an interpolative relation on a set X. In [9], Vickers introduced the system

$$\sigma(X,\prec)=\{U \mid U={\succ}U=\{x \mid x\succ u\in U\}\},$$

and used the fact that $\sigma(X,\prec)$ was a frame, indeed a completely distributive lattice. In general it is not a topology on X but we have

2.1.1. Proposition. *1. Each* ${\succ}a=\{x \mid a\prec x\}$ *is in* $\sigma(X,\prec)$.

2. $\sigma(X,\prec)$ *is a topology on* X *iff* $(X,\prec)$ *is an ld-set.*

3. If $(X,\prec)$ *is an ld-set then* $\precsim$ *is the specialization preorder in* $\sigma(X,\prec)$.

4. If $(X,\prec)$ *is a continuous domain then* $\sigma(X,\prec)$ *is the Scott topology* (for Scott topology see e.g. [3] or [7]).

5. The ld-maps $f:(X,\prec)\to(Y,\prec)$ *are precisely the continuous maps.*

Proof. 1 is obvious. 2: The condition (ld1) in 1.2 says exactly that $X\in\sigma(X,\prec)$, and we obviously have $\sigma(X,\prec)$ closed under intersection of couples iff for any $a,b\in X$, $({\succ}a)\cap({\succ}b)\in\sigma(X,\prec)$ which in turn is easily seen to be the condition (ld2).

3. If $x\in\overline{\{y\}}$ and $a\prec x$ then the open set ${\succ}a$ has to contain y, that is, $a\prec y$. If $x\precsim y$ and $x\in U$ open, then for some $a\prec x$, ${\succ}a\subseteq U$ and since $a\prec y$, we have $y\in U$.

4. If U is in $\sigma(X,\prec)$ then obviously $U=\{x \mid x\geq u\in U\}$ by 3 and (1.3.1); if D is directed and $\bigvee^{\uparrow}D\in U$ take an $a\ll\bigvee^{\uparrow}D$, $a\in U$, to obtain $a\leq d$ for some $d\in D$. On the other hand, if U is Scott open then obviously ${\succ}U\subseteq U$, and if $x\in U$ then $x=\bigvee^{\uparrow}\{y \mid y\ll x\}$ and hence $y\in U$ for some $y\ll x$.

5. If f is continuous then $x\precsim y$ implies $f(x)\precsim f(y)$ by 3; if $y\prec f(a)$ we have $a\in f^{-1}({\succ}y)$ open and hence there is an $a'\prec a$ such that $a'\in f^{-1}({\succ}y)$, that is, $y\prec f(a')$. If f is an ld-map and $a\in f^{-1}({\succ}y)$, that is, $y\prec f(a)$, there is an $a'\succ a$ with $a'\in f^{-1}({\succ}y)$. If $x\in({\succ}a')$, we have $y\prec f(a')\precsim f(x)$; hence $a\in({\succ}a')\subseteq f^{-1}({\succ}y)$. Thus, $f^{-1}({\succ}y)$ is open and since obviously the ${\succ}y$ constitute a basis of $\sigma(X,\prec)$, f is continuous. $\square$

2.2. The Lawson topology on an ld-set $(X,\prec)$ is generated by $\sigma(X,\prec)$ and the (lower Alexandroff) topology $\{U \mid U=\{x \mid x\precsim u\in U\}\}$. (Lawson topology – see e.g. [3] – is usually defined on more special $(X,\prec)$ but the extension is obvious.) It will be denoted by

$$\lambda(X,\prec).$$

For $a \prec b$ set

$$a * b = \{x \mid a \prec x \precsim b\}.$$

2.2.1. Lemma. *1. The sets $a * b$ constitute a basis of $\lambda(X, \prec)$.*

2. U is open in $\lambda(X, \prec)$ iff for every $b \in U$ there is an $a \prec b$ such that $a * b \subseteq U$.

3. $M \subseteq X$ is dense in $\lambda(X, \prec)$ iff for any $a \prec b$ in X there is an $m \in M$ such that $a \prec m \precsim b$.

Proof. 1. We must prove that the intersection $U = (a_1 * b_1) \cap (a_2 * b_2)$ is always open. Let $b \in U$. Then $a_1, a_2 \prec b \precsim b_1, b_2$. Choose an a such that $a_1, a_2 \prec a \prec b$. Then $b \in a * b \subseteq U$.

2 and 3 are immediate consequences of 1. $\square$

2.2.2. Proposition. *Let $(X, \prec)$, $(Y, \prec)$ be ld-sets. A monotone map $f : (X, \precsim) \to (Y \precsim)$ is an ld-map $(X, \prec) \to (Y, \prec)$ iff it is continuous in the Lawson topology.*

Proof is much the same as that of 5 in 2.1.1, using the fact that the $a * b$ constitute a basis. This time, of course, the monotonicity does not follow from the continuity and has to be explicitly assumed. $\square$

2.3. Closed subsets in $\lambda(X, \prec)$. The *trace* of an element x in an ld-set $(X, \prec)$ is a subset T of X such that

- for each $t \in T$, $t \precsim x$, and
- for each $y \prec x$ there is a $t \in T$ such that $y \prec t$.

Notes. 1. If $(X, \prec)$ is a continuous domain then obviously the traces of x are precisely the directed $T \subseteq X$ such that $x = \bigvee^{\uparrow} T$.

2. Losely speaking, the traces of x are the subsets of X that are cofinal in $\prec x$.

3. In the second condition it suffices to require $y \precsim t$; indeed, interpolate $y \prec y' \prec x$.

2.3.1. Proposition. *A subset $A \subseteq X$ is closed in $\lambda(X, \prec)$ iff $x \in X$ is in A whenever it has a trace contained in A.*

Proof. Let A be closed and $x \in X \setminus A$. Then there is a $y \prec x$ such that $y * x \subseteq X \setminus A$. If T is a trace of x we have a $t \in T$ with $y \prec t \precsim x$, hence, a $t \notin A$.

Let A not be closed. Then $X \setminus A$ is not open and hence there is an $x \in X \setminus A$ such that no $y * x$ with $y \prec x$ is contained in $X \setminus A$. Thus, for every $y \prec x$ we can choose a $t(y) \in A$ such that $y \prec t(y) \precsim x$. Then $T = \{t(y) \mid y \prec x\}$ is a trace of x contained in A. $\square$

2.4. The T_0 case. The $\lambda(X, \prec)$ that are T_0 have a very simple characteristics.

Lemma. $\lambda(X, \prec)$ is T_0 iff $\precsim$ is an order iff $\eta_{(X,\prec)}$ is one-one.

Proof. The first two statements can be translated to "if $x \neq y$ then for some a, $a \prec x$ or $a \prec y$ but not both"; the third one is just a reformulation of the second. $\square$

In consequence, we will refer to the $(X, \prec)$ with $\precsim$ an order as the T_0 ld-sets.

3 The ideal functor as a completion

3.1. Embeddings. An ld map $f : (X, \prec) \to (Y, \prec)$ is called *embedding* if

(1) $x \prec y \quad \Rightarrow \quad f(x) \prec f(y)$, and

(2) $f(x) \precsim f(y) \quad \Rightarrow \quad x \precsim y$.

Notes. 1. A composition of embeddings is obviously an embedding. To have a system requested for for the completion as in [2] and [1] we should have, moreover, compositions of embeddings with isomorphisms again embeddings which, in general, we have not (recall 1.2). This could be easily remedied by modifying the assumption (1) to $(x \prec y \ \Rightarrow \ \exists z, f(x) \precsim x \prec y)$ without much changing the facts below (in particular because if $(Y, \prec)$ is a continuous domain, the modification requires the same as (1)). We keep the simpler definition.

2. Each $\eta_{(X,\prec)} : (X, \prec) \to \mathfrak{I}(X, \prec)$ is obviously a *dense* embedding (that is, an embedding with $\eta[X]$ dense in the Lawson topology of $\mathfrak{I}(X, \prec)$).

3. Just the fact that a subset of an ld-set is an ld-set with the $\prec$ same as in the bigger one, even if it is dense, does not make the inclusion an embedding. Consider $Y = I \times \{0, 1\}$, where I is the unit interval, with the relation defined by $(r, i) \prec (s, j)$ if $r < s$ and $i \leq j$, or $r = s = 0$; take $X = \{(x, 0) \mid r < 1\} \cup \{(1, 1)\}$.

3.2. Aside. The maps $\eta : (X \ \prec) \to \mathfrak{I}(X, \prec)$ are topologically very satisfactory. We have

Proposition. *In the Lawson topology, U is open in $(X,\prec)$ iff $U = \eta^{-1}(V)$ for an open V in $\Im(X,\prec)$.*

Proof. It suffices to prove that such are the $a * b$. Set $V_1 = \{J \in \Im(X,\prec) \mid \exists x \in J, a \prec x\}$. Then V is open, even in $\sigma(X,\prec)$, since if $x \in J$ and $a \prec x$ we can choose a $y \in J$ such that $a \prec x \prec y$, and then ${\prec}y \subseteq V_1$ and ${\prec}y \ll J$; we have $\eta^{-1}(V_1) = \{u \mid \exists x \prec u,\ a \prec x\} = \{u \mid a \prec u\}$. Further consider $V_2 = \{J \mid J \subseteq {\prec}b\}$. Then $\eta^{-1}(V_2) = \{x \mid {\prec}x \subseteq {\prec}b\}$ and hence $\eta^{-1}(V_1 \cap V_2) = a * b$. $\square$

3.3. Proposition. *Let $(Y,\leq)$, $(Z,\leq)$ be continuous domains and let $f : (X,\prec) \to (Y,\ll)$ be a dense embedding and $g : (X,\prec) \to (Z,\ll)$ an embedding. Then there is exactly one ld-map $\alpha : (Y,\ll) \to (Z,\ll)$ such that $\alpha f = g$.*

Proof. Set $\alpha(y) = \bigvee^{\uparrow}\{g(x) \mid f(x) \ll y\}$ (if $f(x_i) \ll y$ for $g(x_i)$, $i = 1,2$, choose a y' such that $f(x_i) \ll y' \ll y$, and by density an $f(x)$ such that $y' \ll f(x) \ll y$; then by (2), $x_1, x_2 \precsim x$ and hence $g(x_i) \leq g(x)$; thus the join is directed). Obviously α is monotone. Now let $z \ll \alpha(y)$. Then $z \ll g(x_0)$ for some x_0 with $f(x_0) \ll y$. Choose an $x \prec x_0$ such that $x \ll g(x)$. By (1), $f(x) \ll f(x_0)$ and hence $g(x) \leq \alpha(g(x_0))$. Thus, $y' = g(x_0) \ll y$ and we have $x \ll \alpha(y')$ proving that α is an ld-map.

Obviously $\alpha(f(x)) \leq \bigvee^{\uparrow}\{g(t) \mid f(t) \ll f(x)\} \leq g(x)$. Now if $z \ll g(x)$ there is a t such that $t \prec x$ (and hence $f(t) \ll f(x)$) such that $z \ll g(t)$. Thus we also have $\alpha(f(x)) \geq g(x)$.

Finally, if $\beta f = g$ and β is an ld-map (and hence a domain map), since $y = \bigvee^{\uparrow}\{f(x) \mid f(x) \ll y\}$ by density, we have $\beta(y) = \bigvee^{\uparrow}\{\beta f(x) = g(x) \mid f(x) \ll y\} = \alpha(y)$. $\square$

3.3.1. Corollary. *Let $(Y,\leq)$, $(Z,\leq)$ be continuous domains and let $f : (X,\prec) \to (Y,\ll)$, $g : (X,\prec) \to (Z,\ll)$ be dense embeddings. Then the α such that $\alpha f = g$ (as in 3.3) is an isomorphism. Consequently, up to isomorphism, $\eta_{(X,\prec)}$ is the only dense embedding of $(X,\prec)$ into a continuous domain.*

3.4. As further corollaries we obtain the standard completion facts.

Proposition. *1. If $(X,\prec)$ is a continuous domain then $\eta_{(X,\prec)}$ is an isomorphism.*

2. If $f : (X,\prec) \to (Y,\prec)$ is a dense embedding then $\Im(f)$ is an isomorphism.

3. For a T_0 ld-set $(X,\prec)$ (recall 2.4), $(X\precsim)$ is a continuous domain iff each dense embedding $f:(X,\prec)\to(Y,\prec)$ into a T_0 ld-set is an isomorphism.

Proof. 1 is obvious.

2. $\eta\cdot f:(X,\prec)\to\mathfrak{I}(Y,\prec)$ is a dense embedding and hence, by 3.3, $\mathfrak{I}(f)$ is the only ld-map α such that $\alpha\eta=\eta f$, and hence, by 3.3.1, it is an isomorphism.

3. If $(X,\prec)$ is a continuous domain and $f:(X,\prec)\to(Y,\prec)$ is dense, we have $\eta\cdot f=\mathfrak{I}(f)\cdot\eta$ an isomorphism by 1 and 2. Thus, if $(Y,\prec)$ is T_0, $\eta_{(X,\prec)}$ is a one-one retraction and hence an isomorphism. Thus, also f is an isomorphism. On the other hand if the statement holds then in particular $\eta_{(X,\prec)}$ is an isomorphism. □

3.5. Now we will prove a counterpart of the characteristics of complete uniform spaces as those for which the image under any uniform embedding is closed. In spaces, this is the same as the 3 in 3.4 above. Here we would have to prove, first, that the closure of the image of an embedding is an ld-set into which the restriction of the original embedding is again an embedding. Instead, we will present an easy direct proof.

Proposition. *A T_0 ld-set $(X,\prec)$ is isomorphic to a continuous domain iff for every embedding $f:(X,\prec)\to(Y,\prec)$ into a T_0 ld-set, $f[X]$ is (Lawson) closed.*

Proof. Let $(X,\prec)$ be a continuous domain and let $f:(X,\prec)\to(Y,\prec)$ be an embedding. Let $b\in Y$ and let $T\subseteq f[X]$ be a trace of b. For $y\prec b$ choose, first, $\xi(y)\in X$ such that $y\prec f(\xi(y))\precsim b$ and then, using the property of ld-maps, an $x(y)\prec\xi(y)$ such that still $y\prec f(x(y))$. Then $f(x(y))\prec f(\xi(y))$ and hence

$$y\prec f(x(y))\prec b.$$

The system

$$D=\{x(y)\mid y\prec b\}$$

is directed in $(X,\prec)$: for $x(y_i)$, $i=1,2$, choose $y\prec b$ such that $f(x(y_i))\prec y\prec b$; then $f(x(y_i))\prec f(x(y))$ and hence $x(y_i)\precsim x(y)$. Thus we have a supremum $a=\bigvee^{\uparrow}D$. If $y\prec f(a)$ there is an $x\ll a$ such that $y\prec f(x)$; thus, $x\precsim x(z)$ for some $x\prec b$, and $y\prec f(x(z))\prec b$. If $y\prec b$ then $y\prec f(x(y))\precsim a$ and hence $y\prec f(a)$. Thus, $b\sim f(a)$, and by T_0, $b=f(a)\in f[X]$.

The other implication immediately follows from 3.4.3. □

Remark. Recall the example in the Notes in 3.1. Note that X is a continuous domain while the set X is not closed in Y.

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