DIMATIA-DIMACS conference

The Future of Discrete Mathematics

Štiřín, May 19-25, 1997

Contents

1 Foreword

It is a great pleasure to welcome you at the Future of Discrete Mathematics meeting held at Štiřín castle on May 19–25, 1997. This is the first meeting organized by the newly created center DIMATIA. It is organized jointly with DIMACS, which provided not only a necessary expertise but also essentially contributed to the success of the meeting through the Czech – NSF joint grant No. 055/1997. The conference is further supported by our Czech grants GAČR 0194/1996, GAUK 194/1996 and FR MŠMT, and by DIMATIA associated members LaBRI Bordeaux, Pacific Institute of Mathematical Sciences, Sobolev Institute of Mathematics Novosibirsk and Mathematical Institute Budapest. This is our first joint project, and we all believe this is the first out of many. Thus we included in this brochure a brief description of the DIMATIA center, its scope, its goals (and its dreams).

We thank all speakers and participants for joining us for the inauguration of DIMATIA and we wish you pleasant and fruitful time in Štiřín.

Jan Kratochvíl, Jaroslav Nešetřil (DIMATIA)
Ronald L. Graham, Fred S. Roberts (DIMACS)
for the organizing committee

2 Few words about DIMATIA

The **Center for Discrete Mathematics, Theoretical Computer Science and Applications** was established in September 1996 at Charles University in Prague as a joint project of Charles University, Academy of Sciences of the Czech Republic and Institute of Chemical Technology, Prague. The center is open and actively seeks partners from all parts of the world.

1. Overview and goals. The purpose of the center is to foster research in all fields of discrete mathematics and its modern applications and relationship to computer science, operations research and fields as diverse as biology, chemistry and social sciences. Towards this end the center will organize a continuing program of workshops, conferences and research visits both in Prague and other places together with cooperating institutions. The project will concentrate on research topics that are on the forefront of current research in discrete mathematics and theoretical computer science.

2. Activities. The activities of DIMATIA are planned to stimulate research in the most promising areas of discrete mathematics and its applications. The program seeks to involve senior scientists and outstanding young researchers both from Czech Republic and outside. The following are considered to be the most important activities:

- DIMATIA will organize research workshops focused on specific topics to reflect recent trends in discrete mathematics and theoretical computer science (e.g., a workshop on probabilistic method is planned jointly with DIMACS for 1997, a joint DIMACS-DIMATIA workshop on computational geometry is planned for 1998). DIMATIA will take over the organization of problem oriented Midsummer combinatorial workshops, which were organized by the Department of Applied Mathematics in the past 4 years and which have built up a successful tradition. Workshops are organized in an informal way which stresses the working atmosphere. They usually involve 20 - 30 researchers.

- DIMATIA will also organize larger conferences on more general topics for more general audience (the DIMATIA-DIMACS conference "The Future of Discrete Mathematics" is organized in

May 1997, the 5th International Czecho-Slovak Symposium on Combinatorics and Graph Theory is planned for 1998).

- DIMATIA will continue the tradition of publishing KAM Preprint Series, which will be published and distributed on a larger scale.
- DIMATIA will serve as an international center, encouraging wide international contacts and cooperation by supporting short-term visits of researchers on the base of invitations and bilateral cooperation. For this purpose, DIMATIA has created and refurnished new office spaces, including the basic computer equipment.
- DIMATIA will also encourage long-term visits of senior researchers and young scientists. Postdoctoral positions are announced and supported jointly with our partner institutions. (E.g., following a contract with Humboldt Fellowship we are offering one visiting position from September 1997.)

3. Organizational structure and personnel. DIMATIA is a corporate structure founded by three leading Prague research groups working in discrete mathematics, theoretical computer science and their applications. The research team includes 14 permanent members, mostly mathematicians and computer scientists, but some members of the research team are physicists and chemists. All these members are staff members of the founding institutions – Faculty of Mathematics and Physics of the Charles University (V. Janota, M. Klazar, J. Kratochvíl, L. Kučera, M. Loebl, J. Matoušek, J. Nešetřil, P. Pančoška, P. Valtr), Mathematical Institute of the Academy of Sciences of the Czech Republic (I. Havel, J. Krajíček, P. Pudlák, J. Sgall) and Faculty of Chemical Engineering of the Institute of Chemical Technology, Prague (D. Turzík). This number does not include associated researchers, foreign associates and graduate students.

The center has a Director (J. Nešetřil) to provide scientific leadership and day-to-day operation of the center. In that he is assisted by Deputy Director (J. Kratochvíl) and Scientific Secretary (V. Janota). Charles University provides the center with a full time secretary position.

The center has an International Scientific Advisory Board consisting of representatives of DIMATIA participating organizations. DIMATIA (as opposed to e.g. DIMACS) is an open institution, directed to the cooperation on the whole European scale and open to further collaboration. This is a very special feature of the DIMATIA concept and key to its intended activities. Presently the foreign associates are Sobolev Institute of Mathematics, Novosibirsk, LaBRI Institute, Bordeaux, Pacific Institute of Mathematical Sciences and DIMACS, New Jersey. We expect that further institutions will join DIMATIA shortly (among them Bielefeld and Budapest). The open character of DIMATIA is also expressed by its International Advisory Board. The role of this board is to foster the international character of DIMATIA as a whole European center with close links to the top American institutions. Presently, the foreign members of the board include Prof. V. Beresnev (Sobolev Institute of Mathematics), Prof. R.L.Graham (AT&T Bell Labs), Prof. B. Korte (Universität Bonn), Dr. D. Pappenfuss (Alexander von Humboldt Stifftung), Prof. F.S.Roberts (DIMACS), Prof. E. Sopena (University Bordeaux).

4. Budget. As it is usual an institution of this kind is only partially supported by its host institution. Charles University provides rooms and offices (in one of the most attractive historical districts of Prague). It also supports the center by a secretary position. In addition, each of the founding institutions (Charles University, Academy of Sciences of the Czech Republic and Institute of Chemical Technology, Prague) contribute equally to the operation costs ($ 2000 annually). Also the foreign associates support the activities of DIMATIA. The funds for larger activities (scholarships, conferences, workshops, visits) are supported by our grants and cooperation contracts and are budgeted usually for 2-3 year periods.

To obtain more information about DIMATIA, application forms etc., contact us by e-mail or regular mail at the address given below. Also visit our web page at

`http://www.ms.mff.cuni.cz/acad/kam/dimatia/`

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3 Preliminary Program of the Conference

Monday May 19, 1997

14:00 departures of buses from Malostranské nám.

15:00 expected arrival to Štiřín

15:30-16:00 coffee

16:00 M. Rosenfeld: Three Problems in Combinatorial Number Theory

17:00 Robin Thomas: Permanents, Pfaffian Orientations, and Even Directed Circuits

19:00 dinner

Tuesday May 20, 1997

9:00 Lászlo Lovász: Random Walks, Mixing, and Sampling

10:00-10:30 coffee break

10:30 William T. Trotter: Ramsey Theory and Sequences of Random Variables

11:30 Fred S. Roberts: Meaningless Statements

12:30 lunch

14:00 Jiří Matoušek: Upper Bounds in Geometric and Combinatorial Discrepancy

15:00 Günter Rote: Convex 3-polytopes Can Be Realized with Linearly Many Bits per Coordinate

16:00-16:30 coffee break

16:30 Pavel Valtr: Several Recent Results on Geometric Graphs

17:00 Mary Inaba: Geometric Clustering Problem

17:30 Peter Brass: On Lattice-like Extremal Sets for Combinatorial Geometry Problems

19:00 dinner

Wednesday May 21, 1997

9:00 Joel Spencer: Sophisticated Probability

10:00-10:30 coffee break

10:30 Tomasz Luczak: Random Trees and Random Graphs

11:30 Krystyna T. Balińska: An Algorithmic Approach to a Random Process for Graphs with Bounded Degree

12:30 lunch

14:00 Eugene Luks: Algorithmic Applications of the Simple Groups Classification

15:00 Luděk Kučera: Fast Deflection Routing of Long Messages

15:45-16:15 coffee break

16:15 Mike Fellows: The Shortest Vector Problem and the Prospects for Cryptosystems Based on NP-Hard Problems

16:45 Jiří Sgall: On Pairs of Intersecting Families

17:15 Pavel Pudlák : New algorithms for satisfiability

19:00 conference banquet

Thursday May 22, 1997

9:00 Jaroslav Nešetřil: The Structure of Graph Homomorphisms

10:00-10:30 coffee break

10:30 Jan Kratochvíl: List Colorings and Choosability of Graphs

11:15 Alexander Kostochka: On Colour-critical Graphs and Hypergraphs with Few Edges

12:30 lunch

15:00 afternoon excursion

19:00 dinner

Friday May 23, 1997

9:00 Vera Sós: TBA

10:00-10:30 coffee break

10:30 Graham Brightwell: Recent Results in Partially Ordered Sets

11:30 Eric Sopena: Acyclic Improper Colorings

12:30 lunch

14:00 Jan Krajíček: Proof Complexity and Some Related Algebraico - combinatorial Problems

15:00 pm Arvind Gupta: Monadic Second Order Logic and Complement Problems

16:00-16:30 coffee break

16:30 Josep Diaz: Approximation of #P Problems in RNC

17:00 Gyorgy Elekés: Metric Combinatorics and Combinatorial Algebra

17:30 Gábor Simonyi: Information Theory in Combinatorics

19:00 dinner

Saturday May 24, 1997

9:00 Walter Deuber: TBA

10:00 András Frank: Algorithms for Increasing the Connectivity of Graphs

11:00-11:30 coffee break

11:30 Wolfgang Mader: An Extremal Problem for Subdivisions of K_5 and Topological Subgraphs in Graphs of Large Girth

14:00 lunch

15:30 Martin Klazar: Extremal Problems for Colored Trees

16:00 Miklós Simonovits: General Methods to Prove Exact Results in Extremal Graph Theory

17:00 Peter Mihók: The Structure of Hereditary Properties of Graphs and Minimal Reducible Bounds

17:30 William F. Smyth: Periodicity in Strings

19:00 dinner

Sunday May 25, 1997

9:00 Jerry Griggs: The Distribution of Subset Sums in R^m and in Finite Abelian Groups

10:00-10:30 coffee break

10:30 Bernhard Korte: TBA

11:30 lunch

13:00 departure of buses for Prague

4 Preliminary List of Participants

Robert Babilon, Praha
Krystyna T. Balińska, Poznan, Poland
Josef Balogh, Szeged, Hungary
Peter Brass, Berlin, Germany
Graham Brightwell, London, U.K.
Walter Deuber, Bielefeld, Germany
Josep Diaz, Barcelona, Spain
Gyorgy Elekés, Hungary
Sándor P. Fekete, Koeln, Germany
Mike Fellows, Victoria, Canada
András Frank, Budapest, Hungary
Hubert de Fraysseix, Paris, France
Ronald L. Graham, DIMACS, U.S.A.
Jerry Griggs, Columbia, U.S.A.
Arvind Gupta, Burnaby, Canada
Penny Haxell, Waterloo, Canada
Petr Hliněný, Praha
Mary Inaba, Tokyo, Japan
Martin Klazar, Praha
Alexander Kostochka, Novosibirsk, Russia
Bernhard Korte, Bonn, Germany
Jan Krajíček, Praha
Jan Kratochvíl, Praha
Luděk Kučera, Praha
Lászlo Lovász, New Haven, U.S.A.
Tomasz Luczak, Poznan, Poland
Eugene Luks, Eugene, U.S.A.
Wolfgang Mader, Hannover, Germany
Jiří Matoušek, Praha
František Matúš, Bielefeld, Germany
Patrice Ossona de Mendez, Paris, France
Peter Mihók, Kosice, Slovakia
Jaroslav Nešetřil, Praha
Petr Pančoška, Praha
Gábor Pete, Szeged, Hungary

Victor Pollara, Braunschweig, Germany
Pavel Pudlák, Praha
Fred S. Roberts, Rutgers, U.S.A.
M. Rosenfeld, Seattle, U.S.A.
Günter Rote, Graz, Austria
Jiří Sgall, Praha
Miklós Simonovits, Budapest, Hungary
Gábor Simonyi, Budapest, Hungary
William F. Smyth, Hamilton, Canada
Eric Sopena, Bordeaux, France
Vera Sós, Budapest, Hungary
Joel Spencer, New York, U.S.A.
Claude Tardiff, Bielefeld, Germany
Robin Thomas, Atlanta, U.S.A.
William T. Trotter, Tempe, U.S.A.
Pavel Valtr, Praha
Paola Vocca, Rome, Italy
Shiyu Zhou, U.S.A.

5 Abstracts

Krystyna T. Balińska (Poznań): An algorithmic approach to a random process for graphs with bounded degree
(joint work with Louis V. Quintas)

Graphs with bounded degree are simple graphs having no vertex of degree greater than a given positive integer f. These are called (*f- graphs*) ([J.W. Kennedy and L.V. Quintas, Probability models for random f-graphs, *Combinatorial Mathematics* (New York, 1985), *Ann. N.Y. Acad. Sci.,* **555** (1989) 248-261]). The probability model for f-graphs on n vertices that generalizes the random graph process (i.e. with $f = n-1$) is known as the random f-graph process (RfGP) of order n ([K.T. Balińska and L.V. Quintas, The random f-graph process, *Quo Vadis, Graph Theory?* (Fairbanks, Alaska, 1990) *Ann. Discrete Math.,* **55** (1993) 333-340]).

The main difficulty encountered in the study of the RfGP is that choosing edges for a random f-graph from the set of available edges is dependent on the process.

A general method called *RGA* (Random Graph Algorithms) ([K.T. Balińska, Algorithms for random graphs with bounded degree, *Politechnika Poznańska, Rozprawy*, **314**, Poznań (1996)]) for obtaining an insight into deterministic and probabilistic problems for the RfGP is presented. The method is based on the *transition digraph* $D(n, f)$ of the process and consists of the following classes of algorithms.

Class $\mathcal{A}$: Exact algorithms generating all nodes of $D(n, f)$ and calculating corresponding state probabilities. The distribution of any random variable on f-graphs in the RfGP can be obtained and enumeration problems for f-graphs and $D(n, f)$ can be solved ([K.T. Balińska and L.V. Quintas, Degrees in a digraph whose nodes are graphs, (Keszthely, 1993) *Discrete Math.* **150** (1996) 17-29]).

The main algorithm, IMAGEf, uses the BFS (Breadth First Search) method for generating and visiting nodes of $D(n, f)$. Its time complexity at step t, corresponding to f-graphs of size t of the RfGP, depends on the number of nodes of $D(n, f)$ and on the complexity of the algorithm for testing the isomorphism of two graphs.

In general, this algorithm is exponential in n, however for the case $f = 2$ there exists a $O(n^3)$ polynomial algorithm. IMAGEf has also been extended for constructing the underlying (undirected) graph of $D(n, f)$ and calculating some of its properties (e.g. its diameter).

Class $\mathcal{B}$: Simulation algorithms for generating and identifying graphs up to large orders to solve probabilistic problems for the RfGP. Algorithms of this class are polynomial in n. This method can be regarded as using a sample of random walks along $D(n, f)$ from the initial state to a terminal state of the RfGP. These have been combined with approximation functions for studying some properties of the RfGP as functions of n and f.

Peter Brass (FU Berlin): On lattice-like extremal sets for combinatorial geometry problems

Only in very few cases the structure of the extremal sets for Erdős-like combinatorial geometry problems is known, but many of Erdős' conjectures are motivated by the belief that lattice sections and related sets have many extremal properties. A structural approach to these problems may be preferable to direct counting, since the importance of lattice-constructions shows the connection to number-theoretic problems which are difficult to handle by purely combinatorial methods (e.g. in the many-unit-distances problem).

In this talk we will give a survey of lattice-like constructions and present some new results to their extremality, especially connected with the maximum number of unit distances, of translates of a given pattern, and of point-line-incidences.

Josep Diaz (Barcelona): Approximation of #P problems in RNC

(joint work with P. Spirakis and M.J. Serna)

Using genetic systems we analize the rapid mixing of Markov chains for approximate counting and almost uniform generation of matching and perfect matching problems.

G. Elekes (Budapest): Metric Combinatorics and Combinatorial Algebra
(joint work with L. Ronyai)

We study the structure of polynomials (and rational functions) of two real variables which take few distinct values on large (finite) Cartesian products. As an application, a problem of G. Purdy is solved on finite subsets of the plane which determine few distinct distances.

Sándor P. Fekete (Köln): A new exact algorithm for general orthogonal d-dimensional knapsack problems and a new lower bound for bin packing problems
(joint work with Jörg Schepers)

The *d-dimensional orthogonal knapsack problem* (OKP) has a wide range of practical applications, including packing, cutting and scheduling. We present a new approach to this problem, using a graph-theoretical characterization of feasible packings. This characterization allows us to deal with whole classes of packings that share a certain combinatorial structure, instead of single packings. Combining the use of this structure with other heuristics, we develop a two-level tree search algorithm for finding exact solutions for the d-dimensional OKP. Computational results are reported, including optimal solutions for all two–dimensional test problems from recent literature.

We also describe a general method to obtain good lower bound for packing problems. In particular, we establish a good new lower bound for bin packing problems.

Michael R. Fellows (Victoria): The Shortest Vector Problem and the Prospects for Cryptosystems Based on NP-Hard Problems
(joint work with Rod Downey, Neal Koblitz, Alex Vardy and Geoff Whittle)

In 1996 Ajtai showed that hard instances of the SHORTEST VECTOR problem concerning integer lattices can be efficiently generated.

This is a major breakthrough in the fundamental problem of key generation in cryptography, and even in our understanding of the nature of computational intractability — but wait: is SHORTEST VECTOR a hard problem? This question has remained open for nearly 20 years. Problems about integer lattices seem to be closely related to problems about linear codes. In particular, the SHORTEST VECTOR problem is formally very close to the MINIMUM DISTANCE problem about linear codes, also conjectured for more than 20 years to be NP-hard. Using deep coding theory arguments, Vardy has recently shown that the MINIMUM DISTANCE problem is NP-hard. The talk will describe a fresh attack on this collection of fundamental problems using the techniques of parameterized complexity. Among the results: MAXIMUM LIKELIHOOD DECODING and THETA SERIES FOR INTEGER LATTICES are hard for the parameterized complexity class W[1]. The talk will describe some of the basics of the parameterized complexity framework, some of the broader issues concerning prospects for cryptosystems based on NP-hard problems, and address some general remarks to the future of continued fruitful collaborations between algebra and combinatorics in work such as this.

András Frank (Budapest): Algorithms for increasing the connectivity of digraphs

In an earlier paper, co-authored by T. Jordán, we proved a min-max formula for the necessary number of new edges to be added to a digraph in order to increase its connectivity. The original proof is not algorithmic and our present goal is to exhibit a constructive proof that gives rise to a combinatorial polynomial time algorithm. The same method yields a simple algorithmic proof of Győri's theorem on intervals. We also describe a two-phase greedy algorithm to increase the rooted connectivity of a digraph at minimum cost.

Jerrold R. Griggs (University of South Carolina): The Distribution of Subset Sums in R^m and in Abelian Groups

We consider two separate problem areas that concern the distribution of the 2^n subset sums $\sum_{i\in I} a_i$, over all index sets I. The first problem concerns maximizing the number of subset sums equal to the same target point t, over all choices of n vectors $a_1, \ldots, a_n \in \mathbf{R}^m$ and all $t \in \mathbf{R}^m$, where some additional condition on the a_i's must be satisfied. Such situations arise naturally in connection with a certain model of database security: The goal is to maximize the number of queries of a statistical database that are answerable without compromising the database. Well-known problems that arise in this way include the Littlewood-Offord problem and the Erdős-Moser problem. We shall focus on the problem of maximizing the concentration of subset sums when each m of the vectors a_i form a basis.

The second subset sum project is joint work with Bjorn Poonen on spanning sets in finite abelian groups G. Let $a_1, \ldots, a_n$ be distinct nonzero elements of G. We prove that if $n > |G|/2$, then the set of nontrivial subset sums of the a_i's includes every element of G. Our main result is that this remains true, when n is reduced to $|G|/2$, for groups G of even order > 8, where it is best-possible. For groups G of odd order, the minimum required size n remains open. Dias da Silva and Hamidoune have solved the case that $G = \mathbf{Z}_p$, p prime, settling an old problem of Erdős and Heilbronn. In that case, the answer is $\lfloor 2(p-2)^{1/2} \rfloor$.

Arvind Gupta (Simon Fraser University): Monadic Second Order Logic and Complement Problems
(joint work with Damon Kaller and Tom Shermer)

Graphs of bounded tree-width have an underlying tree-like structure that often allows tree-based algorithms for problems to also work on this larger class. This class is the same as the partial k-trees for fixed k and includes such well-studied graph families as outerplanar graphs, series-parallel graphs and Halin graphs. Since 1985 a concerted effort by many researchers has lead to a logical characterization of those graph properties that have linear time algorithms on this class; many of these properties are $\mathcal{NP}$-complete for general graphs. One of the most common characterization is the Counting Monadic Second Order Logic (CMS). Indeed many well known problems can be expressed in CMS or one of its variants.

Succinctly, the theory states that any problem expressible in CMS will have a linear time solution on bounded tree-width graphs. That is, for some problem, let Π be the set of yes-instances of the problem and suppose Π is CMS-definable. Then for any constant k and G_k the class of partial k-trees, the intersection $\Pi \cap G_k$ can be recognized by a *tree automaton.* The tree-automaton is similar to a finite automaton except it works up a tree (from leaves to root) with state changes dependent on child labels and child states. The resulting algorithm then takes a dynamic-programming approach similar to the solution of the problem on a tree.

Despite the elegance of the CMS formalism, it is not always clear which naturally-stated graph problems are CMS-definable. This is an important issue because the ability of CMS logic to define a problem is intimately related to the power of dynamic programming in solving the problem. This talk discusses the role of *complement* problems in elucidating the structure of CMS-definable graph problems. Again consider the set Π of yes-instances and define $\overline{\Pi}$ to consist of the graph-theoretic complements of all graphs in Π. We are interested in answering the following question: *If* Π *is CMS-definable, when is* $\overline{\Pi}$ *CMS-definable?* We show that in some cases $\overline{\Pi}$ is CMS-definable, but in other cases $\overline{\Pi} \cap G_k$ is CMS-definable but not $\overline{\Pi}$. Furthermore, using a pumping lemma we show that in some cases not even $\overline{\Pi} \cap G_k$ is CMS-definable.

Mary Inaba (Tokyo): Geometric clustering problem
(joint work with Hiroshi Imai)

Clustering is a very important problem in statistics, image understanding, computer graphics, etc. There are two types of clustering problem, one is clustering on a weighted graph (or a (dis)similarity matrix), and the other is a geometric one. Geometric clustering has nicer structures than the graph version due to constraints induced by geometry.

This talk first points out combinatorial optimization aspects of the geometric clustering problem, and summarizes recent results by our group on the k-clustering problem for a set S of n points $p_i = (x_i)$ $(i = 1, \ldots, n)$ in the d-dimensional space with variance-based errors as clustering criteria. A main problem is to find a k-clustering of S

into S_j $(j = 1, \ldots, k)$ minimizing

$$\sum_{j=1}^{k} \sum_{p_i \in S_j} \|x_i - \bar{x}(S_j)\|^2$$

where $\|\cdot\|$ is the L_2 norm and $\bar{x}(S_j)$ is the centroid of points in S_j, i.e., $\frac{1}{|S_j|}\sum_{p_i \in S_j} x_i$.

Next, relations of those results with the existing local improvement technique, called k-means, are described, together with results of computational experiments and its application to the color quantization problem. We also relate it to some mathematical programming problem called geographic optimization problem by Iri, Murota, Ohya. We further consider an improvement of the k-means algorithm via overrelaxation, motivated from the viewpoint of mathematical programming.

Martin Klazar (Praha): Extremal problems for colored trees

We discuss a generalization of the extremal problems connected with *Davenport-Schinzel sequences* to trees. For example, in the case of has no immediate repetition of a symbol. For this particular problem the maximu What is the maximum number of vertices of a tree that can be vertex colored by at most n colors so that (1*) no subgraph is a subdivision of the 6-vertex path with color pattern *aabbab*, (2*) the coloring is proper and (3*) no subgraph is a subdivision of the 4-vertex star with the (unique) proper coloring pattern. Condition (3*), which is void for paths (i.e. DS sequences), We do not know if for the tree generalization the maximum size remains $O(n)$.

Alexander Kostochka (Novosibirsk): On colour-critical graphs and hypergraphs with few edges
(joint work with M. Stiebitz)

The aim of the talk is to describe the currently known bounds for $f(r, k, n)$—the minimum possible number of edges in r-uniform k-critical hypergraphs on n vertices. In particular, it is proved that

$f(r,k,n) = (k+o(k))n$ for each $r > 2$ and new lower bounds for $f(2,k,n)$ improving recent results by M. Krivelevich are given.

Jan Krajíček (Oxford and Praha): Proof complexity and some related algebraico - combinatorial problems

We give some assorted examples of results (recent as well as not so recent) on complexity of proof systems, and of open problems they motivate.

A proof system (tacitly for propositional logic here) is a non - deterministic algorithm such that the property of being an accepting computation is polynomial - time, and that accepts exactly the set of propositional tautologies in some complete language (or some other closely related coNP-complete set like polynomial systems over a finite field unsolvable in that field).

The statement that there is no polynomial - time proof system is just the statement that NP $\neq$ coNP. A research in proof complexity aims currently at proving lower bounds for stronger and stronger natural proof systems. For example, it is open whether every tautology has a polynomial - size proof in the usual text - book calculus based on modus ponens. In fact, no non - trivial lower bounds are known for this calculus at all.

The topics we plan to discuss include: provability and unprovability (by short proofs) of counting principles in propositional calculi in various languages working with bounded depth formulas only, related algebraic proof systems (systems manipulating polynomials), and various open problems of algebraico - combinatorial nature motivated by these results.

Jan Kratochvíl (Praha): List colorings and choosability of graphs
(joint work with Zs. Tuza and M. Voigt)

Given a graph G and a system $L : V(G) \to B$ of lists $L(u)$ of admissible colors for particular vertices, we say that (G, L) is *list colorable* if there is a proper coloring choosing for every vertex u a color from

$L(u)$. A graph is called *k-choosable* if it is list colorable for every system of lists of sizes $\geq k$.

It is well known that determining the choosability of a given graph is a PSPACE-complete problem, and deciding list colorability is NP-complete. We will investigate the complexity of restricted variants of list colorings and choosability, also for a more general notion of (p, q, r)-choosability.

In particular, we will consider list colorings nearly disjoint lists adjacent vertices having nearly disjoint lists and we will prove bounds for this variant of choosability, which are tight upto a multiplicative constant.

Luděk Kučera (Lyon and Praha): Fast deflection routing of long messages

In order to obtain lowest possible latency, routing algorithms should try to avoid message waiting for resources (network links) blocked by other messages and multiplexing of more messages over one physical channel. This requirement becomes an imperative in the case of long messages. The only type of protocols able to guarantee this type of routing under heavy load are algorithms based on deflection (also called nonminimal adaptive or hot potato) routing.

The paper deals with problems connected with the use of deflection algorithms. In contrast the case of nonadaptive or partially adaptive routing, it is very infrequent that a deflection routing becomes deadlocked and similarly livelock is not a serious problem. On the other hand, there are two other phenomena, called together a *deflection jam*, that limit throughput of deflection algorithms used to route long messages. They have been observed for many deflection heuristics, interconnection network topologies and both virtual cut-through and wormhole routing.

Under a light load orthodox deflection algorithms route messages in an almost optimal way, but if the load reaches a certain threshold value, which is still relatively low, it might happen that the communication suddenly breaks down almost completely after thousands or even millions of steps of smooth information exchange. A low load deflection jam need not to lock messages for ever, but the network throughput decreases to a very small fraction of the original value and the saturation tends to be persistent.

An analysis presented in the paper shows how to modify naive deflection algorithm to preserve their very low latency while extending the range of a sustainable load well beyond the above threshold. The throughput of such algorithms is usually limited by a high load deflection jam, which occurs when the subnetwork of free channels becomes highly disconnected and represent the ultimate bound for efficient deflection-type routing.

László Lovász (Yale and Budapest): Random walks, mixing, and sampling
(joint work with Peter Winkler)

Random walk techniques have many applications to the problem of sampling, i.e., to the problem of generating a random element from a given distribution. Applications of such methods include simulation in statistical mechanics, combinatorial enumeration, integration, volume computation, generation of contingency tables, and optimization over convex bodies.

The basis of these methods is the fact that if we do random walk on a (non-bipartite, connected) graph, the distribution of the current node after t steps will tend to a limit distribution (as t tends to infinity) in which the probabilities of nodes are proportional to the degrees. Crucial in these applications is the question of how long we have to walk before getting close to the limit distribution.

It is not necessary that every walk stops after the same number of steps. In fact, making the stopping rule dependent on the history, any prescribed distribution can be achieved exactly. There exist optimal stopping rules (minimizing the expected number of steps before stopping) of a rather simple kind. The time used by an optimal stopping rule is a relatively well-behaved and easily computable value. While an optimal rule to the stationary distribution may be impossible to implement in practice, there is a very simple, easily implementable rule that gives a good approximation and is only a (decent) constant factor slower than the optimal rule.

Various algorithmic situations suggest a number of other “mixing measures”. One of these is the ”reset time”, which may be defined as the minimum mean length of a stopping rule that, when started from the stationary distribution, generates a node that has the same distribution but is independent from the starting node. Another one is the "forget time": this is defined as the minimum mean length of any stopping rule that yields the same distribution, independently from the starting node.

A surprising (and not quite easy) fact is that the forget time of a chain is equal to the reset time of the time-reversed chain. In particular, the forget time and the reset time of a time-reversible chain are equal. There are a number of unexpected further identities and "approximate identities" between mixing measures.

Tomasz Luczak (Poznan): Random trees and random graphs

We describe some simple self-similarity arguments which can be used to study random trees. In particular, we find the asymptotic number of trees of large height, which immediately leads to a precise estimate of the diameter of the random graph in the subcritical phase.

Eugene Luks (Eugene): Algorithmic Applications of the Simple Groups Classification

The Classification of Finite Simple Groups stands as a monumental mathematical achievement, its implications far-reaching, and occasionally surprising. We survey several applications to the computational complexity of problems that have seemingly elementary formulations. In some instances, the Classification is currently needed just to attain polynomial time, in others to establish parallelizability. There are also significant consequences in low-level complexity, facilitating speedups of an order-of-magnitude or more for much of the group-theoretic machinery. This includes the very basic, and well-worked, problem of testing membership in a permutation group. Particularly useful tools stem from Classification-dependent observations of Cameron about the structure of primitive permutation groups; such groups frequently occur as the base case of combinatorial divide-and-conquer algorithms. There is even an impact on the best result for testing graph-isomorphism as a result of a sharpening of Babai's combinatorial bounds on the orders of primitive groups. It is noteworthy that the Classification is most often invoked via succinct, uncomplicated corollaries. However, some algorithms demand a case-by-case analysis of the classes of simple groups.

Wolfgang Mader (Hannover): An extremal problem for subdivisions of K_5 and topological subgraphs in graphs of large girth

We prove that every graph with $n \geq 3$ vertices and $3n-5$ edges contains a subdivision of K_5. This is best possible for every n and had been conjectured by Dirac in 1964. We shall also characterize the graphs with $3n-6$ edges without a subdivision of K_5. A key result in the proof is that every graph with $n \geq 6$ vertices, $2n-5$ edges, and girth at least 5, but the Petersen graph, contains a subdivision of $K_5 - e$ (e an edge). So we will study also the interrelation between the existence of subdivisions and girth.

Jiří Matoušek (Praha): Upper bounds in geometric and combinatorial discrepancy

Geometric discrepancy theory studies the question "How uniformly can an n-point set be distributed in a given region of space?". Combinatorial discrepancy is concerned with 2-coloring points of a set system in such a way that the colors on each set are as balanced as possible. We discuss a technique (the so-called entropy method) that leads to asymptotically optimal bounds in several problems from both areas. We also point out some open problems where new methods seem to be called for.

P. Mihók (Košice): Additive and monotone properties of graphs are uniquely factorizable into irreducible factors

Let us denote by $\mathcal{I}$ the class of all mutually nonisomorphic finite simple graphs. If $\emptyset \neq \mathcal{P} \subset \mathcal{I}$ be a nonempty isomorphism-closed subset of $\mathcal{I}$, then $\mathcal{P}$ denote the property of graphs. A property $\mathcal{P}$ is said to be *monotone* whenever $H \subseteq G$ and $G \in \mathcal{P}$ implies $H \in$

$\mathcal{P}$ and *additive* if it is closed under the disjoint union of graphs. (For more details see [M.Borowiecki, I.Broere, M.Frick, P.Mihók and G.Semanisin: *Survey of hereditary properties of graphs* Discussiones Math. - Graph Theory, **17**, 1997, to appear] and [T.R.Jensen and B.Toft: *Graph Colouring Problems*, Wiley - Interscience Publications, New York, 1995]).

Let $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n$ be properties of graphs, a vertex $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n)$ - partition of a graph G is a partition $(V_1, V_2, \ldots, V_n)$ of the vertex set $V(G)$ such that the induced subgraph $G[V_i] \in \mathcal{P}_i$ for $i = 1, 2, \ldots, n$.

Let us denote by $U(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n)$ the set of all graphs which posses unique vertex $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n)$ - partition and $\mathcal{R} = \mathcal{P}_1.\mathcal{P}_2.\ldots.\mathcal{P}_n$ the set of all vertex $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n)$ - partitionable graphs. It is easy to see, that if $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n$ are additive and monotone properties of graphs, then $\mathcal{R} = \mathcal{P}_1.\mathcal{P}_2.\ldots.\mathcal{P}_n$ is additive and monotone,too. The property $\mathcal{R} = \mathcal{P}_1.\mathcal{P}_2.\ldots.\mathcal{P}_n$, $n \geq 2$ is said to be reducible, i.e. $\mathcal{P}$ is reducible if there exist $\mathcal{P}_1, \mathcal{P}_2$ such that $\mathcal{R} = \mathcal{P}_1.\mathcal{P}_2$ and irreducible otherwise.

A graph G is said to be $\mathcal{P}$ -maximal if $G \in \mathcal{P}$ but for any edge e of its complement $G + e \notin \mathcal{P}$.

To characterize the reducible properties of graphs the following problems seems to be natural.

Problem 1 UNIQUE FACTORIZATION PROBLEM [T.R.Jensen and B.Toft: *Graph Colouring Problems*, Wiley - Interscience Publications, New York, 1995]
Let $\mathcal{R}$ be a reducible property of graphs and $\mathcal{R} = \mathcal{P}_1.\mathcal{P}_2.\ldots.\mathcal{P}_n$, $n \geq 2$ be a factorization of $\mathcal{R}$ into irreducible factors. Is the factorization unique (apart from the order of factors)?

This basic question is related to the following problems:

Let $\mathcal{R} = \mathcal{P}_1.\mathcal{P}_2.\ldots.\mathcal{P}_n$, $n \geq 2$ be a factorization of the property R into irreducible factors. Do the following statements hold?

Problem 2 THE STRUCTURE OF MAXIMAL GRAPHS
If the property $\mathcal{R}$ is reducible, then all $\mathcal{R}$-maximal graphs are joins of at least n graphs. Is this necessary condition also sufficient?

Problem 3 THE EXISTENCE OF UNIQUELY PARTITIONABLE GRAPHS
$U(\mathcal{P}_1.\mathcal{P}_2.\ldots.\mathcal{P}_n) \neq \emptyset$, moreover $U(\mathcal{P}.\mathcal{P}.\ldots.\mathcal{P}) \neq \emptyset$ if and only if $\mathcal{P}$ is irreducible.

We shall present an affirmative solution of all of above presented problems.

Jaroslav Nešetřil (Praha): The Structure of Graph Homomorphisms

Graph homomorphisms appear naturally in the context of various combinatorial problems, especially related to coloring and construction (and reconstruction) algebraical problems. On the other hand, homomorphisms yield a "minimal calculus" for graphs, thus providing a natural setting for seemingly isolated problems.

In the talk, we want to list some of the recent results and open problems in this area related to the structure of color classes, universality and duality theorems.

Petr Pančoška (Chicago and Praha): Spectra, graphs and proteins. Towards understanding of protein folding

(joint work with V. Janota and J. Nešetřil)

With proper transformation of protein three-dimensional structures known from x-ray crystallography, various features of protein conformation can be encoded into mathematical objects (vectors, matrices, graphs, strings, fractals and others). Examples of novel graph representations of segment topology of globular proteins and polypeptide backbone distance matrix will be shown together with practical applications in analysis of protein optical spectra and protein fold classification. Problems related to the analysis of mathematical properties of the resulting graphs will be formulated. When the chemical composition of the sets of edges of graph representation of the backbone distance matrix was analyzed using the basic theorems of coding theory, a simple method for identification of structurally correlated parts of protein primary structure was formulated. Examples of structural interpretation of this result will be given for a representative set of nohomologous protein structures. Example of application for analysis of protein function will be demonstrated using the database of somatic mutations of protein p53 in human tumors.

Pavel Pudlák : New algorithms for satisfiability
(joint work with Ramamohan Paturiand Francis Zane)

We present and analyze two simple algorithms for finding satisfying assignments of k-CNFs (Boolean formulae in conjunctive normal form with at most k literals per clause). The first is a randomized algorithm which, with probability approaching 1, finds a satisfying assignment of a satisfiable k-CNF formula F in time $O(n^2|F|2^{n-n/k})$. The second algorithm is deterministic, and its running time approaches $2^{n-n/2k}$ for large n and k. The randomized algorithm is the best known algorithm for $k > 3$; the deterministic algorithm is the best known deterministic algorithm for $k > 4$. The key idea used in these upper and lower bounds is what we call the *Satisfiability Coding Lemma*. This basic lemma shows how to encode satisfying solutions of a k-CNF succinctly.

Fred S. Roberts (Rutgers): Meaningless Statements

A statement involving scales of measurement is called meaningless if its truth or falsity can depend upon the particular versions of scales which are used in the statement. This is an idea which is closely related to the theme of invariance, which has played such a central role in the history of the mathematical sciences, for instance in the Erlanger Program of Felix Klein, in geometry, in physics, and so on. We develop the theory of meaningful and meaningless statements. This theory has had a wide variety of applications and we concentrate here on examples relevant to fundamental ideas in discrete mathematics: limitations on conclusions in combinatorial optimization, and in particular in graph coloring and scheduling; limitations on conclusions in linear programming and game theory; and limitations on the possible consensus functions which are appropriate in group decisionmaking.

Moshe Rosenfeld (Tacoma): 3 problems in combiatorial number theory

Let n be a positive integer. The Factor-Difference-Set, $D(n)$ of n, is defined by: $D(n) = \{|a-b| : ab = n\}$. We conjecture that for every integer k, there are k integers $n_1 < n_2 < \ldots < n_k$ such that

$|\bigcap_{i=1}^{k} D(n_i)| \geq k$. In the first part I'll trace the origin of this problem and discuss some related results. This work was done with Paul Erdős.

Let (A, B) be a partition of $\{1, 2, \ldots, 2n\}$ into two disjoint n-sets. We say that A and B are gracefully matched if one can sequence A and B so that all differences $\pm(a_i - b_i) \bmod (2n+1)$ will be distinct. We conjecture that for $n \geq 4$ all partitions of $\{1, 2, \ldots, 2n\}$ admit a graceful matching. We use a theorem of Weil to prove that if $A = Q(q), B = N(q)$ where $q = p^r$, r an odd prime, $Q(q)$ the quadratic residues in $GF(q)$, $N(q)$ the non-residues in $GF(q)$ can be gracefully matched. Applications and related open problems will be discussed. This is joint work with J. Kratochvíl, J, Nešetřil and Tamas Szonyi.

Consider an $n \times n$ square chess board. Select randomly a set of squares and "cover" it. We say that the covered set A can be doubled if there is a translation of A that will occupy only uncovered squares. Suppose you start with a set A consisting of a single square, double A then continue doubling the new set. How many times can we repeat the doubling before we run out of space? This problem is related to the Shannon Capacity of cycles, and special subspaces of $GF^n(q)$. We discuss this problem and a nice uniqueness proof communicated to me by Rafael Robinson.

Günter Rote (Graz): Convex 3-polytopes can be realized with linearly many bits per coordinate

By a theorem of Ernst Steinitz (1922), every 3-connected planar graph with n vertices can be realized as a convex polytope. We can do this with integer vertex coordinates between $-C$ and C, where $C = 43^n$.

Onn and Sturmfels (1994) have shown this for $C = 2^{169n^3}$.

Das and Goodrich (1995) have given an algorithm that works for triangulated 3-polytopes and selects rational vertex-coordinates with polynomially many bits for the numerator and the denominator, i.e., $C = 2^{n^{O(1)}}$. However, their algorithm takes only a linear number of steps (including arithmetic operations on numbers of size at most C).

The approach of Das and Goodrich is more closely related to the original proof of Steinitz, who builds up the polytope from simpler polytopes by making local changes. (Steinitz's original proof yields a doubly-exponential bound for C.)

We follow the approach of Onn and Sturmfels, who exploit the connection between 3-polytopes and *stresses* on the edges of a plane projection of a polytope, which goes back to Maxwell (1864). We fix a triangular face as exterior face, interpret all remaining edges as springs with elasticity $w = 1$, and compute the equilibrium of this mechanical system. This amounts to solving a system of linear equations, and leads to vertex coordinates of size at most 42^n. The same procedure was also used by Tutte (1960) for obtaining a nice drawing of a planar graph.

The next step calculates the polytope which projects onto this drawing. In contrast to Onn and Sturmfels, who find such a polytope by solving a system of inequalities for the z-coordinates, we do this directly by exploiting the geometric significance of the edge weights.

Finally, the case that the graph contains no triangles is not treated by realizing the dual graph and then going to the polar polytope. Instead, we carefully select the shape of a 4-sided or 5-sided face to ensure that an equilibrium on the interior vertices can be extended to the whole graph.

The question whether the bound C can be improved to a polynomial is open.

Jiří Sgall (Praha): On Pairs of Intersecting Families
(joint work with Pavel Pudlák)

We study pairs of families $\mathcal{A}, \mathcal{B} \subseteq 2^{\{1...r\}}$ such that $|A \cap B| \in L$ for any $A \in \mathcal{A}$, $B \in \mathcal{B}$. We are interested in the maximal product $|\mathcal{A}| \cdot |\mathcal{B}|$, given r and L.

We give tight bounds for $L = \{1 \ldots k\}$ and partial results for $L = \{k, k+1\}$.

This latter case is motivated by a conjecture about the relation between communication complexity and rank of a matrix.

Miklós Simonovits (Budpest): General methods to prove exact results in extremal graph theory

In the talk I will present a few general methods to prove **exact** results in extremal graph theory.

Given a family $\mathcal{L}$ of – so called – forbidden graphs, denote by $\text{ext}(n, \mathcal{L})$ the maximum number of edges a graph G_n of order n can have under the conditions that G_n contains no forbidden subgraphs $L \in \mathcal{L}$. Generally we wish to determine $\text{ext}(n, \mathcal{L})$, or at least get good asymptotics as $n \to \infty$. Besides, it is important also to determine the (asymptotical) structure of the graphs attaining the maximum.

Mostly we can get good asymptotics but not exact results in such cases. Here we shall regard cases where for $n > n_0(\mathcal{L})$ we have the exact values of the function $\text{ext}(n, \mathcal{L})$.

Some of the results presented here are joint with J. Griggs and Thomas Rubin.

Gábor Simonyi (Budapest): Information theory in combinatorics

Information theoretic concepts and methods helped to solve some combinatorial problems during the last decade. Two outstanding examples are Kahn and Kim's sorting algorithm in [J. Kahn and J. H. Kim, Entropy and sorting, *Proc. 24th Annual ACM Symposium on the Theory of Computing*, 178–187] and the solution of Rényi's qualitative 2-independence problem by Gargano, Körner, and Vaccaro, cf. [L. Gargano, J. Körner, U. Vaccaro, Capacities: from information theory to extremal set theory, *J. Comb. Theory, ser A*, **61** (1992), 173–192]. The key role in both of the above cases is played by a graph functional that also depends on a probability distribution on the vertex set of the graph at hand. In the first case this functional is Körner's notion of graph entropy, a probabilistic refinement of the chromatic number, cf. [G. Simonyi, Graph entropy: a survey, *in* "Combinatorial Optimization", DIMACS Series in Discrete Math. and Theoretical Comp. Sci., Vol. 20 (W Cook, L. Lovász, and P. D. Seymour eds.), 399–441]. In the second case it is an analogous notion for the clique number which has very close connections to the Shannon capacity of graphs. The talk is intended to describe these and other connections between information theory and combinatorics and also formulate some open problems.

W. F. Smyth (McMaster): Periodicity in strings

Periodicity in strings can be generalized in various interesting ways, some of which have applications, to molecular biology, for example. In this talk I discuss some of these generalizations: covers, k-covers, weak repetitions, nontandem repetitions, approximate repetitions. Some algorithms that compute approximate periodicity are described, and some open problems are indicated.

Eric Sopena (Bordeaux): Acyclic Improper Colorings
(joint work with Pierre Boiron and Laurence Vignal)

Many variations and extensions of graph colorings have been considered in the literature (see [Jensen, T.R. and Toft, B.: *Graph Coloring Problems.* Wiley Interscience, 1995] for a general overview). In particular, *improper* colorings (sometimes called *genreralized*, *defective* or *relaxed* colorings) have been extensively studied. A coloring of a graph G is said to be improper whenever two adjacent vertices may be assigned the same color. In such colorings, the adjacency constraint is thus replaced by some constraint on the structure of *monochromatic subgraphs.* We will say that such an improper coloring is *acyclic* if for every two colors i and j, the subgraph induced by the edges whose endpoints are respectively i- and j-colored is acyclic. This notion generalizes the usual notion of acyclic coloring.

Our motivation when introducing such colorings was the study of homomorphisms of oriented graphs [Kostochka, A.V., Sopena, E. and Zhu, X.: Acyclic and oriented chromatic numbers of graphs. *J. Graph Theory*, to appear], [Nešetřil, J., Raspaud, A. and Sopena, E.: Colorings and girth of oriented planar graphs. *Discrete Math.*, to appear], [Raspaud, A. and Sopena, E.: Good and semi-strong colorings of oriented planar graphs. *Inform. Processing Letters* 51:171–174, 1994], [Sopena, E. The chromatic number of oriented graphs. *J. Graph Theory*, to appear]. We prove that the existence of some special acyclic improper colorings of a graph allows to derive an upper bound on its oriented chromatic number [Sopena, E. The chromatic number of oriented graphs. *J. Graph Theory*, to appear]. Acyclic improper coloring are also related, in a similar way, to homomorphisms of edge-colored graphs, recently introduced by Alon and Marshall [Alon, N.

and Marshall, T.H.: Homomorphisms of edge-coloured graphs and Coxeter groups. *Manuscript*, 1996].

We shall review some results on acyclic improper colorings of outerplanar graphs, planar graphs and graphs with bounded degree and mention some open questions.

Joel Spencer (New York): Sophisticated Probability

It wasn't too many decades ago that the probability needed for random structures and/or probabilistic methods was quite elementary: expectation, variance and the Chernoff bounds could carry one a long long way. No longer. We discuss some examples: Large deviation results using inequalities of Janson and Talagrand. Martingales in surprising forms. Entropy arguments. Asymptotic approximation of random processes by continuous time branching processes. Most recently, Fourier Analysis to characterize coarse threshold behavior.

Robin Thomas (Atlanta): Permanents, Pfaffian orientations, and even directed circuits
(joint work with N. Robertson and P.D. Seymour)

Given an n by n 0-1 matrix A, when can some of the 1's be changed to -1's in such a way that the permament of A equals the determinant of the modified matrix? When does a real n by n matrix A have the property that every real matrix B with the same sign pattern (that is, the corresponding entries either have the same sign, or are both zero) is non-singular? When is a hypergraph with n vertices and n hyperedges minimally non-bipartite? When does a bipartite graph have a "Pfaffian orientation"? Given a digraph, does it have a directed circuit of even length? Given a digraph, does it have a subdivision with no even directed circuit?

It is known that all the above problems are equivalent. We prove a structural characterization of the feasible instances, which implies a polynomial-time algorithm to solve all of the above problems. The structural characterization says, roughly speaking, that a bipartite graph has a Pfaffian orientation if and only if it can be obtained by piecing together (in a specified way) planar bipartite graphs and one sporadic non-planar bipartite graph.

The structural characterization was independently obtained by W. McCuaig.

W.T.Trotter (Tempe): Ramsey theory and sequences of random variables
(joint work with Peter Winkler)

We consider probability spaces which contain a family $\{E_A : A \subseteq \{1, 2, \ldots, n\}, |A| = k\}$ of events indexed by the k–element subsets of $\{1, 2, \ldots, n\}$. Our goal, albeit one which is not precisely defined, is to develop a "Ramsey theory" for probability spaces. We begin by studying notions of patterns and uniformity. This line of research was originally motivated by a question of Brightwell and Scheinerman involving the fractional dimension of partially ordered sets, and we set out to develop a theoretical base sufficient to settle their question. This approach led naturally to the following question:

A pair (A, B) of k–element subsets of $\{1, 2, \ldots, n\}$ is called a *shift pair* if the largest $k-1$ elements of A coincide with the smallest $k-1$ elements of B. For a shift pair (A, B), $\mathrm{Prob}[A\overline{B}]$ is the probability that event E_A is true and E_B is false. We investigate how large the minimum value of $\mathrm{Prob}[A\overline{B}]$, taken over all shift pairs, can be. As $n \to \infty$, this value converges to a number λ_k, with $\frac{1}{2} - \frac{1}{2k+2} \leq \lambda_k \leq \frac{1}{2} - \frac{1}{4k+2}$. We show that λ_k is a strictly increasing function of k, with $\lambda_1 = \frac{1}{4}$ and $\lambda_2 = \frac{1}{3}$.

For $k = 1$, our results have the following natural interpretation. If a fair coin is tossed repeatedly, and event E_i is true when the $i^{\underline{th}}$ toss is heads, then for all i and j with $i < j$, $\mathrm{Prob}[E_i\overline{E_j}] = \frac{1}{4}$. Furthermore, as we show in this paper, for any $\varepsilon > 0$, there is an n such that for any sequence $E_1, E_2, \ldots, E_n$ of events in an arbitrary probability space, there are indices $i < j$ with $\mathrm{Prob}[E_i\overline{E_j}] < \frac{1}{4} + \varepsilon$. The results and techniques we develop in this research, together with further applications of Ramsey theory, are then used to answer Brightwell and Scheinerman's question by showing that the supremum of fractional dimensions of interval orders is exactly 4.

Generalizing the $\frac{1}{4} + \varepsilon$ result to random variables $X_1, X_2, \ldots, X_n$ with values in an m–element set, we obtain a finite version of de

Finetti's theorem without the exchangeability hypothesis: for any fixed m, k and ε, every sufficiently long sequence of such random variables has a length-k subsequence at variation distance less than ε from an i.i.d. mix.

Pavel Valtr (Rutgers and Praha): Several recent results on geometric graphs

A *geometric graph* is a graph $G = (V, E)$ drawn in the plane so that the vertex set V consists of points in general position and the edge set E consists of some of the straight line segments between points of V. We will talk about recent extremal-type results on geometric graphs. E.g., we will discuss upper bounds on the maximum number of edges in a geometric graph on n vertices with no k pairwise disjoint (pairwise crossing) edges.