Midsummer Combinatorial Workshop VI

Prague
July 26-30, 1999

Contents

Preface

The Sixth Prague Midsummer Combinatorial Workshop was held from July 26 to July 30, 1999 at Malostranské náměstí building of Charles University which is depicted on the cover of this publication. The workshop was organized by the Department of Applied Mathematics (KAM) of Charles University jointly with the DIMATIA centre. Only a small but distinguished group of mathematicians was invited and we were particularly happy to have John Gimbel, Meir Katchalski and Hubert de Fraysseix (with his mother) among us.

The workshop benefited from two facts: In the framework of the joint DIMATIA-DIMACS REU program, 5 selected undergraduate students from the USA and 2 students from Charles University took part in the workshop, together with student guides Jana Maxová and Paul Dreyer. The other factor was the proximity of the ESA '99 meeting which took place just before the workshop (July 11-15).

The workshop followed an informal daily routine with morning and early afternoon discussions. This report reflects some of the discussions during the workshop. Perhaps you can digest from these proceedings some of the atmosphere at the workshop and you can also see that the fruitful exchange of ideas led directly to some new results and papers.

This volume was edited by Jan Vondrák. Most of the problems described here were supplied by the authors in electronic form; in a few cases, slight typographical changes were necessary. We apologize for any possible inaccuracies which might have occurred in the editing process.

The conference photos were taken in the "Silent Valley" during our trip to the Koliba restaurant where we had a conference dinner.

This summer workshop was partly supported by a Charles University grant GAUK 158, a Czech grant GAČR 201/99/0242 and Kontakt 337.

Based on our past experience and being encouraged by several participants, we hope to organize the Seventh Prague Combinatorial Workshop in the summer of 2000. We hope to meet you all there!

Jaroslav Nešetřil

Obstructions To Treewidth and Pathwidth

Janka Chlebíková

The obstruction sets for treewidth 1, 2, and 3 are known so far (see [1] and [6]). Over 75 minimal forbidden minors for treewidth at most four of widely varying structures is presented in [5]. The obstruction sets for pathwidth 1 and 2 were described in [3]. In [4] a structural characterization of graphs from $obs(TW(w))$ (resp. $obs(PW(w))$) with $(w+3)$ vertices is given.

We have found a structural characterization of graphs from $obs(TW(w))$ (resp. $obs(PW(w))$) with a fixed number of vertices in terms of subgraphs of the complement. Our approach essentially simplifies the characterization of graphs from $obs(TW(w))$ (resp. $obs(PW(w))$) with $(w+3)$ vertices as was given in [4]. This method also solves an open problem from [4]: for any $w \geq 3$ a graph from $obs(TW(w)) \setminus obs(PW(w))$ can be constructed.

Definition 1 *Let $\mathcal{T}^1 = \{K_2\}$, $\mathcal{T}^2 = \{K_3, P_3\}$ and the graphs from $\mathcal{T}^{r+1}$ are constructed from graphs from $\mathcal{T}^r$ as follows: for each graph H from $\mathcal{T}^r$ and each independent vertex set A of H with at least $|V(H)|-(r+1)$ vertices denote $B = V(H) \setminus A$. Let $C \cup \{v_0\}$ be the set of new vertices such that $|C| = r+1-|V(B)|$. Define the graph G from $\mathcal{T}^{r+1}$ by the following way:*

$$V(G) = V(H) \cup C \cup \{v_0\} \quad \textit{and} \quad E(G) = E(H) \cup \{\{v_0, u\}, u \in B \cup C\}.$$

Theorem 2 *Given $r \geq 1$, the following conditions are equivalent for any graph G:*

1. *$TW(G) \geq |V(G)| - r$;*
2. *$\overline{G}$ contains no graph from $\mathcal{T}^r$ as a subgraph.*

Theorem 3 *Given $r \geq 2$. A graph G with $(w+r+1)$ vertices is in $obs(TW(w))$ if and only if G satisfies the following three conditions:*

$T_1(r)$: *$\overline{G}$ contains no graph from $\mathcal{T}^r$ as a subgraph.*

$T_2(r)$: *If H is a minor of G with $|V(G)|-1$ vertices, then $\overline{H}$ contains some graph from $\mathcal{T}^{r-1}$ as a subgraph.*

$T_3(r)$: *For every $e \in E(G)$, $\overline{G \setminus e}$ contains some graph from $\mathcal{T}^r$ as a subgraph.*

Lemma 4 *Given $r \geq 2$ and a graph G with the property $T_2(r)$. Let F be a graph such that $\overline{F}$ contains $\overline{G}$ as a subgraph. Then F has the property $T_2(r)$.*

Remark 5 *According to Lemma 4 the family of graphs $\mathcal{M}^r$ (for fixed r) is supergraph-closed. It easily follows that $\mathcal{M}^r$ can be characterized in terms of subgraphs of the complement. A graph G possesses $T_2(r)$ if and only if $\overline{G}$ contains some graph from $\min \mathcal{M}^r$ as a subgraph ($\min \mathcal{M}^r$ stands for the set of minimal elements of $\mathcal{M}^r$ with respect to subgraph relation).*

Problem 6 *Is there any structural characterization of graphs from* $\min \mathcal{M}^r$ *at least for small values of* r, $r \geq 3$. *Is the set* $\min \mathcal{M}^r$ *always finite?*

According the proofs of the following Theorems 7 and 8 (see [2]) closer informations about $\min \mathcal{M}^r$ help us to find graphs from the obstruction set, $obs(TW(w))$.

Theorem 7 *Let* $r \geq 2$ *be given and* F *be a graph with property* $T_1(r)$ *and* $T_2(r)$. *Then for every* w, $w \geq |V(F)| - (r+1)$ *there exists a graph* $G \in obs(TW(w))$ *with* $(w+r+1)$ *vertices such that some subgraph of* F *with vertex set* $V(F)$ *is an induced subgraph of* G.

Theorem 8 *Let* H *be a graph from* $obs(TW(w))$ *for some* w. *Then for every* $k > w$ *there exists a graph* G *from* $obs(TW(k))$ *such that*

$$|V(G)| - k = |V(H)| - w$$

and H *is an induced subgraph of* G.

Remark 9 *The results analogous to Theorems 2, 3, 7 and 8 are valid also for pathwidth, where* $\mathcal{T}^r$ *is replaced by* $\mathcal{P}^r = \{H \in \mathcal{T}^r, H \text{ is a comparability graph}\}$.

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Computational Complexity of Half-edge Coloring

Jiří Fiala

Given a cubic graph $G = (V, E)$, we call a mapping $c : E \times V \to \{a, b, c\}$ a *proper half-edge coloring* if for each vertex $v \in V$ the following holds: If we denote e_1, e_2, e_3 three edges v belongs to then c restricted to the set $\{(e_1, v), (e_2, v), (e_3, v)\}$ is injective on the set $\{a, b, c\}$. Such a coloring for a given graph always (trivially) exists.

We say that the half-edge coloring c on an edge $e = (u, v)$ satisfies constraint $A(e) \subseteq \{a, b, c\}^2$ if $(c(e, u), c(e, v)) \in A(e)$.

In general, the question whether G admits a proper half-edge coloring satisfying constrains $\{A(e)\}$ is NP-complete, since after setting all $A(e) = \{(a, a), (b, b), (c, c)\}$ the question is equivalent to the looking for a proper three-edge-coloring of G.

There is an open problem to determine for each parameter set $\mathcal{A} \subseteq \mathcal{P}(\{a, b, c\}^2)$ the computational complexity of the problem of existence of a proper half-edge coloring satisfying constraints s.t. $A(e) \in \mathcal{A}$ for all edges e.

It is already known that if $\{(a, a), (b, b), (c, c)\} \in \mathcal{A}$ then the problem is NP-complete. There are also two polynomially solvable cases $\mathcal{A} = \{\{(a, b), (a, c), (b, c)\}\}$ and $\mathcal{A} = \{\{(a, b)\}, \{(a, c)\}, \{(b, c)\}\}$.

Three Problems

John Gimbel

Problem 1. An Sg-polytope is a compact orientable surface of genus g made by gluing convex polygons together at edges. The dual of an Sg-polytope is a graph where each vertex corresponds to a face, and two vertices are adjacent if their corresponding faces share a common boundary. Is there an Sg-polytope whose dual contains a complete graph on five vertices?

I believe no such Sg-polytope exists. I presented a non-rigorous proof of this at the workshop. The proof involves folding a piece of paper in a variety of ways. Pavel Valtr has declared my proof extremely non-rigorous.

Problem 2. The domination number, $d(G)$, of a graph G is the fewest number of vertices in a set S having the property that each vertex not in S is adjacent to a vertex in S. Equivalently, we can label the vertices with $\{0, 1\}$ so that the sum of labels on each closed neighborhood is at least one. If we sum all labels, the minimum value of such sums is the domination number. Now, suppose we take this same definition, and replace $\{0, 1\}$ with the interval $[0, 1]$. This gives us the fractional domination number, $f(G)$, of G. Clearly, $f(G) \leq d(G)$ for any graph G. Is it true that for any graph G of order n, $d(G) \leq log(n)f(G)$, for some appropriately chosen logarithm?

Problem 3. Given graph G, the co-chromatic number, $z(G)$, is the fewest number of colors needed to color the vertices of G so that each color class induces either a complete or empty graph. Also, let $\chi(G)$ be the chromatic number of G. Clearly, $z(G) \leq \chi(G)$, for any graph G. Now, let G_n be the random graph on n vertices, with edge probability $\frac{1}{2}$. Is it true that $\chi(G_n) - z(G_n)$ goes to infinity? That is, is there some parameter $d(n)$ which goes to infinity where almost surely $\chi(G_n) - z(G_n) \geq d(n)$?

This problem was originally posed by Erdos and myself. At one point he offered \$100 if it could be shown to be true. Also, he offered \$1000 if it could be shown to be false. On one occasion he remarked that he had perhaps offered too much money.

Steiner Diagrams

Winfried Hochstättler

Problem 1 (STEINER Diagrams): Let V be a set of nodes and $w : V \times V \to \mathbb{Z}^+$ a non-negative integer weight function on the (bidirected) complete digraph on V. Assume that w respects the triangle inequality. We consider the following optimization problem.

INPUT: Let $A \subseteq V \times V$ be a set of arcs such that the digraph $G = (V, A)$ is acyclic, i.e.it has no directed cycles.

OBJECTIVE: Find a set B of arcs such that $H = (V, B)$ is acyclic and for every arc $a = (u, v)$ there is a directed path from u to v in H such that $\sum_{b \in B} w(b)$ is smallest possible.

Thus, we search for a diagram that spans the *demand edges* A. We could show that this problem is NP-complete even if A spans all vertices (no STEINER-points), but is polynomially solvable if $|A|$ is bounded. Our "algorithm" enumerates a huge – but polynomial – set of "condensed" diagrams. Our problem is:

Find a reasonable algorithm.

Problem 2: This problem is derived from the special case of the STEINER diagram problem where the number of STEINER points is bounded.

INPUT: Given a graph $G = (V, E)$, a non-negative integer weight function $w_0 : E \to \mathbb{Z}^+$ on the edges and k integer weight functions on the vertices $w_1, \ldots, w_k : V \to \mathbb{Z}^+$.

OBJECTIVE: Find a set of edges $F \subseteq E$ and k subsets of the vertices $V_1, \ldots, V_k$ such that

$$\forall e = (u, v) \in E \setminus F \exists i \in \{1, \ldots, k\} : \{u, v\} \subseteq V_i$$

and

$$\sum_{e \in F} w_0(e) + \sum_{i=1}^{k} \sum_{v \in V_i} w_i(v)$$

is minimal.

In other words: Cover the edges of E either directly or by both of its endpoints as cheap as possible.

We can show that the problem is solvable in polynomial time using a network flow algorithm if $k \leq 2$ and is $\mathcal{NP}$-complete if k is unbounded.

What is the complexity for $k = 3$ or for some other bound on k?

U. Blasum, W. Hochstättler, P. Oertel: Steiner Diagrams, *Tech Report 99–342.* available from http://www.zpr.uni-koeln.de/~paper/.

Problem 3: These are the missing cases for a cographic version of a theorem of Dirac on large circuits: Let G be an essentially 4-connected graph with n-vertices, m edges, minimum degree at least three and girth $d \in \{5, 6, 7, 8\}$ and assume $2d \leq m - n + 2$. Prove or disprove that under these conditions G has a cocircuit of size at least $2d$.

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Touching Convex Sets

Meir Katchalski

Touching convex sets in the plane: Two sets touch if they intersect and are weakly separated by a straight line (Two unit discs whose intersection is a point are touching).

Conjecture: If $a1, ..., a7$ and $b1, ..., b7$ are convex sets in the plane and every a touches every b then either three of the a's or three of the b's have nonempty intersection.

Representing convex sets in the plane: A set of points A represents a family B of convex sets if when we replace each member of B by the convex hull of the points of A that it contains we obtain a family B' of convex polygons such that any subfamily C of B has nonempty intersection iff. the corresponding subfamily C' of B' has nonempty intersection. (For a family B of closed segments,the set of endpoints of the segments represents B).

Conjecture: There is a constant c such that for any planar family B of n convex set4s such that the intersection of any three of the sets is empty, B can be represented by a set A of not more then $cn^{4/3}$ points. (A result on incidences of points and lines implies that the 4/3 in the exponent cannot be reduced).

The problems are motivated by results with Pach and Liu.

Polynomials Associated with Nowhere-Zero Flows

Martin Kochol

We study relations between nowhere-zero $\mathbb{Z}_k$- and integer-valued flows in graphs and the functions $F_G(k)$ and $I_G(k)$ evaluating the numbers of nowhere-zero $\mathbb{Z}_k$- and k-flows in a graph G, respectively. It is known that $F_G(k)$ is a polynomial for $k > 0$. We show that $I_G(k)$ is also a polynomial and that $2^{m(G)}F_G(k) \geq I_G(k) \geq (m(G)+1)F_G(k)$, where $m(G)$ denotes the rank of the cocycle matroid of G. We give also some formulas estimating the growths of $F_G(k)$ and $I_G(k)$.

Asymmetric Graphs and Coloring Problems

Jaroslav Nešetřil

I have presented three problems. One on the first day, inspired by the presentation of Hubert de Fraysseix, and two problems on the very last day of the workshop.

The first problem is related to the automorphism group of a graph:

1. Critical Oriented Asymmetric Graphs Do Not Exist. We say that a graph G is *asymmetric* if the only automorphism $G \to G$ is the identity. We say that a graph G is *critical asymmetric* if G is asymmetric and every vertex deleted subgraph $G - x$ fails to be asymmetric.

Undirected graphs have many examples of critical asymmetric graphs. By the same token the same is true for the relations as undirected graphs may be identified with symmetric relations.

But suppose that we consider oriented graphs, or simple directed graphs, i.e. directed graphs which do not contain an oriented cycle of length 2, or, in other words, those directed graphs which are orientations of an undirected graph.

Question: Is it true that K_1 is the only critical asymmetric oriented graph?

This is an old problem of mine. I proved it for oriented trees and Wojcik (Comment. Math. Univ. Carol. 1995) proved it for acyclic graphs.

2. Finiteness of Critical 4-chromatic Graphs. The title is of course nonsense. There are infinitely many critical 4-chromatic graphs (critical means here vertex critical for vertex colorings). What are your favourite examples?

Now suppose that we consider only those graphs with all vertices bounded by some number b (i.e. *b-bounded graphs*). If $b \leq 3$ then the situation is easy by the Brooks theorem: K_4 is the only such example. But what if $b = 4$?

We still get infinitely many examples. One such example is if we consider graphs G_l which are snarks with high girth$\geq l$. These snarks were established (despite all the evidence and earlier conjectures in the opposite) by M. Kochol.

The line graph $L(G_l)$ is clearly a 4-chromatic graph and although it need not be critical, any of its critical subgraph has at least $l + 1$ vertices. This clearly implies an infinite family of 4-chromatic 4-bounded critical graphs.

What about triangle free graphs?

There exists a construction of Galai of 4-chromatic 4-regular critical triangle free graphs which have, say, $10k + 5$ vertices for any $k > 1$.

However this leads us to the following problem isolated by Gert Sabidussi and myself (see for example the recent workshop on coloring problems organized in Budapest by Andras Gyarfas):

Question: Is it true that there are only finitely many 4-chromatic 4-regular critical graphs with girth greater than 4?

3. The Pentagon Problem. Again we consider a standard coloring problem:

Let G be any 3-bounded graph (see the previous problem) with a very high girth (say the girth is greater than 10^{10}). Then G has chromatic number ≤ 3.

But the experience tels us much more: the graph G seems to be not only 3-colorable, we seem to have a lot of freedom when assigning colors to the points. Here is a particular questions which should express this phenomenon:

Question: Let G be a 3-bounded graph with high girth (your choice). Is it true that we can color the vertices of G by 5 colors $\{1, 2, ..., 5\}$ so that any edge gets one of the following combinations of colors: $12, 23, 34, 45, 51$ (and none of $13, 35, 52, 24, 41$)?

Alternatively, one can formulate this by the following notion:

A *homomorphism* of $G = (V, E)$ into $G' = (V', E')$, notation $G \to G'$ is any mapping $f : V \to V'$ such that $f(x)f(y) \in E'$ whenever $xy \in E$.

The above question then asks whether $G \to C_5$ for every 3-bounded graph G of sufficiently high girth.

One should be warned that this question has a negative answer with C_5 beeing replaced by C_13 (which follows from a result of Bollobas) and by C_11 which is a yet unpublished result by A.Kostochka, P. Smolíková and the author.

The 3-Sum Problem

Marco Pellegrini

3SUM : Given a set S of n integers, are there three elements $a, b, c \in S$ such that $a + b + c = 0$?

3SUM' : Given three sets of integers A,B and C of total size n are there $a \in A$, $b \in B$ and $c \in C$ such that $a + b = c$?

Problems 3SUM and 3SUM' are easy to state problems whose efficient solution (or lack of it) has implications for many applied problems in computational geometry (see e.g. Gajentaan and M. Overmars 1995). Problems 3SUM and 3SUM' can be solved by an algorithm that takes $O(n^2)$ steps in the standard RAM model of computation (Gajentaan and Overmars 1995, Mirzaian and Arjomandi 1985). A non-trivial $\Omega(n^2)$ lower bound is known only in a very restricted model of computation (Erickson and Seidel 1995). It would be nice either to find an $o(n^2)$ algorithm or to prove a non-trivial lower bound in a stronger model of computation.

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The Rate of Euclidean Superimposed Codes

Miklós Ruszinko

A family of n-dimensional unit norm vectors is a *Euclidean superimposed code*, if the sums of any two distinct at most m-tuples of vectors are separated by a certain minimum Euclidean distance d. Formally, let $\mathcal{C}$ be a finite set of unit norm vectors in $\mathbf{R^n}$ (a *spherical code*). For a subset $\mathcal{A}$ of $\mathcal{C}$ – following the notation of [1] – $f(\mathcal{A})$ stands for the sum of vectors $\mathbf{x} \in \mathcal{A}$, $f(\mathcal{A}) = \sum_{\mathbf{x}\in\mathcal{A}} \mathbf{x}$. Let m and T be positive integers, $m \leq T$, and let d be a real number, $0 < d \leq 1$. The finite set, $\mathcal{C}$, of unit norm vectors in $\mathbf{R^n}$ is an *Euclidean superimposed code* with parameters (n, m, T, d) if $|\mathcal{C}| = T$ and for arbitrary two distinct subsets $\mathcal{A}$ and $\mathcal{B}$ of $\mathcal{C}$ with $0 \leq |\mathcal{A}|, |\mathcal{B}| \leq m$ the Euclidean distance of the vectors $f(\mathcal{A})$ and $f(\mathcal{B})$ is at least d. More precisely, denote by $\mathcal{C}^m$ the set of the sums of all at most m-tuples of vectors of $\mathcal{C}$, i.e.,

$$\mathcal{C}^m = \{f(\mathcal{A}) : \ \mathcal{A} \subseteq \mathcal{C}, \ |\mathcal{A}| \leq m\}.$$

Set

$$d_E(\mathcal{C}^m) = \min_{\substack{\mathcal{A}\neq\mathcal{B} \\ 0\leq|\mathcal{A}|,|\mathcal{B}|\leq m \\ \mathcal{A},\mathcal{B}\subseteq\mathcal{C}}} ||f(\mathcal{A}) - f(\mathcal{B})||,$$

where

$$||\mathbf{x}|| = \left(\sum_{i=1}^{n} x_i^2\right)^{1/2}$$

is the usual Euclidean norm. Equivalently, the set $\mathcal{C}$ of finite unit norm vectors in $\mathbf{R^n}$ is an *Euclidean superimposed code* with parameters (n, m, T, d) if $d_E(\mathcal{C}^m) \geq d$. Given n, m and d, let $T(n, m, d)$ denote the maximum size of an *Euclidean superimposed code,* i.e.,

$$T(n, m, d) = \max\{T : \ \mathcal{C}(n, m, T, d) \neq \emptyset\}.$$

As it was shown in [1], $T(n, m, d)$ increases exponentially in n. Therefore – due to coding theoretic traditions –

$$R(m, d) = \limsup_{n\to\infty} \frac{\log T(n, m, d)}{n}$$

is the exponent of the growth. It is also called the *rate* of the code. Here by log the logarithm of base 2 is denoted.

In 1988 Ericson and Györfi [1] proved that the rate of such a code is between $(\log m)/(4m)$ and $(\log m)/m$ for m large enough. Recently, Füredi and Ruszinkó [2] exponentially improved the upper bound for T. They showed that $R(m, d) \leq \left((\log m)/(2m)\right)(1 + o(1))$. Thus we pose the problem to determine the exact value of $R(m, d)$, or at least to give sharper bounds. Note that the problem has information theoretic motivations.

References

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Point Matching

Bill Steiger

Given a set S with $n = w+b$ points in general position in the plane, w of them being white, and b of them black. The goal is to find a matching $M(S)$ that only matches points of the same color, that has NO crossings between the segments that join matched points, and which leaves the fewest number of points of S unmatched. $M(S)$ is a largest monochromatic, non-crossing matching. Write $|M(S)|$ for the number of points in the matching and $M(n) = max(|M(S)|$, the max taken over all sets S containing n points in general position in the plane. I discussed the recent theorem of Dumitrescu and Steiger showing that

$$\frac{5}{6}n \leq M(n) \leq \frac{155}{156}n.$$

Thus, although there is a set for which every monochromatic, non-crossing matching must exclude at least $(1/156)^{th}$ of the points, every set admits a non-crossing matching that excludes at most $(1/6)^{th}$ of the points.

The proof of the upper bound is probabilistic. Therefore a main open question asks for an explicit construction of a set S of size n for which $|M(S)| < (1-c)n$, $c > 0$. Another asks whether it is possible to achieve the upper bounds in the theorem if the black and white sets are both of size approximately $n/2$; in the probabilistic construction has b is approximately $1.5w$.

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